**PHYS 813: Statistical Mechanics and Thermodynamics**
**Exam I**

March 26, 2009, 75 minutes, closed book.

Please start each solution on a fresh sheet of paper, use only one side of the paper. Try to show how well you understand the problems. Always justify your reasoning. Assemble your solutions in the increasing numerical order. Derive all thermodynamic formulas beyond definitions of thermodynamic functions.

1. Consider a simple system with energy $E$ and volume $V$, containing $N$ particles.
   (a) Formulate (briefly) the main postulates of thermodynamics.
   (b) Formulate (briefly) the main postulates of statistical mechanics.
   (c) By considering two simple systems separated by a partition which transmits energy, relate the entropy $S$ to the number of microstates $\Omega$.
   (d) Using the relation from point (c), derive the postulates of thermodynamics from statistical mechanics (for $\partial S/\partial E$ give heuristic argument).

2. Consider a system of $N$ independent distinguishable particles, where $N$ is large. Each particle can be in two possible states with energies $\pm\epsilon$.
   (a) Assuming a microcanonical ensemble, find the number of possible states of the system for a given total energy $E$.
   (b) Find the entropy $S$ and the temperature $T$ of the system.
   (c) Assume that $E > 0$, i.e., the number of particles in the state $\epsilon$ is larger than at $-\epsilon$ (this can be achieved by laser pumping). What temperature range does this state correspond to? What is the condition that such temperatures are meaningful?
   (d) Draw $S/kN$ as a function of $E/\epsilon N$ and relate $T$ to various parts of this graph.

3. The material constants are often connected by rigorous relations. Find a relation connecting the isothermal compressibility $\kappa_T$, adiabatic compressibility $\kappa_S$, heat capacity at constant pressure $C_p$, and heat capacity at constant volume $C_V$ for a system with a fixed number of particles. The coefficients are defined as follows:

$$\kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T, \quad \kappa_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S, \quad C_p = T\left(\frac{\partial S}{\partial T}\right)_p, \quad C_V = T\left(\frac{\partial S}{\partial T}\right)_V.$$

   *Hint:* Consider $S$ and $V$ as functions of $T$ and $p$.