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with Bose-Einstein condensation**

J. BOSSY, J. OLLIVIER, H. SCHOBER and H. R. GLYDE

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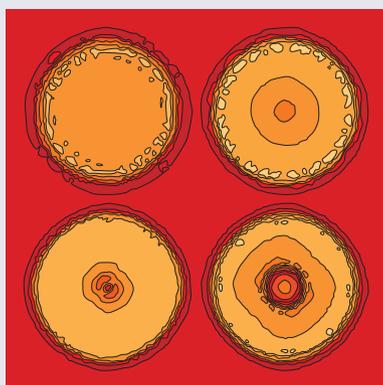
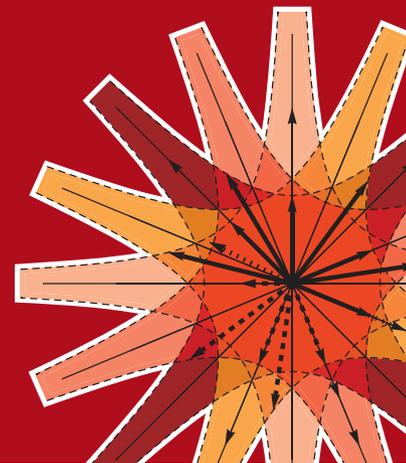
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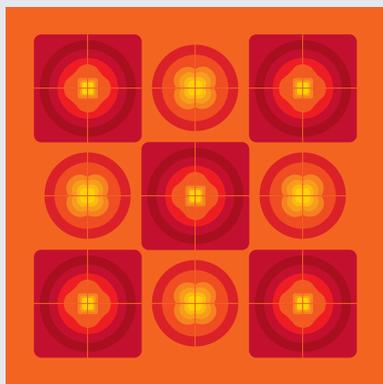
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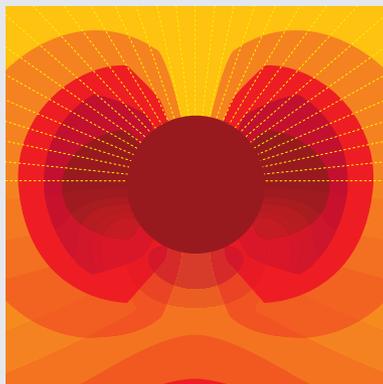
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**Image:** Ornamental multiplication of space-time figures of temperature transformation rules (adapted from T. S. Bíró and P. Ván 2010 *EPL* **89** 30001; artistic impression by Frédérique Swist).

# Phonon-roton modes in liquid $^4\text{He}$ coincide with Bose-Einstein condensation

J. BOSSY<sup>1</sup>, J. OLLIVIER<sup>2</sup>, H. SCHOBER<sup>2,3</sup> and H. R. GLYDE<sup>4</sup>

<sup>1</sup> *Institut Néel, CNRS-UJF - BP 166, F-38042 Grenoble Cedex 9, France, EU*

<sup>2</sup> *Institut Laue-Langevin - BP 156, F-38042 Grenoble, France, EU*

<sup>3</sup> *Université Joseph Fourier, UFR de Physique - F-38041 Grenoble Cedex 9, France, EU*

<sup>4</sup> *Department of Physics and Astronomy, University of Delaware - Newark, DE 19716-2593, USA*

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**Abstract** – We present neutron scattering measurements of the phonon-roton (P-R) and layer modes of liquid  $^4\text{He}$  confined in MCM-41 under pressure up to 38 bar. The data shows unambiguously that the P-R mode exists at low temperature only. As temperature is increased, there is a gradual transfer of intensity from the P-R mode to the normal liquid response, which lies at a lower energy at higher pressure. The transfer takes place with no observable mode broadening. The loss of P-R modes is identified with the loss of Bose-Einstein condensation (BEC). The mode giving rise to the specific heat,  $c_V$ , of liquid  $^4\text{He}$  in porous media (*e.g.*, gelsil) at higher temperature is the layer mode since the energy of the mode extracted from  $c_V$  and the layer mode energy are the same.

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There is a rich and long-standing debate on the nature of phonon-roton (P-R) modes in liquid  $^4\text{He}$ . The first picture is that they are collective density modes of a strongly interacting cold Bose liquid. This picture was initiated by Landau [1,2] and developed by Feynman [3] and others [4–7]. It forms the basis used today to calculate P-R mode energies with high accuracy [8–10]. In this picture gradual thermal broadening of modes with increasing temperature is expected [11–14]. The second is that they are density modes but Bose-Einstein condensation (BEC) plays a critical role in establishing them as sharply defined modes at low temperature, especially at higher wave vectors in the roton region  $Q \simeq 2 \text{ \AA}^{-1}$  and beyond. When there is BEC, the density and single-particle (SP) modes have the same energy [15–18]. There are therefore no independent, low-energy SP modes to which the P-R mode can decay. The P-R mode can decay only to itself, which becomes vanishingly small at low temperature. This picture originated with Bogoliubov [15] and was extended to Bose liquids by Gavoret and Nozières [16] and others [17,18]. The single-particle and density responses observed in the dynamic structure factor,  $S(Q, E)$ , are coupled via the condensate. In this picture we expect a difference [19] in  $S(Q, E)$  below and above  $T_{BEC}$ , the

temperature at which BEC vanishes and above which independent SP modes can exist.

We present new data on liquid  $^4\text{He}$  under pressure in MCM-41 which shows that the P-R mode at higher wave vectors exists at low temperature only. As the temperature is increased, there is a transfer of intensity from the P-R mode to new intensity at low energy. Above a specific temperature, denoted  $T_{BEC}$ , there is no longer an observable P-R mode.  $T_{BEC}$  is identified as the temperature at which BEC and as a result P-R modes no longer exist in the liquid. Above  $T_{BEC}$ , all the intensity is at low energy, which is interpreted as the response of the normal liquid (NL) where there is no BEC.

A simple transfer of intensity from the P-R mode to NL response with no mode broadening is observed at 34 bar because, under pressure, a) the P-R mode disappears at low temperature ( $T_{BEC} \simeq 1.5 \text{ K}$ ) before thermal broadening of the mode becomes significant and b) the “normal” liquid response lies at low energy ( $E \simeq 0$ ) that can be readily distinguished from the P-R mode. The P-R mode is therefore not simply a sharp density mode in a cold Bose liquid which broadens with increasing temperature. Rather it depends for its existence as a sharp mode on BEC. The transfer of intensity is not so

clear in bulk liquid  $^4\text{He}$  at lower pressure because there is mode broadening as well as intensity transfer and the P-R and NL intensities overlap in energy making it difficult to distinguish the NL response from the broadened P-R mode. In the bulk liquid, the loss of BEC coincides with the loss of superflow ( $T_{BEC} = T_\lambda$ ). Above  $T_{BEC}$ , the normal liquid response in bulk liquid  $^4\text{He}$  changes little with temperature [20,21].

Secondly we present measurements of the layer modes in liquid  $^4\text{He}$  under pressure in both MCM-41 and gelsil. This shows that the mode responsible for the specific heat,  $c_V$ , of liquid helium in porous media at higher pressure is the layer mode, as at SVP in Vycor [22–24]. Thus, the temperature at which  $c_V$  peaks is governed by the layer mode and the existence of layer modes has no established connection with BEC.

The measurements were performed on the IN5 and IN6 time-of-flight spectrometers at the Institut Laue-Langevin which are described fully at [www.ill.fr](http://www.ill.fr). Incident neutron wavelengths of  $5.0 \text{ \AA}$  and  $4.1 \text{ \AA}$ , with corresponding energy resolutions of  $0.1 \text{ meV}$  and  $0.17 \text{ meV}$ , were used, respectively. The MCM-41 of  $47 \pm 3 \text{ \AA}$  pore diameter was fabricated by the “Laboratoire de Matériaux Minéraux”, Mulhouse, France, and is fully described in ref. [25]. The gelsil of  $25 \text{ \AA}$  mean pore diameter was fabricated by 4F International and is fully described in ref. [26]. In these porous media,  $^4\text{He}$  remains liquid up to 38 bar where it solidifies to an amorphous solid [25]. The sample cells are fully described in refs. [25,26].

Figure 1 (top) shows the scattering intensity, proportional to the dynamic structure factor,  $S(Q, E)$ , from the helium in the sample cell at 25.3 bar and  $T = 0.4 \text{ K}$ . This is scattering from the liquid  $^4\text{He}$  in the MCM-41 pores and the bulk crystalline solid  $^4\text{He}$  between the grains of the MCM-41. The volume in the pores and between the grains is  $V_P = 0.83 \text{ cm}^3$  and  $V_{IG} = 1.87 \text{ cm}^3$ , respectively,  $V_P/V_{IG} = 0.44$ . The helium in the pores is liquid except the first 1–2 layers on the pore walls which is amorphous solid. Elastic scattering from the amorphous solid layers is seen in fig. 1 at  $E = 0$  and  $1.8 < Q < 2.5 \text{ \AA}^{-1}$ . The P-R mode of the liquid  $^4\text{He}$  in the MCM-41 pores is observed at energies,  $0.6 \lesssim E \lesssim 1.2 \text{ meV}$ . The P-R dispersion curve is only faintly visible in the phonon region ( $Q \simeq 0.5 \text{ \AA}^{-1}$ ), not visible in the “maxon” region ( $Q \simeq 1.1 \text{ \AA}^{-1}$ ) but is intense in the roton region,  $1.8 \leq Q \leq 2.3 \text{ \AA}^{-1}$  and  $E \simeq 0.6 \text{ meV}$ . The intensity at higher energy arises from phonons in the polycrystalline solid between the grains. The lower-energy optic phonons in the polycrystalline solid have an apparent “roton”-like appearance at an energy  $E \simeq 1.1 \text{ meV}$  on a time-of-flight instrument.

Figure 1 (bottom) shows the scattering intensity,  $S(Q, E)$ , at fixed wave vector  $Q = 2.10 \text{ \AA}^{-1}$  (the roton  $Q$ ) *vs.*  $E$  as a function of pressure at  $T = 0.4 \text{ K}$ . The large peak at  $E \simeq 0$  is the elastic scattering. The peak at  $E \simeq 0.6 \text{ meV}$  arises from the roton in the liquid confined in the MCM-41 pores and that at  $E = 1.1\text{--}1.4 \text{ meV}$  from the phonons in the bulk solid around the MCM-41.

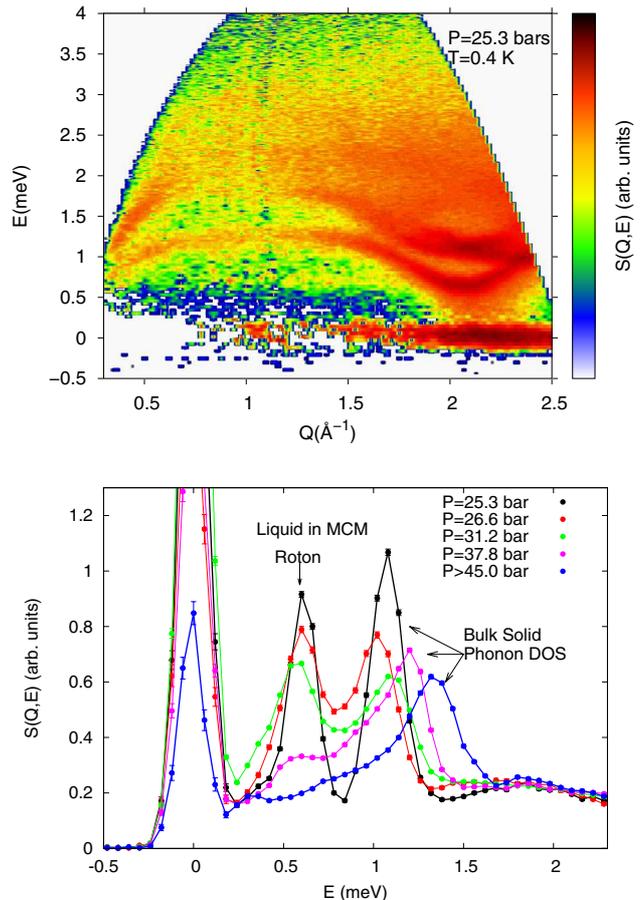


Fig. 1: (Colour on-line) Top: neutron scattering intensity,  $S(Q, E)$ , from liquid  $^4\text{He}$  in the MCM-41 pores and from the bulk solid helium between the grains of the MCM-41 as a function of momentum ( $Q$ ) and energy ( $E$ ) transfer. There is elastic scattering at  $E = 0$  and  $1.8 \leq Q \leq 2.5 \text{ \AA}^{-1}$ , inelastic scattering from the phonon-roton (P-R) mode of liquid  $^4\text{He}$  in MCM-41, especially in the roton region ( $Q \simeq 1.8\text{--}2.3 \text{ \AA}^{-1}$ ) centered at  $E \simeq 0.6 \text{ meV}$ , and inelastic scattering from phonons in bulk solid helium, particularly for  $1.8 \leq Q \leq 2.3 \text{ \AA}^{-1}$  and  $E \simeq 1.1 \text{ meV}$ . Bottom:  $S(Q, E)$  at constant  $Q = 2.1 \text{ \AA}^{-1}$  (the roton  $Q$ ) *vs.*  $E$  as a function of pressure at  $T = 0.4 \text{ K}$ . The intensity arising from the roton of liquid  $^4\text{He}$  in MCM-41 decreases with increasing pressure until at 37.8 bar there is little or no roton.

As pressure is increased, the intensity in the roton peak decreases until at  $p = 37.8 \text{ bar}$  there is essentially no roton peak. Immediately above 37.8 bar, the liquid solidifies [25]. At  $p > 45 \text{ bar}$ , where there is amorphous solid in the pores [25], no roton is observed. The roton energy decreases with increasing pressure, as in bulk liquid  $^4\text{He}$  [21,27]. In contrast the energy of the phonon density of states (DOS) increases with increasing pressure as expected in an ordinary solid. The key result is that the roton intensity decreases gradually with increasing pressure and there is no roton in the solid phase,  $p > 38 \text{ bar}$ .

Figure 2 shows the net scattering intensity,  $S(Q, E)$ , at  $Q = 2.1 \text{ \AA}^{-1}$  from the liquid in the MCM-41 pores at 34 bars and low temperature,  $T = 0.2 \text{ K}$ . The scattering from

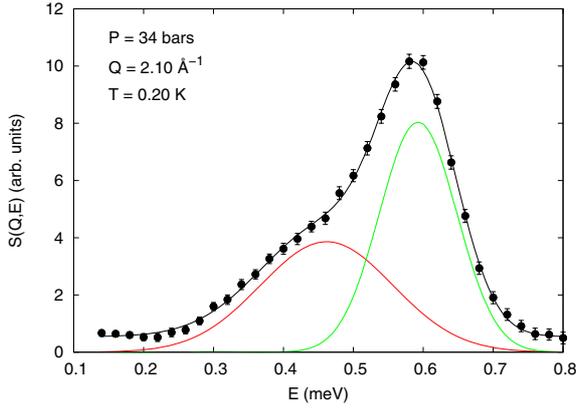


Fig. 2: (Colour on-line) Net  $S(Q, E)$  of liquid  $^4\text{He}$  confined in MCM-41 at 34 bar and  $T = 0.2\text{ K}$  (solid circles) at the roton wave vector,  $Q = 2.1\text{ \AA}^{-1}$ , vs.  $E$ . Also shown is a fit consisting of a phonon-roton mode,  $S_R(Q, E)$ , (green line centered at  $E \simeq 0.6\text{ meV}$ ) and a layer mode,  $S_L(Q, E)$ , (red line centered at  $E \simeq 0.45\text{ meV}$ ),  $S(Q, E) = S_R(Q, E) + S_L(Q, E)$ .

the amorphous solid layers on the pore walls and the solid between the MCM-41 grains has been subtracted. The net liquid  $S(Q, E)$  can be fitted as the sum of two modes, a roton mode,  $S_R(Q, E)$ , centered at  $E \simeq 0.6\text{ meV}$  plus a broad layer mode,  $S_L(Q, E)$ , peaked at  $E \simeq 0.45\text{ meV}$  as shown in fig. 2. More complex representations of  $S(Q, E)$  are possible. A P-R mode and a broad layer mode (which could be more than one layer mode [28]) are observed widely in porous media such as Vycor [29–31] and aerogel [32]. The layer mode propagates in the 2–4 layers of liquid closest to the pore walls. The P-R mode, the same mode as found in bulk liquid helium, propagates in these layers and in the liquid in the interior of the media. The association of layer modes with the liquid layers near the walls and P-R modes chiefly with the liquid in the interior is determined from measurements of mode intensities as a function of filling [32].

Specifically, we write

$$S(Q, E) = S_R(Q, E) + S_L(Q, E), \quad (1)$$

in which the roton mode  $S_R(Q, E)$  is represented by a damped harmonic oscillator (DHO) function,

$$S_R(Q, \omega) = (Z_Q/\pi)[n_B(\omega) + 1] \times \left[ \frac{\Gamma_Q}{(\omega - \omega_Q)^2 + \Gamma_Q^2} - \frac{\Gamma_Q}{(\omega + \omega_Q)^2 + \Gamma_Q^2} \right], \quad (2)$$

where  $n_B(\omega)$  is the Bose function, and  $S_L(Q, E)$  by a Gaussian function. A best fit to the data in fig. 2 gives a roton energy  $\omega_Q = 0.59 \pm 0.02\text{ meV}$  and width  $2\Gamma_Q = 0.071 \pm 0.004$ . Interestingly, the roton has a finite apparent intrinsic width in MCM-41 at 34 bar at  $T = 0.20\text{ K}$ .

The magnitude of  $S_L(Q, E)$  in eq. (1) depends on the volume of liquid in the liquid layers near the walls. From the fits to the data in fig. 2, we found integrated intensities

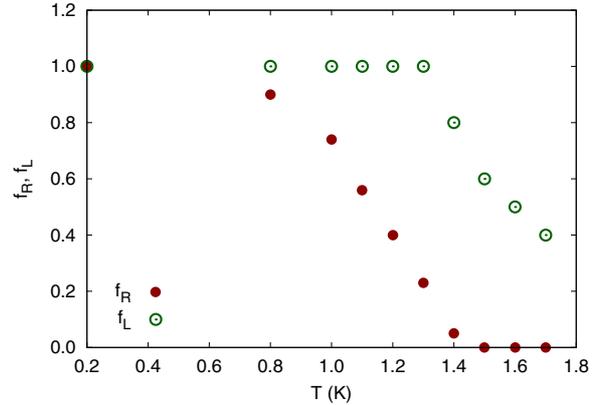
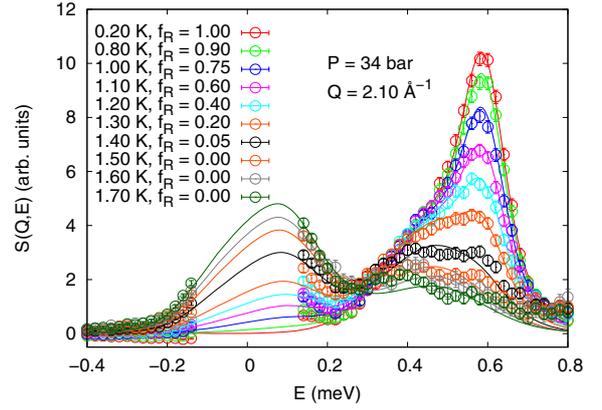


Fig. 3: (Colour on-line) Top: net  $S(Q, E)$  of liquid  $^4\text{He}$  confined in MCM-41 at  $Q = 2.1\text{ \AA}^{-1}$  vs. temperature at constant pressure  $p = 34\text{ bar}$  (open circles). Also shown are fits (lines) to the data using the model in eqs. (3) and (4) in which  $S_R(Q, E)$ ,  $S_N(Q, E)$  and  $S_L(Q, E)$  are all independent of  $T$ . The temperature dependence in eqs. (3) and (4) arises from allowing the weights  $f_R$  and  $f_L$  in the P-R mode and layer mode to decrease with  $T$ . At  $T = 0.2\text{ K}$ ,  $f_R = f_L = 1$ . Bottom: the weights,  $f_R$  and  $f_L$  in the P-R mode and layer mode, respectively, vs. temperature. Up to  $1.3\text{ K}$ ,  $f_L = 1$ . The weight in the P-R mode goes to zero at  $T = 1.5\text{ K}$ .

of  $S_R(Q) = 1.08$  and  $S_L(Q) = 0.91$ . This means that in MCM-41 the volume of liquid near the walls is large enough that  $S_L(Q)$  is comparable to  $S_R(Q)$ . In large pore media  $S_L(Q)$  is relatively much smaller [32]. Note that while there is some overlap, the roton and layer modes propagate in different regions of the liquid.

Figure 3 (top) shows the temperature dependence of the net  $S(Q, E)$  at 34 bar. In fig. 3, the intensity from the solid at  $T = 0.2\text{ K}$  that was subtracted was multiplied by the thermal factor  $[n_B(\omega) + 1]$  to allow for the temperature dependence of the background, a negligible effect for  $E > 0.15\text{ meV}$  up to  $1.7\text{ K}$ . As temperature is increased, the intensity in the P-R mode decreases. The intensity that was in the P-R mode at low temperature is transferred to new intensity at low energy centered near  $E \simeq 0$  at higher temperatures. At  $T = 1.5\text{ K}$  the P-R mode is no longer observed, as we show below. This is similar

to the transfer of intensity seen in the bulk liquid at lower pressure [20,33,34]. However, it is much clearer in the present data at 34 bars for two reasons. Firstly in fig. 3 (top), the transfer of intensity takes place with no observable thermal broadening of the P-R mode. This is because the roton has already a small intrinsic width,  $2\Gamma_Q = 0.07$  meV at low temperature ( $T = 0.20$  K), and the roton intensity vanishes at  $T = 1.5$  K where the thermal broadening of the roton mode is small [20] (*e.g.*, in bulk liquid  $^4\text{He}$  at 20 bars at  $T = 1.5$  K,  $2\Gamma_Q \simeq 0.08$  meV). Thus, it is clear that the temperature dependence is intensity loss without broadening. Secondly, as temperature increases, the intensity lost from the P-R mode appears at low energy near  $E \simeq 0$  well separated from the P-R mode. The layer mode survives to higher temperature, above 1.7 K, the maximum temperature investigated here.

As in earlier work [30,35–38], we interpret the temperature at which the roton disappears as  $T_{BEC}$ , the maximum temperature at which BEC exists in the liquid. Thus, at 34 bar  $T_{BEC} = 1.5$  K. At SVP, the P-R mode in porous media exists up to a temperature  $T_{BEC}$  which lies below but is close to  $T_\lambda$  in all media investigated [30,35,37,39,40]. At higher pressure (*e.g.*, 32–34 bar),  $T_{BEC}$  is reduced to about 1.5 K in all porous media investigated (Vycor, gelsils and MCM-41) [26,38,41]. While  $T_{BEC}$  is similar in all porous media, the superfluid transition temperature  $T_c$  is very sensitive to the pore geometry and size [24,42–44] and lies below  $T_{BEC}$ . For example,  $T_c$  goes to zero at 34 bar in 25 Å gelsil [43] where  $T_{BEC} \simeq 1.5$  K. The temperature range  $T_c < T < T_{BEC}$ , where there is BEC but no superflow, is interpreted as a temperature range of localized BEC [30,35–38,45] in which the BEC is spatially localized to islands (*e.g.*, 25 Å in diameter) separated by normal liquid.

The new feature of the present data in fig. 3 is that the intensity in the P-R mode decreases as temperature is increased without observable mode broadening and this intensity is transferred to new response centered near  $E \simeq 0$ . The intensity near  $E \simeq 0$  is interpreted as normal liquid response,  $S_N(Q, E)$ . Thus, the present data shows that the temperature dependence of  $S(Q, E)$  is not a broadening of the P-R mode. Rather,  $S(Q, E)$  below and above  $T_{BEC}$  is quite different and lies in a different energy range.

To clarify the temperature dependence of  $S(Q, E)$  in fig. 3 (top) we fitted a simple model to the data consisting of a P-R mode,  $S_R(Q, E)$ , a layer mode,  $S_L(Q, E)$ , and the normal liquid response,  $S_N(Q, E)$ . The  $S_R(Q, E)$ ,  $S_L(Q, E)$  and  $S_N(Q, E)$  are all assumed to be independent of temperature. In the model only the weights in the modes change with temperature. The weights  $f_R$  and  $f_L$ , in  $S_R(Q, E)$  and  $S_L(Q, E)$ , respectively, decrease with temperature and this weight is transferred to  $S_N(Q, E)$ . The weights  $f_R$  and  $f_L$  are obtained by fits to data as a function of temperature. Up to  $T = 1.3$  K we can get a good fit to the data with a model in which there is transfer from the roton to the normal liquid response only, *i.e.*, only

$f_R$  changes with temperature with the weight in the layer mode independent of  $T$ ,

$$S(Q, E) = [f_R(T)S_R(Q, E) + (1 - f_R(T))S_N(Q, E)] + S_L(Q, E). \quad (3)$$

The fits of eq. (3) to data up to  $T = 1.3$  K are shown as lines in fig. 3 (top) and the best-fit values of  $f_R$  are shown in fig. 3 (bottom).

Above 1.3 K, between 1.4 K and 1.7 K, the maximum temperature investigated, we obtain a better fit if the weight  $f_L$  in the layer mode also decreases with  $T$ . That is, above 1.3 K we expand the model to

$$S(Q, E) = [f_R(T)S_R(Q, E) + (1 - f_R(T))S_N(Q, E)] + [f_L(T)S_L(Q, E) + (1 - f_L(T))S_N(Q, E)], \quad (4)$$

in which there is in addition transfer of intensity from the layer mode to  $S_N(Q, E)$ . The best fit values of  $f_R$  and  $f_L$  are again shown in fig. 3 (bottom). We see that the weight in the P-R mode  $f_R$  decreases uniformly with increasing temperature and  $f_R = 0$  at  $T = 1.5$  K. The temperature dependence of  $f_R$  is similar to that of the condensate fraction [46],  $n_0(T)$ . The integrated intensity in the normal response is  $S_N(Q) = 1.10$  so that, with  $S_R(Q) = 1.08$  and  $S_L(Q) = 0.91$ , the total  $S(Q)$  of the model eq. (4) is approximately independent of temperature as observed in bulk liquid  $^4\text{He}$ . These fits confirm that there is a complete transfer of intensity from the P-R mode at low  $T$  to normal liquid response centered at  $E \simeq 0$  at  $T_{BEC} = 1.5$  K and that  $S(Q, E)$  is quite different above and below  $T_{BEC}$ .

To illustrate how BEC can play a role in establishing well defined P-R modes in  $S(Q, E)$  we turn to the field theory formulation [15–18] of  $S(Q, E)$  in which  $n_0$  appears explicitly. In this formulation both the single-particle Green function,  $G(Q, \omega)$ , and the density dynamical susceptibility,  $\chi(Q, \omega)$ , that we observe in  $S(Q, \omega)$ , are evaluated. Specifically,  $S(Q, \omega)$  is given by  $S(Q, \omega) = -(1/\pi)[n_B(\omega) + 1]\chi''(Q, \omega)$ , where  $\chi''$  is the imaginary part of  $\chi$ . When there is a condensate,  $\chi(Q, \omega)$  and  $G(Q, \omega)$  have a common denominator [17] and therefore have the same poles and mode energies. There is only a single joint density/single-particle mode. Thus there are no independent single-particle modes lying at low energy to which the density P-R mode can decay. As a result the P-R mode is uniquely sharply defined at low temperature (*e.g.*, compared to phonons in solid  $^4\text{He}$  or zero sound in normal liquid  $^3\text{He}$ ). We expect sharply defined modes below  $T_{BEC}$  where there is BEC but not above  $T_{BEC}$  where there will be independent single-particle modes.

Secondly, when there is a condensate, the single particle  $G(Q, \omega)$  appears in  $\chi(Q, \omega)$  with a weight proportional to  $n_0$ . This may be seen qualitatively from the intermediate DSF,

$$S(\mathbf{Q}, t) = \frac{1}{N} \langle \rho(\mathbf{Q}, t) \rho^\dagger(\mathbf{Q}, 0) \rangle,$$

where  $\rho^\dagger(\mathbf{Q}) = \sum_k a_{k+Q}^\dagger a_k$  is the density operator and  $a_k^\dagger$  is the single-particle creation operator. When there is a

condensate (macroscopic occupation of the  $k=0$  state), the  $a_0$  becomes a number,  $a_0 = \sqrt{N_0}$  and  $a_0^\dagger = \sqrt{(N_0 + 1)} \simeq \sqrt{N_0}$  where  $n_0 = N_0/N$ . Thus,  $\rho^\dagger(\mathbf{Q}, 0)$  separates as

$$\rho^\dagger(\mathbf{Q}) = \sqrt{N_0} A_Q^\dagger + \rho'^\dagger(\mathbf{Q}), \quad (5)$$

where  $A_Q^\dagger = a_Q^\dagger + a_{-Q}$  and  $\rho'^\dagger(\mathbf{Q})$  is the ‘‘regular’’ density operator that does not include the  $k=0$  state. In this case

$$\chi(Q, \omega) = n_0 G(Q, \omega) + \chi_{INT}(Q, \omega) + \chi'_R(Q, \omega) \quad (6)$$

in which first term is proportional to  $n_0$  and  $\chi_{INT}(Q, \omega)$  is a cross term proportional to  $\sqrt{n_0}$ .

When the P-R mode energy is high and near the edge of the two P-R band, the weight in the P-R mode is small. This is the case at higher wave vectors beyond the roton,  $Q \geq 2.5 \text{ \AA}^{-1}$ . At wave vectors beyond the roton, eq. (6) for  $\chi(Q, \omega)$  describes the temperature dependence of  $S(Q, E)$  well [47]. The sharp mode is represented by the  $G(Q, \omega)$  term with weight in  $S(Q, \omega)$  proportional to the condensate fraction,  $n_0(T)$ , which tracks the observed weight in the P-R mode well. The sharp P-R mode is no longer observed [48] above  $T_{BEC}$  where  $n_0 = 0$ .

In the present data, the weight of the roton in  $S(Q, E)$  appears to be proportional to  $n_0(T)$ . However, at the roton wave vector, as in bulk liquid helium, almost all of the weight in  $S(Q, E)$  lies in the roton mode. At the same time,  $n_0$  is small, especially at high pressure (less than 3%). In this case the simple model equation (6) would not reproduce the present data. Somehow there must be a transfer of intensity into the P-R mode from the remainder of  $\chi(Q, \omega)$ . Thus, at this stage, the field-theoretic formulation of  $S(Q, E)$  provides a framework for understanding why the intensity in the roton can be proportional to  $n_0$  and why the response is very different below and above  $T_{BEC}$ . However, an explicit model that reproduces the data of fig. 3 remains to be constructed.

Finally, as noted above, liquid  $^4\text{He}$  in porous media and in films on surfaces supports layer modes that propagate in the liquid layers adjacent to the media walls. The layer mode is observed at wave vectors  $1.7 < Q < 2.3 \text{ \AA}^{-1}$  and has a roton-like energy minimum,  $\Delta_L$ , at an energy approximately 0.15 meV below the P-R roton energy,  $\Delta$ . Figure 4 shows the roton energy,  $\Delta$ , in MCM-41 and the layer mode energy,  $\Delta_L$ , in MCM-41 and gelsil observed here *vs.* pressure. The  $\Delta$  in MCM-41 and in gelsil [38,41] are consistent with bulk roton energies [21,49]. Indeed, in all porous media investigated to date, the P-R mode energies are found to be the same as those in the bulk within precision at SVP [32] and at higher pressure [38,41]. As shown initially in Vycor, it is the excitation of the lower-energy layer mode that is responsible for the specific heat of liquid  $^4\text{He}$  at high temperature in porous media [24]. For example, in early measurements [22] at SVP in Vycor,  $c_V$  was identified as arising from excitation of a layer mode. The layer mode energy extracted from  $c_V$  ( $\Delta_L = 0.53 \pm 0.02 \text{ meV}$ ) agrees with the layer

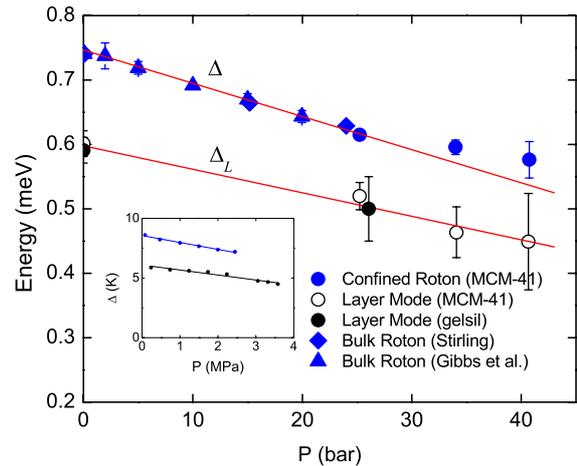


Fig. 4: (Colour on-line) The roton and layer mode energies *vs.* pressure. The blue symbols are the roton energies ( $\Delta$ ); triangles and diamonds in bulk liquid  $^4\text{He}$  at  $p \leq 24$  bar (from Gibbs *et al.* [21] and Stirling [49]) and solid circles in MCM-41 ( $25 < p < 40$  bar). The roton energy in all porous media investigated is the same as in the bulk [32,39]. The solid and open black circles are the layer mode energies ( $\Delta_L$ ) in gelsil and MCM-41, respectively. In the inset, the upper line is the bulk roton energy and the lower line the energy of the mode extracted by Yamamoto [45] from the  $c_V$  of liquid  $^4\text{He}$  in gelsil, which agrees with the layer mode energy  $\Delta_L$  that we observe here (inset from Yamamoto *et al.* [45]). (1 meV = 11.6 K.)

mode energy subsequently observed in Vycor [29,30] ( $\Delta_L = 0.55 \pm 0.01 \text{ meV}$ ).

The inset in fig. 4 shows both the bulk roton energy  $\Delta$  and the energy of the mode extracted from  $c_V$  in gelsil [45] which lies below  $\Delta$ . The energy of the mode extracted [45] from  $c_V$  agrees well with the layer mode energy,  $\Delta_L$  observed here, as was the case in Vycor at SVP. For this reason we believe, it is excitation of the layer mode and not the P-R mode that gives rise to  $c_V$  at high temperature in gelsil. If this is the case, then the temperature at which  $c_V$  peaks is determined by the layer mode energy and is not related to the P-R mode. The temperature at which  $c_V$  peaks has been used to identify  $T_{BEC}$ . However, there is no demonstrated connection between the existence of layer modes and BEC. For this reason, the  $c_V$  in porous media has, at this time, no demonstrated connection with BEC or  $T_{BEC}$ .

In summary, the present data shows that the P-R mode of liquid  $^4\text{He}$  at higher  $Q$  values exists only at low temperature. As temperature is increased, the weight in the P-R mode decreases and this weight is transferred from the P-R mode to new response at low energy, interpreted as normal liquid (NL) response. The transfer of intensity takes place with no observable mode broadening because the temperature is low. The normal liquid response lies at low energy well below the P-R mode. The temperature at which the P-R mode vanishes is identified as  $T_{BEC}$ , the temperature at which the liquid no longer supports localized patches

of BEC and becomes a fully normal liquid. An independent measurement of the condensate fraction,  $n_0(T)$  in MCM-41 to confirm  $T_{BEC}$  would be most interesting but is difficult. The temperature dependence of the weight in the below  $T_{BEC}$  is similar to that of the condensate fraction  $n_0(T)$  observed in bulk  $^4\text{He}$  and a connection between  $n_0$  and this weight is made.

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