PHYS 825: PROBLEM Set I

1. Review the Hydrogen atom.

   (a) Write out the electron wave function for the ground state.

   (b) Write out the expression for the energy levels.

   (c) Write out the electron wave functions $\psi_{nlm}(\lambda, \theta, \varphi)$ in terms of $R_{nl}(r)$, $P_l^m(\theta)$ and $e^{im\varphi}$. Explain n, l and m.

2. Do problem 1, chapter 19 of Ashcroft & Mermin.

3. State Blochs theorem for electron Wave functions in a periodic potential. Write Bloch's equation in its two forms. Explain why it holds and refer to a text and page in the text where a proof of Bloch's theorem can be found.

4. Show that the Fourier transform (FT) of the Coulomb potential $\frac{1}{r}$ is $\frac{4\pi}{Q^2}$. That is, if $\upsilon(Q) = \int d\mathbf{r}\ e^{-i\mathbf{Q}\cdot\mathbf{r}}\upsilon(r)$ is FT of the potential $\upsilon(r)$, then for $\upsilon(r) = \frac{1}{r}$, show that $\upsilon(Q) = \frac{4\pi}{Q^2}$.

   (Hint: multiply $\upsilon(r) = \frac{1}{r}$ by $e^{-\lambda r}$) to do the integral and then let $\lambda \longrightarrow 0$).

5. Show that

$$< e^{i\mathbf{Q}\cdot\mathbf{r}} > \equiv (\frac{\eta}{\pi})^{\frac{3}{2}} \int d\mathbf{u}\ e^{-\eta u^2} e^{i\mathbf{Q}\cdot\mathbf{r}} \tag{1}$$

   where $\mathbf{r} = \mathbf{R} + \mathbf{u}$ and $\mathbf{R}$ is a constant vector and η is a constant, is

$$< e^{i\mathbf{Q}\cdot\mathbf{r}} > = e^{\frac{-Q^2}{4\eta}} e^{i\mathbf{Q}\cdot\mathbf{R}} \tag{2}$$

6. Show that

$$\begin{aligned} < \frac{1}{r} > &\equiv (\frac{\eta}{\pi})^{\frac{3}{2}} \int d\mathbf{u}\ e^{-\eta u^2} \frac{1}{r} \\ &= (\frac{\eta}{\pi})^{\frac{3}{2}} \int d\mathbf{r}\ e^{-\eta(\mathbf{r}-\mathbf{R})^2} \frac{1}{r} \\ &= \frac{1}{R}\ erf(\sqrt{\eta}R) \end{aligned}$$

   where "erf" is the error function.