

# What is Quantum Transport?

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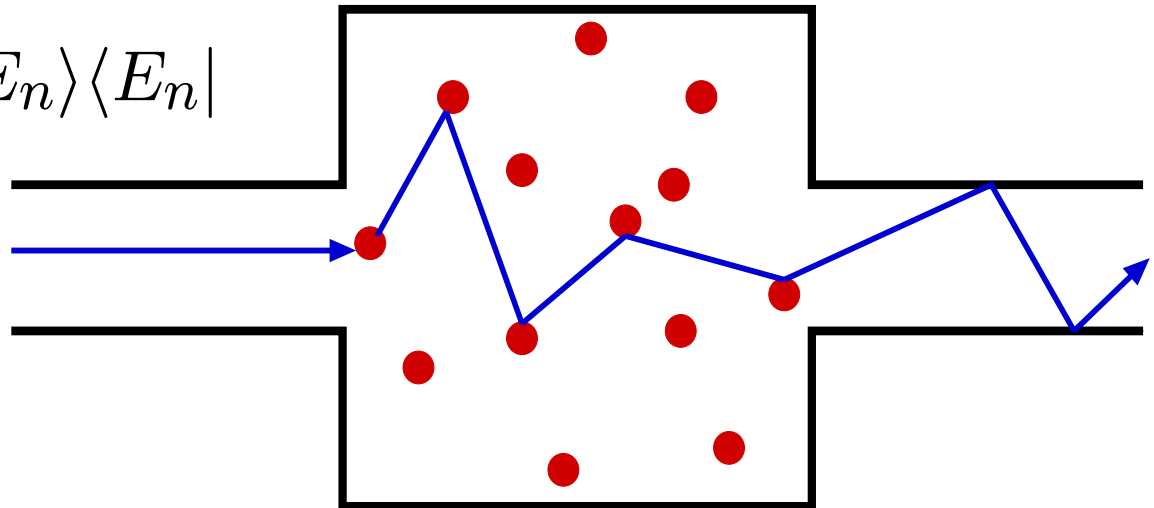


# Semiclassical Transport (is boring!)

- Bloch-Boltzmann electron in decent macroscopic solid:

$$\hat{\rho} = |a|^2 |\Psi_1\rangle \langle \Psi_1| + |b|^2 |\Psi_2\rangle \langle \Psi_2|$$

$$\hat{\rho} = \frac{1}{Z} \sum_n e^{-\beta E_n} |E_n\rangle \langle E_n|$$



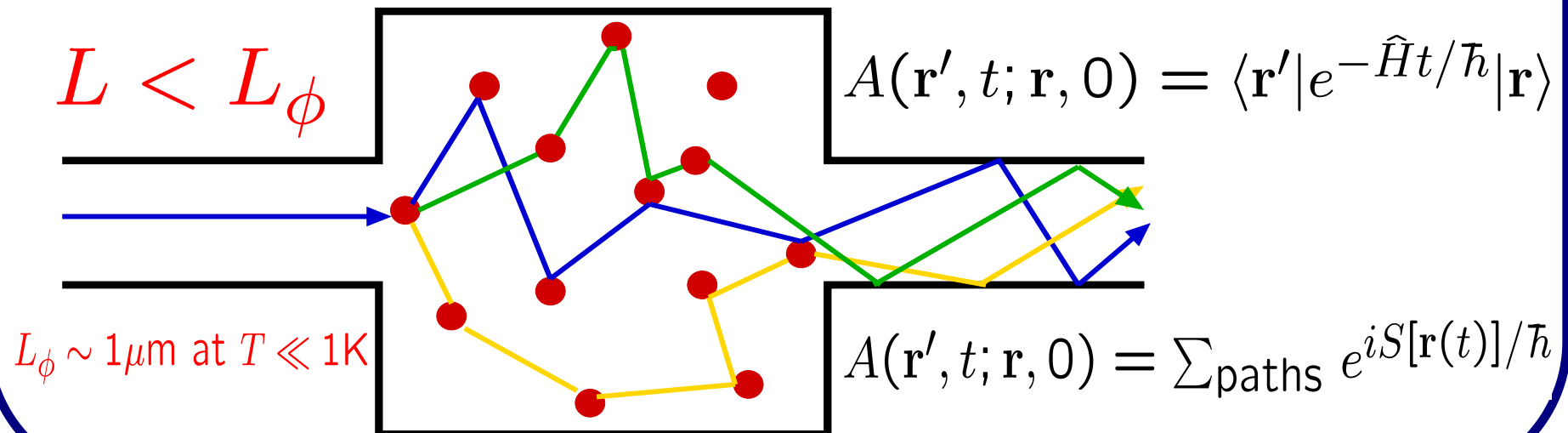
- Use **Semiclassical Boltzmann equation**: **Electron as Newtonian pinball**, but having effective mass determined by the band structure and interactions and exhibiting **random quantum scattering**.

# Quantum Transport

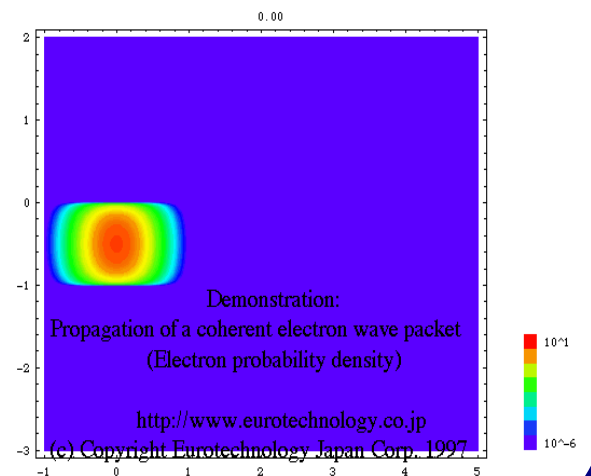
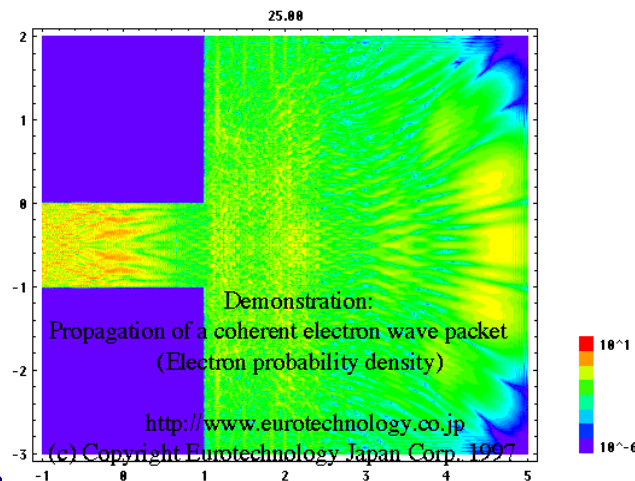
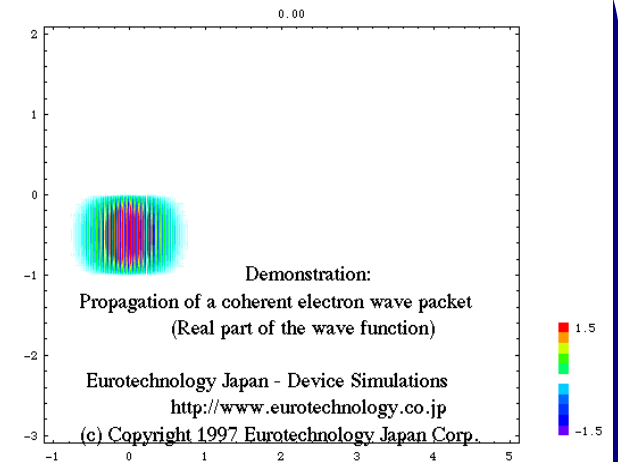
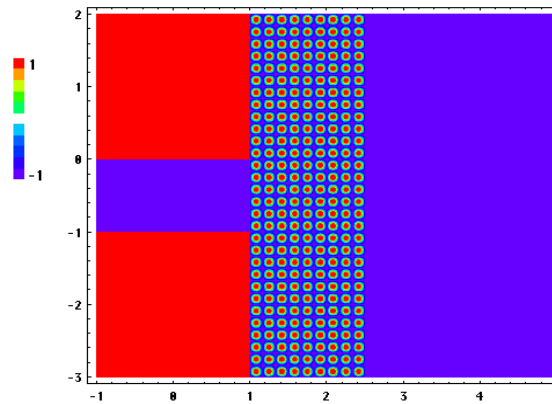
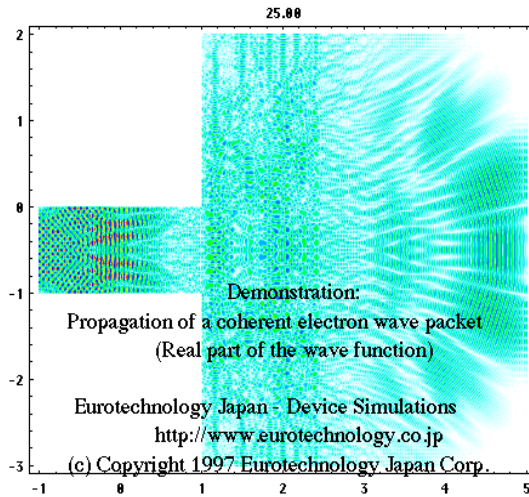
- An electron in **mesoscopic** solids (“**giant molecules**”) is in a pure state:

$$|\Psi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle$$

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = |a|^2|\Psi_1\rangle\langle\Psi_1| + |b|^2|\Psi_2\rangle\langle\Psi_2| + ab^*|\Psi_1\rangle\langle\Psi_2| + a^*b|\Psi_2\rangle\langle\Psi_1|$$



# Visual Quantum Transport: Electron Wave Function



# Four Nobel Truths about Quantum States

E. Schrödinger, Naturwissenschaften **23**, 807, 823, 844 (1935).

1. Superpositions:  $|\Psi\rangle = \cos(\frac{\theta}{2})e^{-i\phi/2}|\uparrow\rangle + \sin(\frac{\theta}{2})e^{i\phi/2}|\downarrow\rangle$

2. Interference:  $|\Psi\rangle = \cos(\frac{\theta}{2})|\uparrow\rangle + \sin(\frac{\theta}{2})e^{i\phi}|\downarrow\rangle$

3. Entanglement:  $|\text{GHZ}^\pm\rangle = \frac{|\uparrow\rangle\otimes|\uparrow\rangle\otimes|\uparrow\rangle \pm |\downarrow\rangle\otimes|\downarrow\rangle\otimes|\downarrow\rangle}{\sqrt{2}}$

$$|\text{EPR}^\pm\rangle = \frac{|\uparrow\rangle\otimes|\downarrow\rangle \pm |\downarrow\rangle\otimes|\uparrow\rangle}{\sqrt{2}}$$

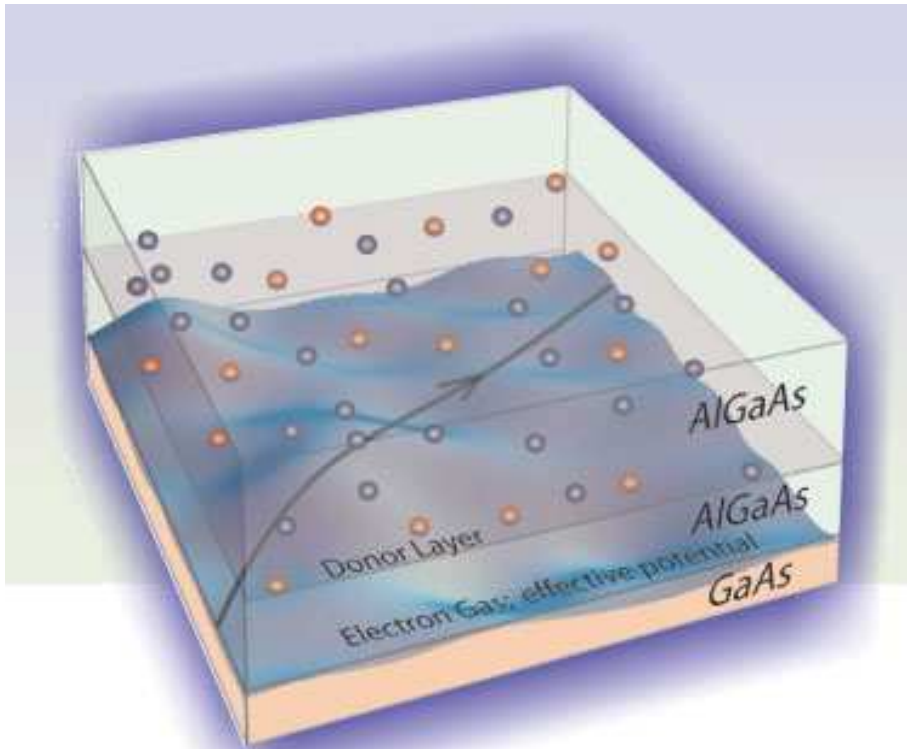
$$|\text{Bell}^\pm\rangle = \frac{|\uparrow\rangle\otimes|\uparrow\rangle \pm |\downarrow\rangle\otimes|\downarrow\rangle}{\sqrt{2}}$$

Such as the many-body states in:  
superconductors, strongly  
correlated systems, quantum critical  
systems, quantum computers, ...

4. Nonclonability and Uncertainty → No Quantum Copier Machine!

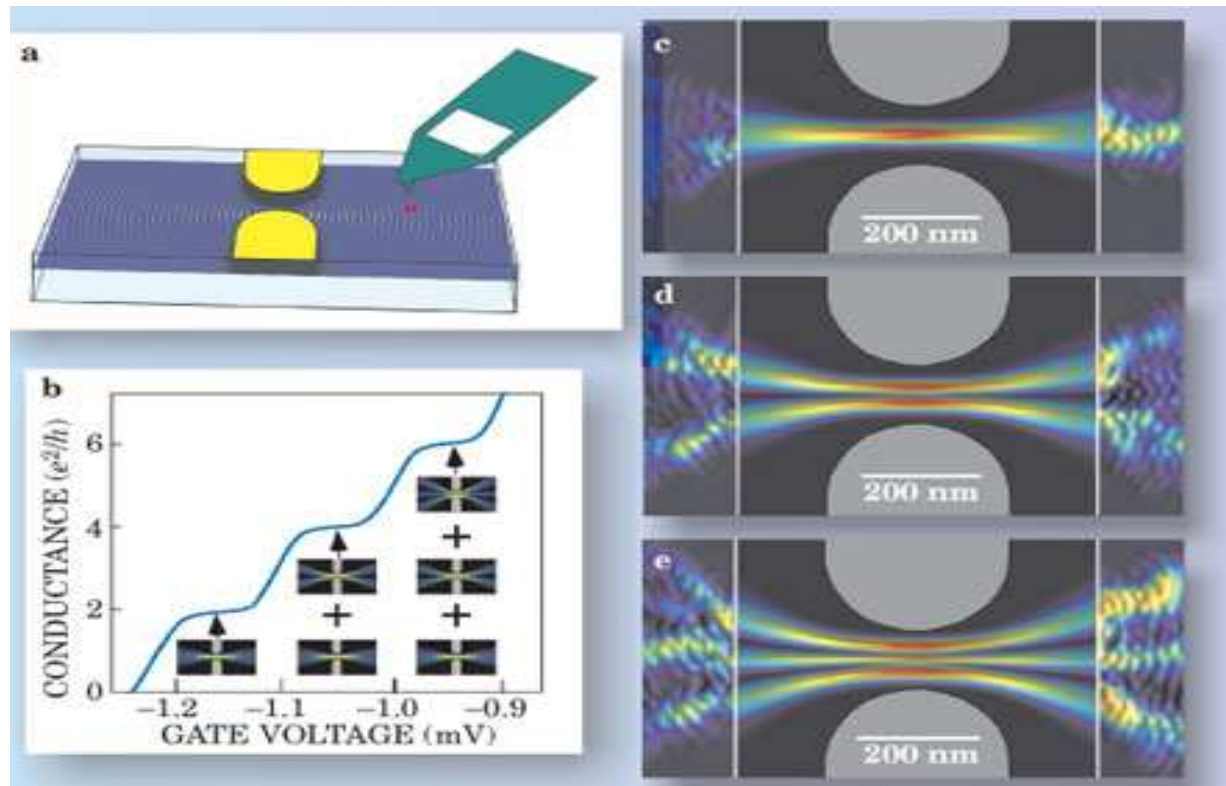
$$U_{\text{QCM}} : |\Psi\rangle_{\text{orig}} \otimes |\phi_0\rangle \mapsto |\Psi\rangle_{\text{orig}} \otimes |\Psi\rangle_{\text{orig}}$$

# Semiconductor Heterostructures



A two-dimensional electron gas formed at the interface between gallium arsenide and aluminum gallium arsenide in a **semiconductor heterostructure**. The AlGaAs layer (green) contains a layer (purple) of silicon donor atoms (dark blue). Electrons from the donor layer fall into the GaAs layer (pink) to form a 2DEG (blue) at the interface. The ionized Si donors (red) create a potential landscape for the electron gas; the resulting small-angle scattering smoothly bends electron trajectories, as shown.

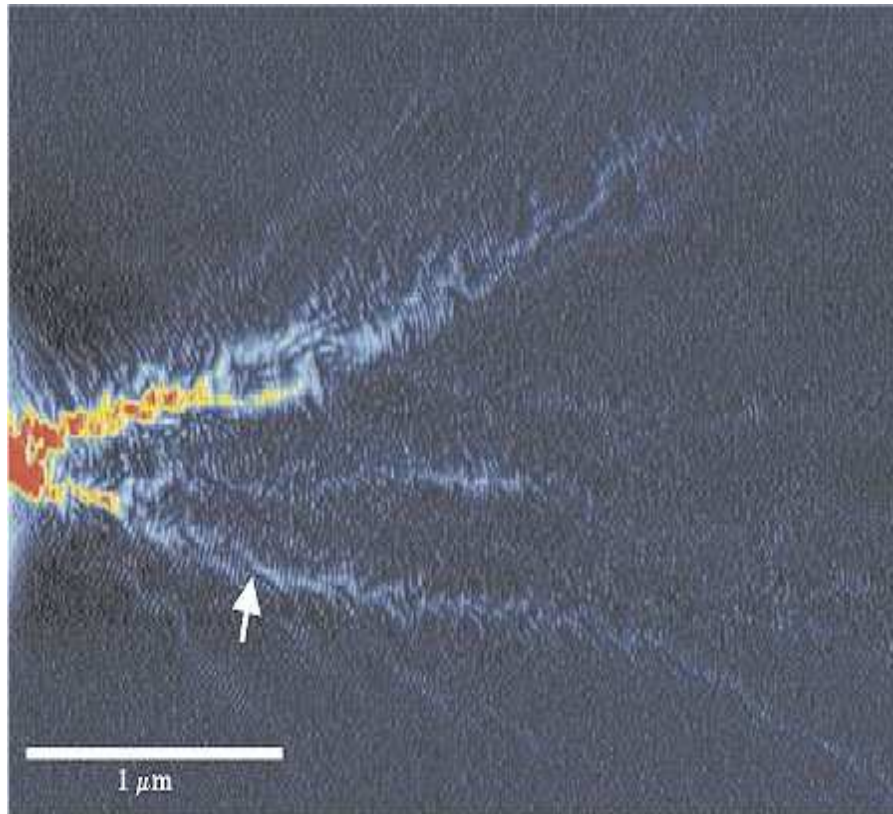
# Quantum Point Contact



$T = 1.7 \text{ K}$

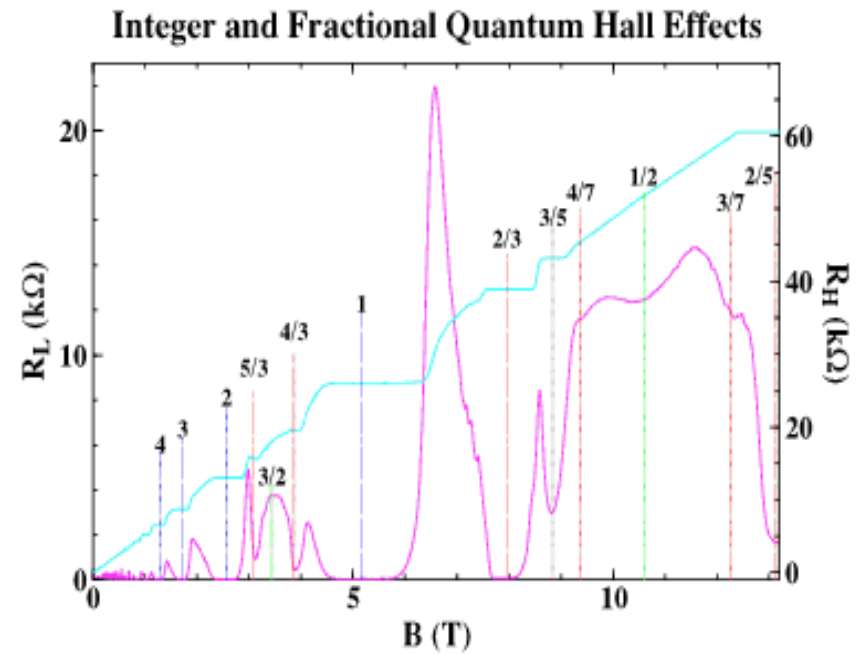
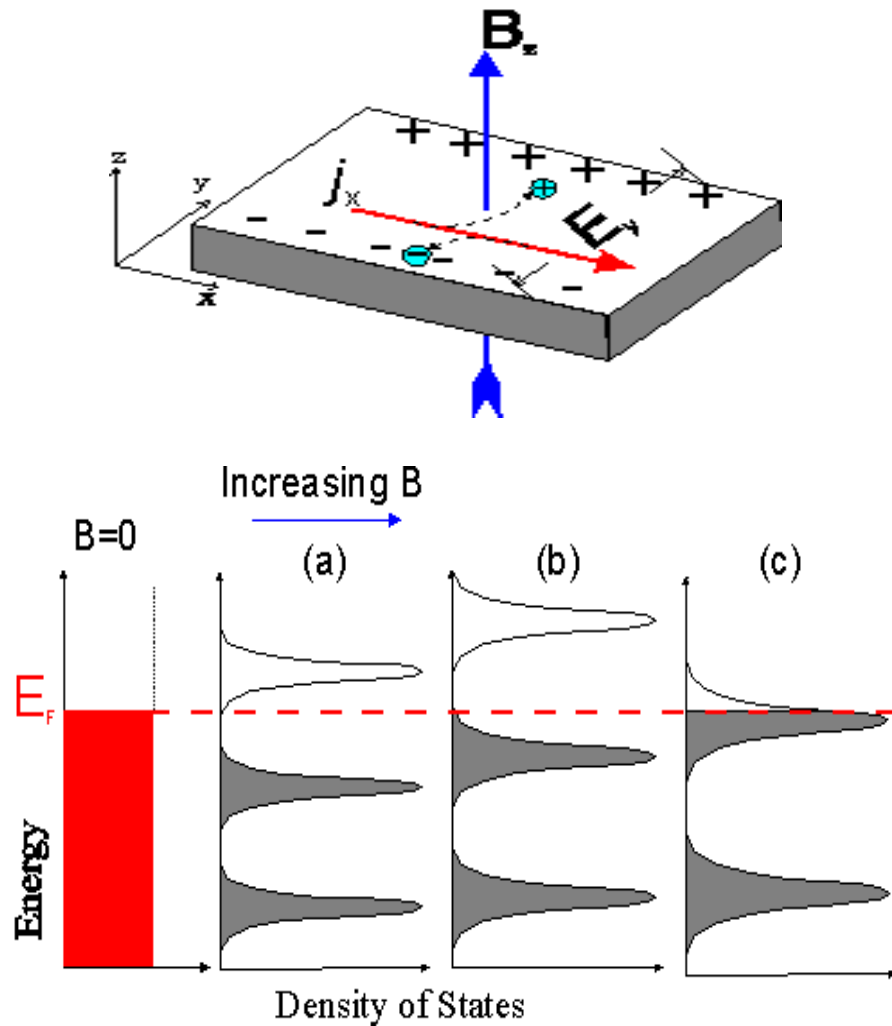
$$G = G_Q \text{Tr} t t^\dagger = G_Q \sum_{n=1}^M T_n, \quad G_Q = \frac{2e^2}{h}$$

# Imaging Quantum Point Contact Flow

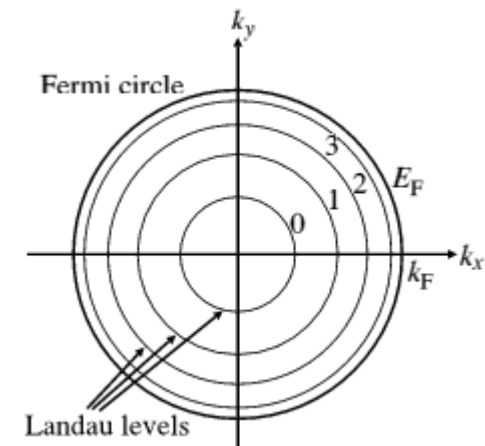
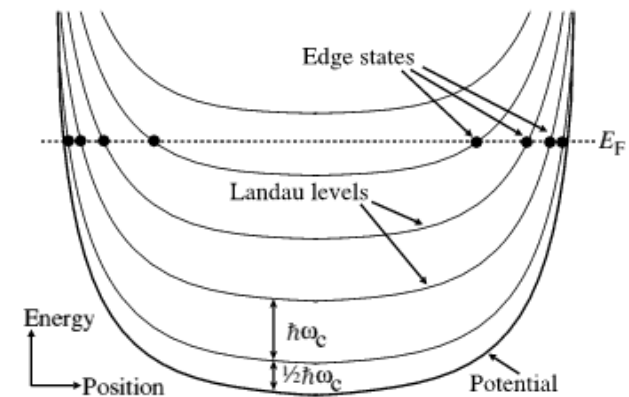
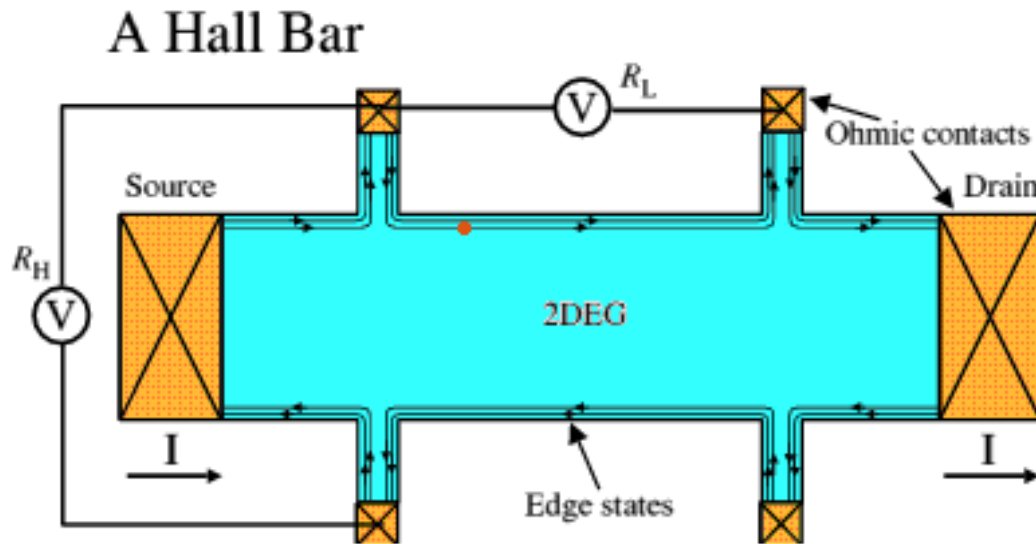


Electron flow through a two-dimensional electron gas from a quantum point contact on the first conductance step. The image shows surprisingly narrow branches that are produced by small-angle scattering from charged donor atoms in the donor layer. The interference fringes, demonstrating quantum mechanical coherence, extend throughout the image. The arrow points to a cusp produced by the focusing effect of a nearby impurity atom [M. A. Topinka *et al.*, *Nature* **410**, 183 (2001)].

# Quantum Hall Effect in 2DEG



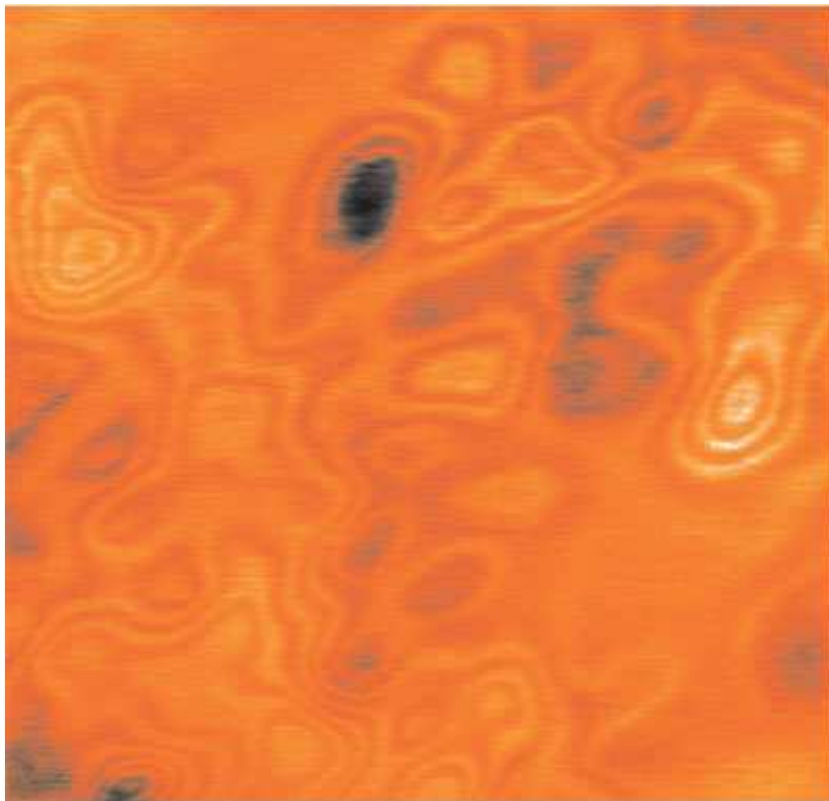
# Macroscopic vs. Mesoscopic Quantum Hall Transport



$$I_p = \sum_q [G_{q \leftarrow p} V_p - G_{p \leftarrow q} V_q]$$

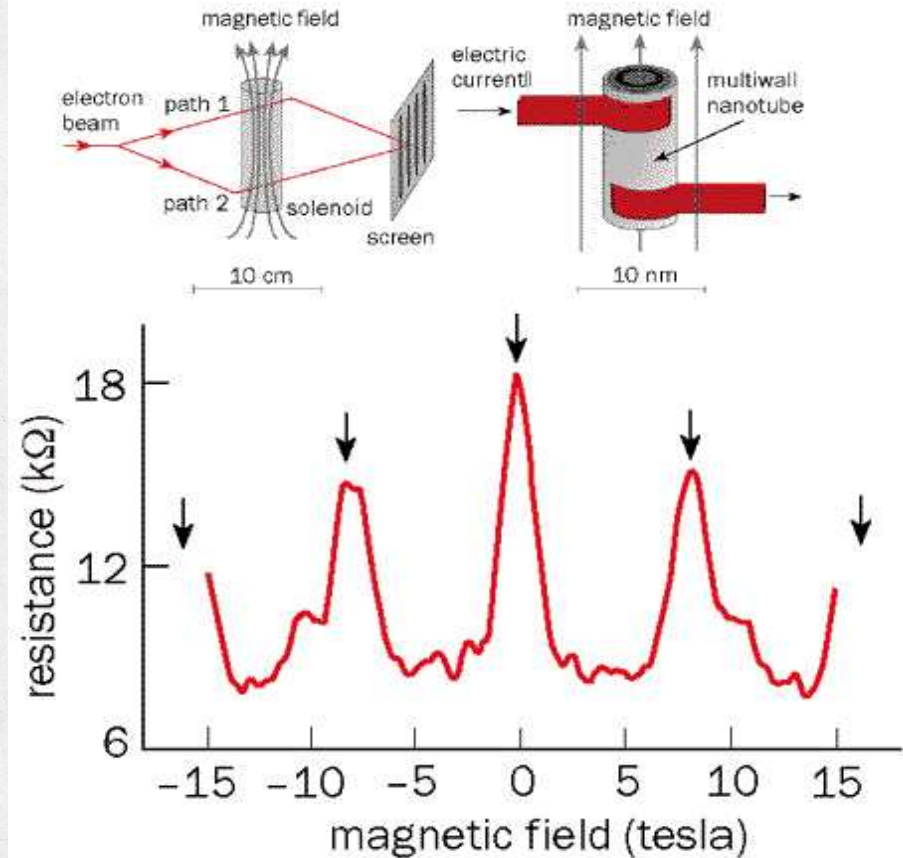
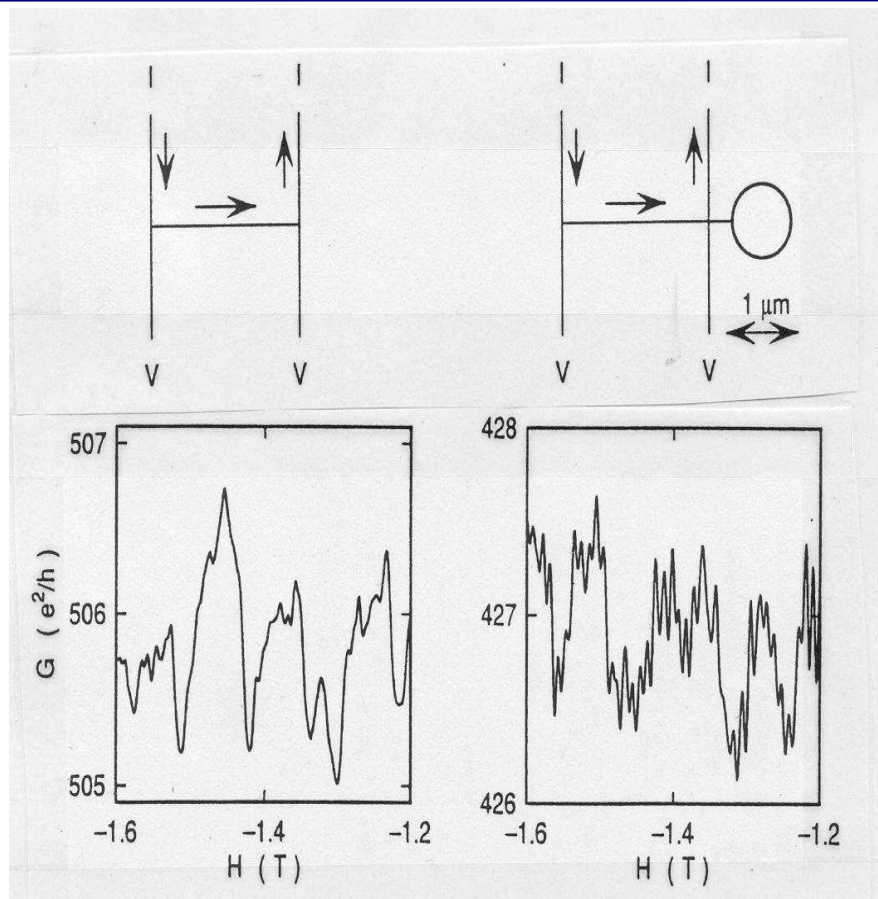
$$I_p = \sum_q G_{pq} (V_p - V_q)$$

# STM Imaging of Quantum Hall Liquid



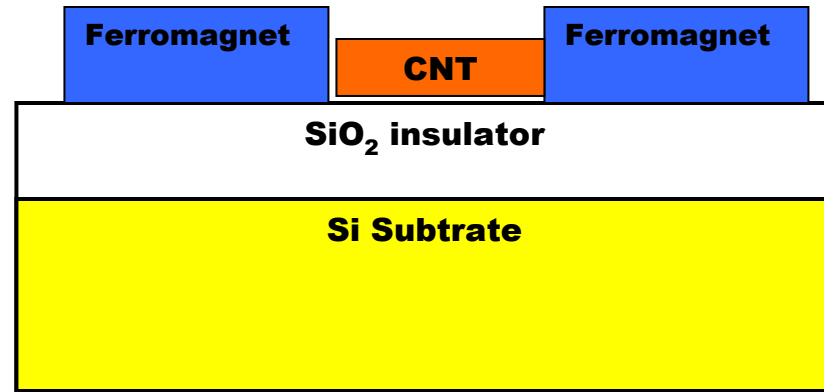
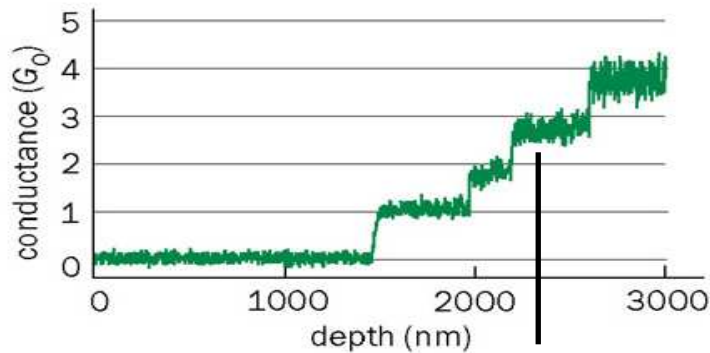
Subsurface charge accumulation in a two-dimensional electron gas maps the electrostatic potential experienced by the 2DEG in the quantum Hall regime with a filling factor  $\nu = 1$ . A positive voltage on the tip of a scanning tunneling microscope above the sample pulls in electrons to create a few-electron bubble in the 2DEG. The closed contours in this  $2.5 \times 2.5\text{-}\mu\text{m}^2$  image are caused by the quantization of electronic charge inside the bubble: The contours, which arise as individual electrons move in and out of the bubble, surround high and low regions of the random electrostatic potential.

# Encounters with Non-Locality: Aharonov-Bohm Effects

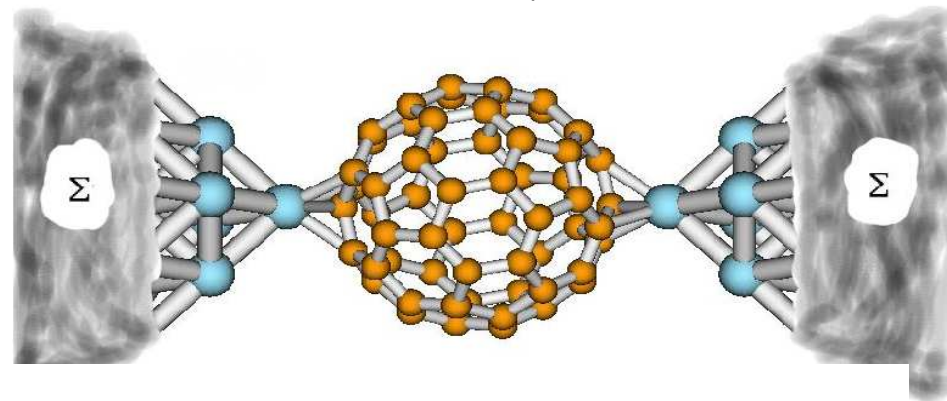


□ **Quantum Transport in Mesoscopic Systems:** Sample-specific conductance replaces traditional material-specific conductivity.

# Molecular Electronics: Quantum Transport at Room Temperature

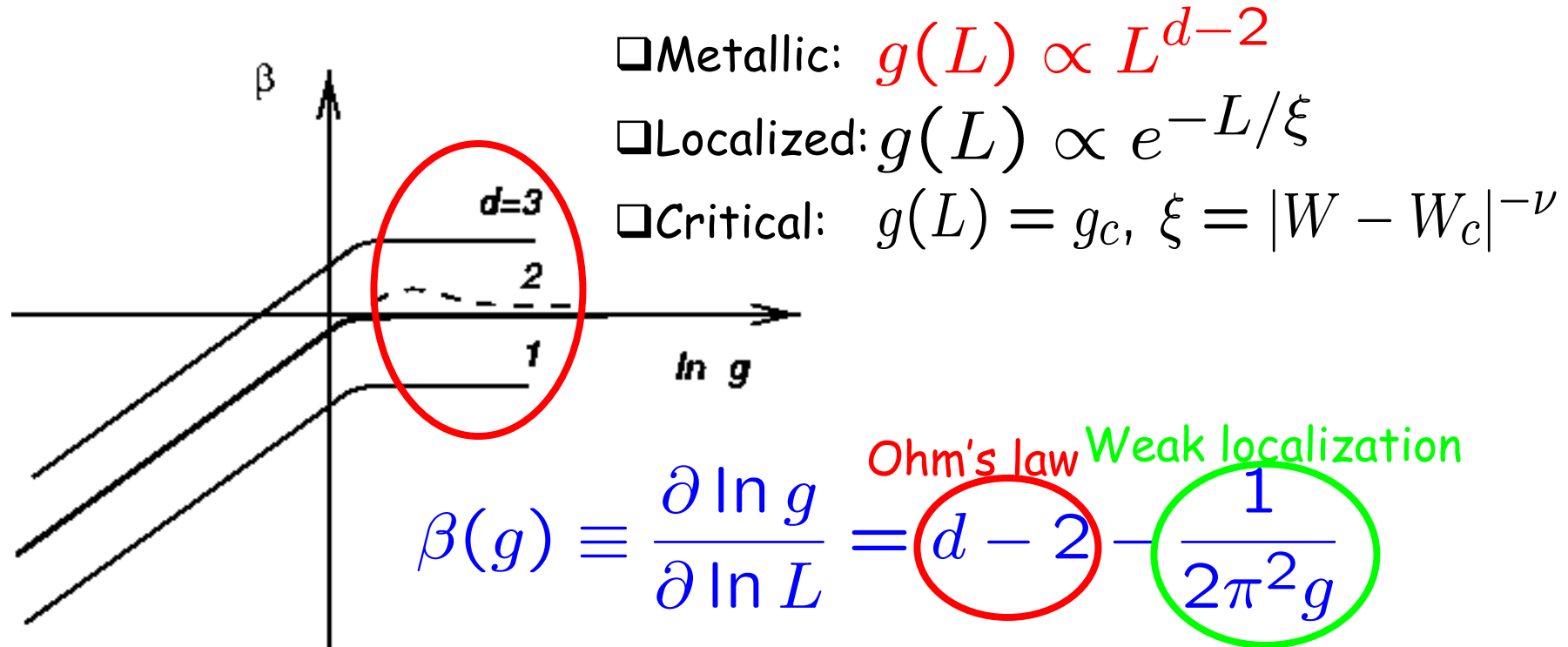


CNT Spintronics



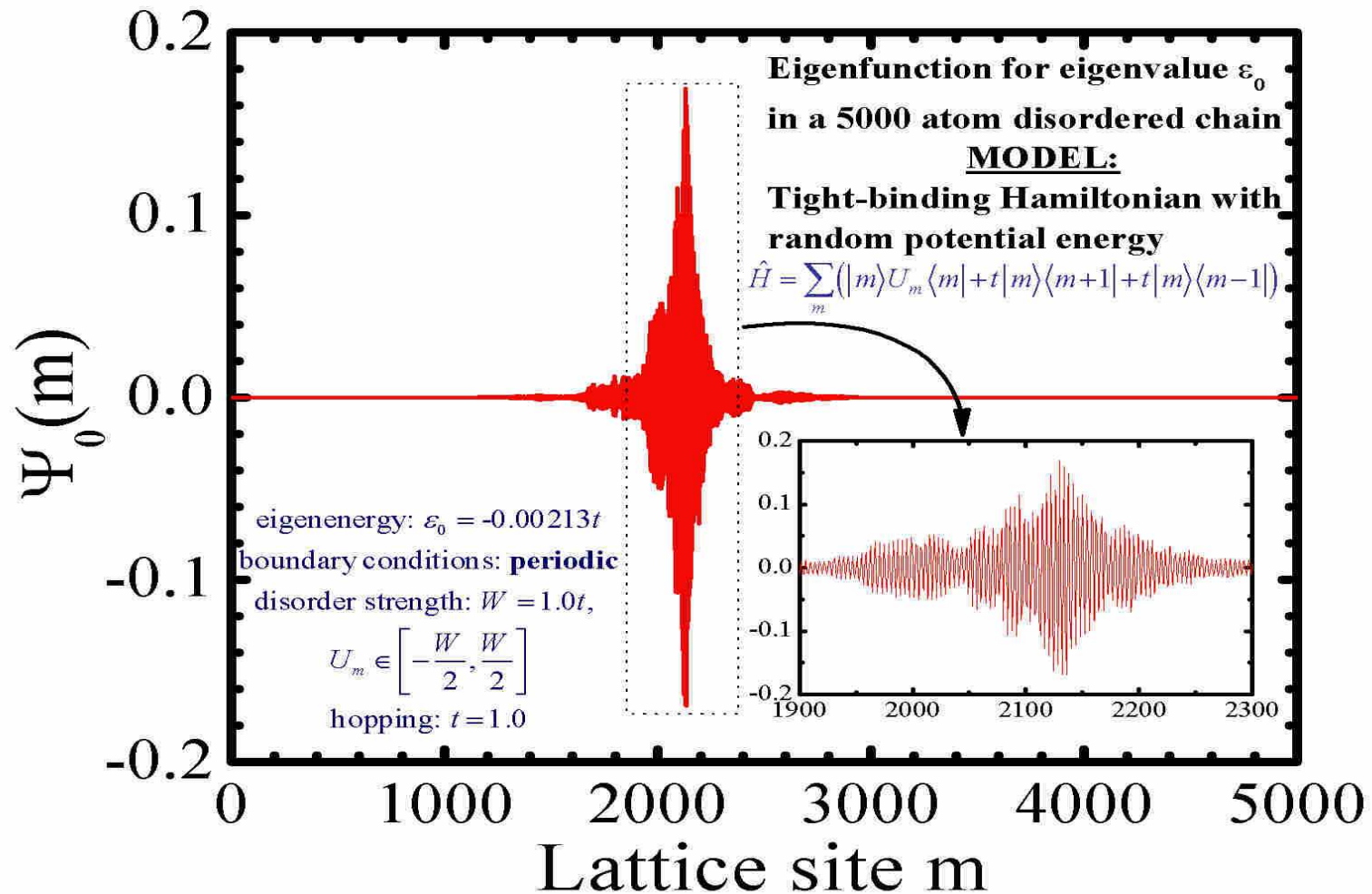
$$I(V) = \frac{2e}{h} \int dE [f_L(E) - f_R(E)] T(E, V)$$

# Quantum Transport History: Anderson Localization

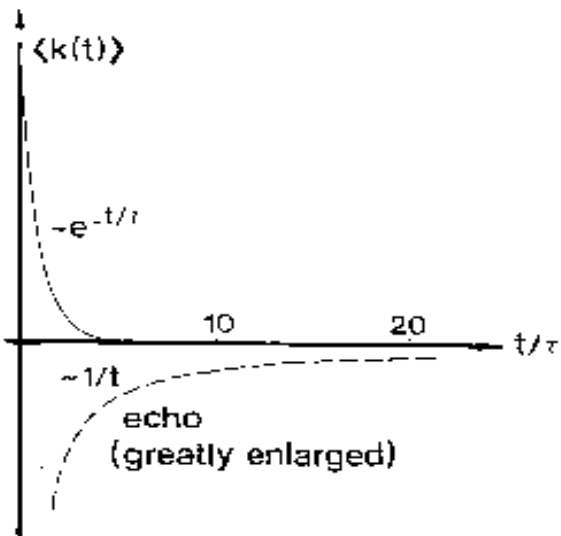
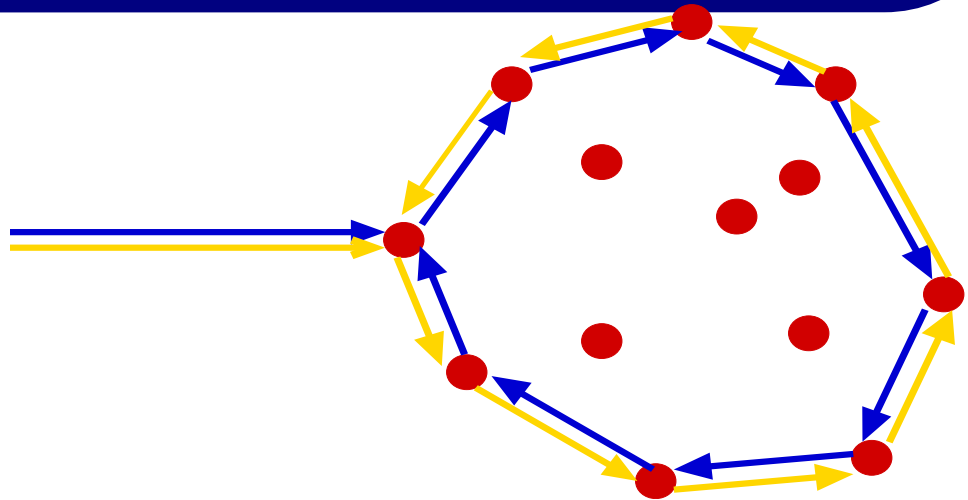
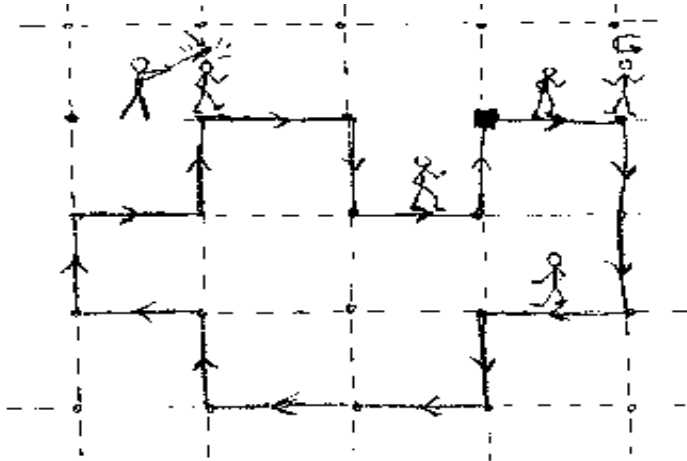


- 1D: All states are localized at arbitrary disorder  $\xi = 4\beta\ell$
- 2D: All states are localized  $\xi \sim \ell e^{k_F\ell}$
- 3D: Metal-Insulator transition occurs only at strong disorder.

# Localized Wave Function in 1D



# Quantum Transport History: Weak Localization

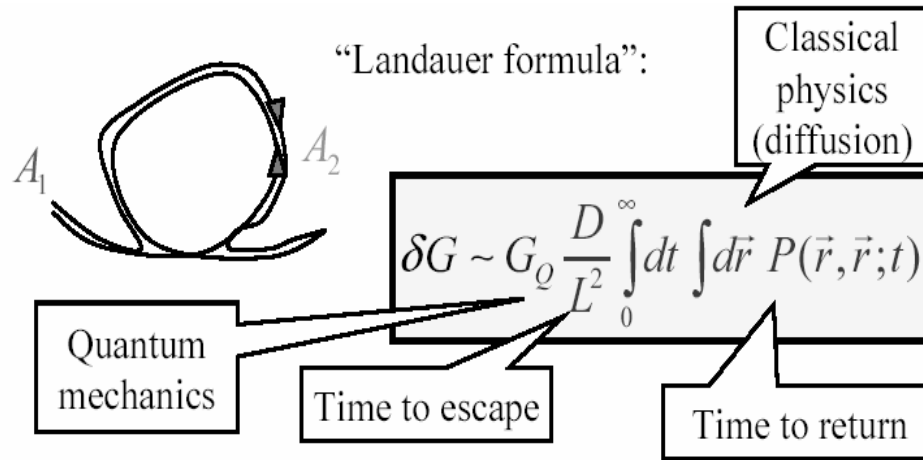


$$|A_{\text{blue}} + A_{\text{gold}}|^2 > |A_{\text{blue}}|^2 + |A_{\text{gold}}|^2$$

$$4|A_{\text{blue}}|^2 > 2|A_{\text{blue}}|^2$$

□ Weak localization realizes, in effect, a time-of-flight experiment with the conduction electrons in a disordered metal  $\Rightarrow$  It is best viewed as a multi-slit Young experiment with wave interference from randomly located scatterers which it, therefore, probes.

# Weak Localization Theory



1D:

$$\delta G \sim \begin{cases} -G_0 & L \ll L_\phi \\ -G_0 L_\phi / L & L \gg L_\phi \end{cases}$$

$P(r, r'; t)$  -probability density to propagate from  $r'$  to  $r$  during the time  $t$ .

2D:

$$\delta G \sim -G_0 \frac{W}{L} \log \frac{\min(L, L_\phi)}{l}$$

Obeys diffusion equation!

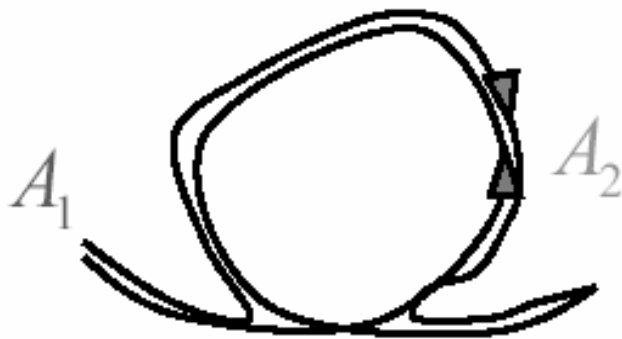
$$\left( \frac{\partial}{\partial t} - D \nabla_r^2 \right) P(r, r'; t) = \delta(r - r') \delta(t)$$

3D:

$$\delta G \sim -G_0 \frac{A}{L} \left( \frac{1}{l} - \frac{1}{\min(L, L_\phi)} \right)$$

# Weak Localization: Magnetic Field Dependence

Let us switch on the magnetic field:



$$A_1 \rightarrow A_1 e^{i\varphi}; A_2 \rightarrow A_2 e^{-i\varphi}$$

$$\varphi = 2\pi\Phi / \Phi_0$$

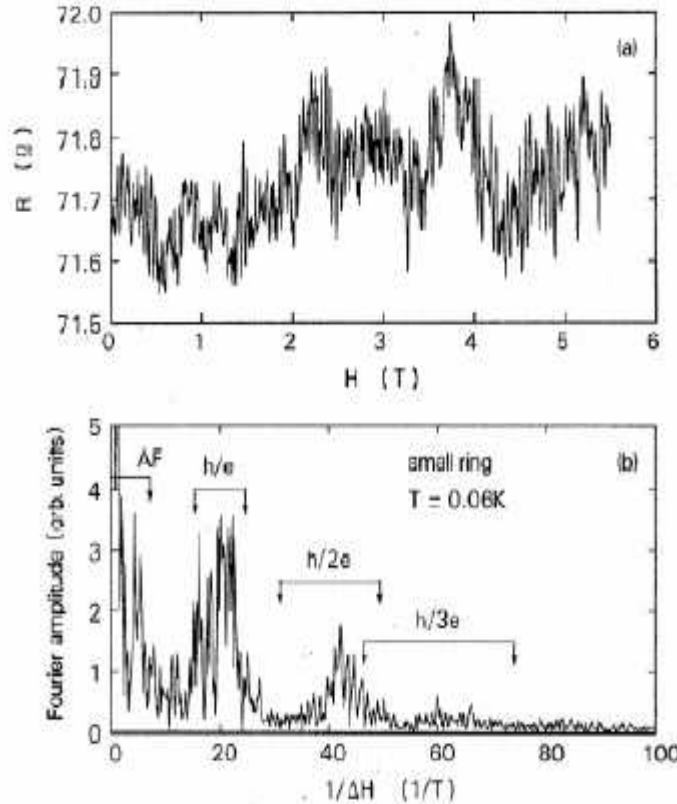
$$|A|^2 = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2|A_1 A_2| \cos 2\varphi$$

- Average over different loops: Interference term disappears
- Magnetic field destroys Weak Localization  $\Rightarrow$  This is how it is observed in experiments!

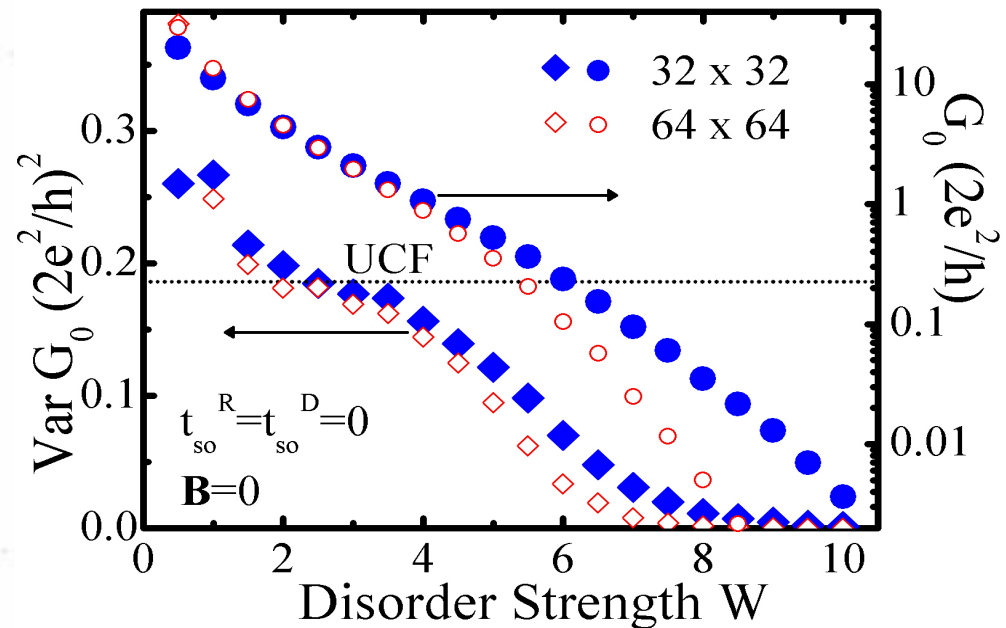
# Quantum Transport History: Universal Conductance Fluctuations

**Theory:** Altshuler (1984), Lee and Stone (1985)

**Experiment:** Wasburn, Umbach, Laibowitz, and Webb (1985)

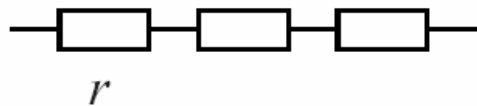


$$\sqrt{\langle G^2 \rangle - \langle G \rangle^2} \simeq 2e^2/h$$



# Conductance Fluctuations: Classical vs. Mesoscopic Physics

Compare: A linear chain of independently fluctuating resistors.



$$R = \sum_n r_n$$

$$\langle (\delta R)^2 \rangle = \left\langle \sum_n (\delta r_n)^2 \right\rangle = N (\delta r)^2$$

$$\langle (\delta G)^2 \rangle \propto N^{-3}$$

Experimentally:

$$\langle (\delta G)^2 \rangle \sim G_Q^2$$

Much stronger! Why?

Quantum interference between the resistors!

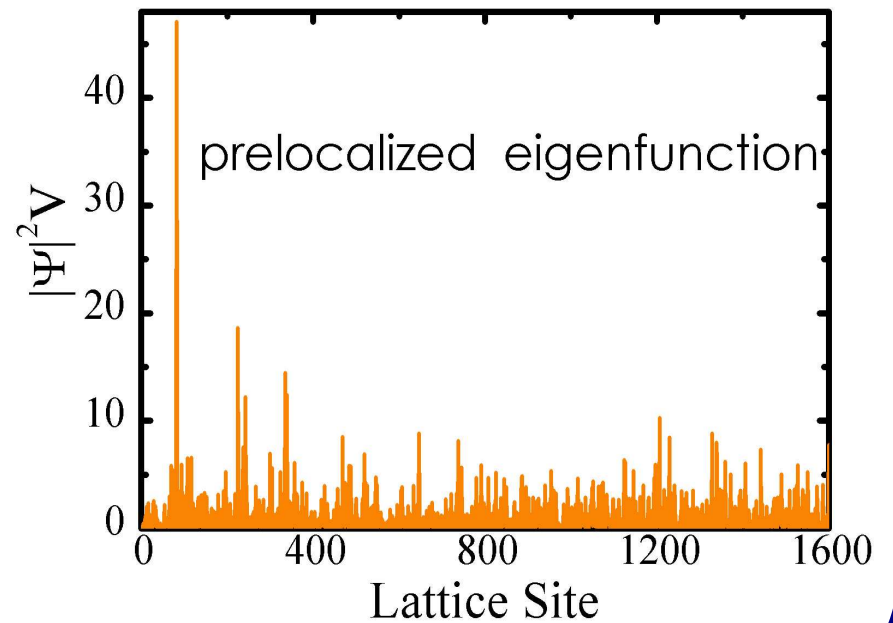
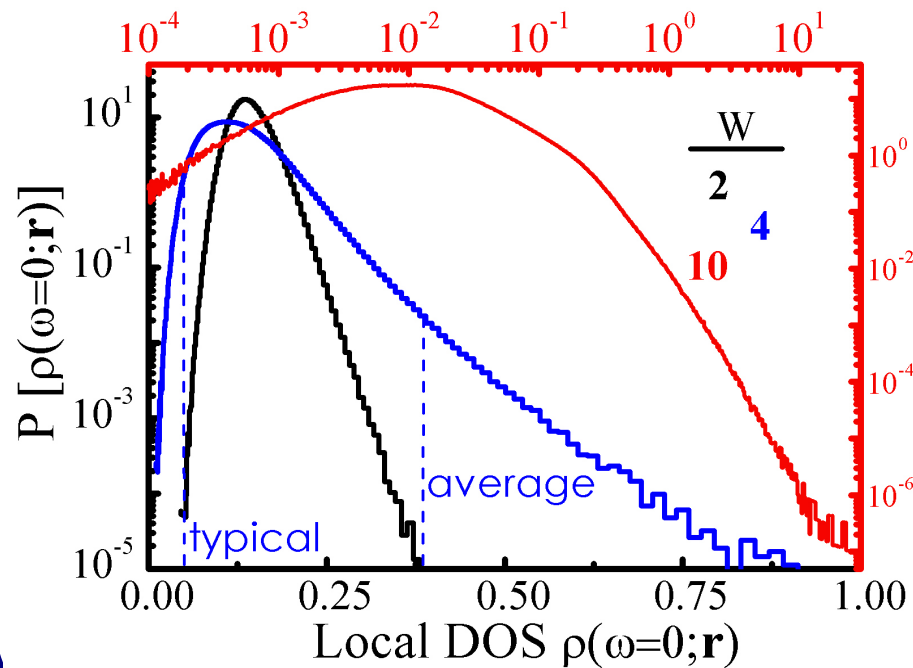
$$G = G_Q \sum_n T_n$$

$$\text{var } G = G_Q^2 \sum_{m,n} (\langle T_m T_n \rangle - \langle T_m \rangle \langle T_n \rangle)$$

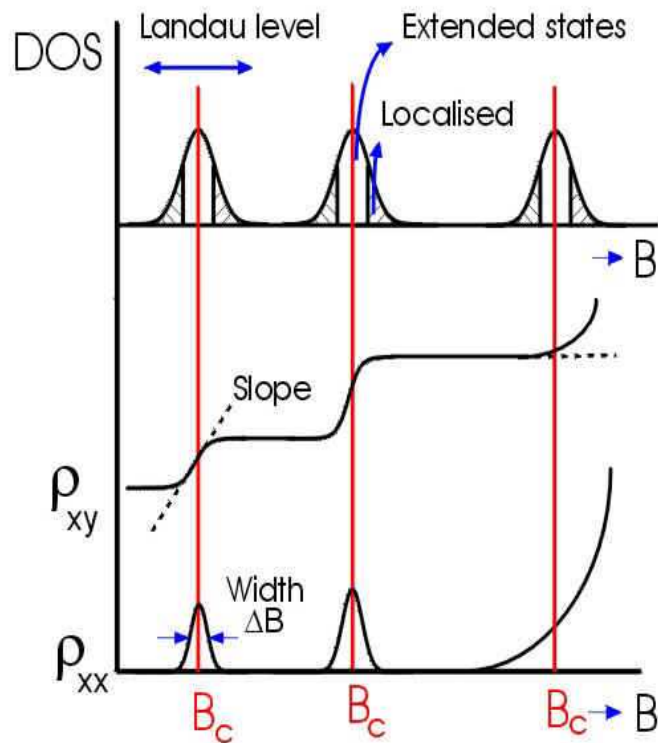
Need to know what is the correlation between different transmission eigenvalues.

# Mesoscopic Fluctuations

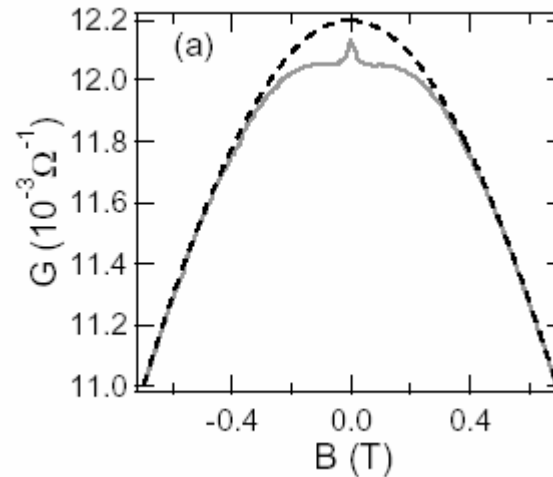
□ **Mesoscopic fluctuations**  $\Rightarrow$  broad distributions of physical quantities in open ( $g, \rho(\mathbf{r}), \tau_c, \dots$ ) or closed ( $|\Psi(\mathbf{r})|^2, \alpha, K_n, \dots$ ) phase-coherent systems.



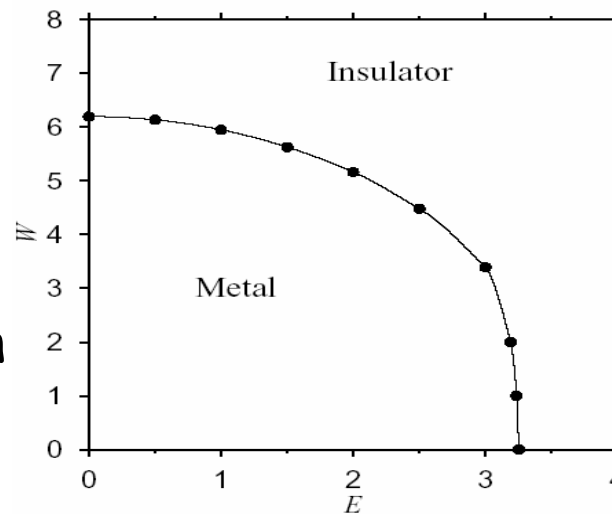
# Delocalization in Two Dimensions? Two Exceptions: QHE and Weak Antilocalization



• Delocalized states in 2DEG in the center of a Landau level

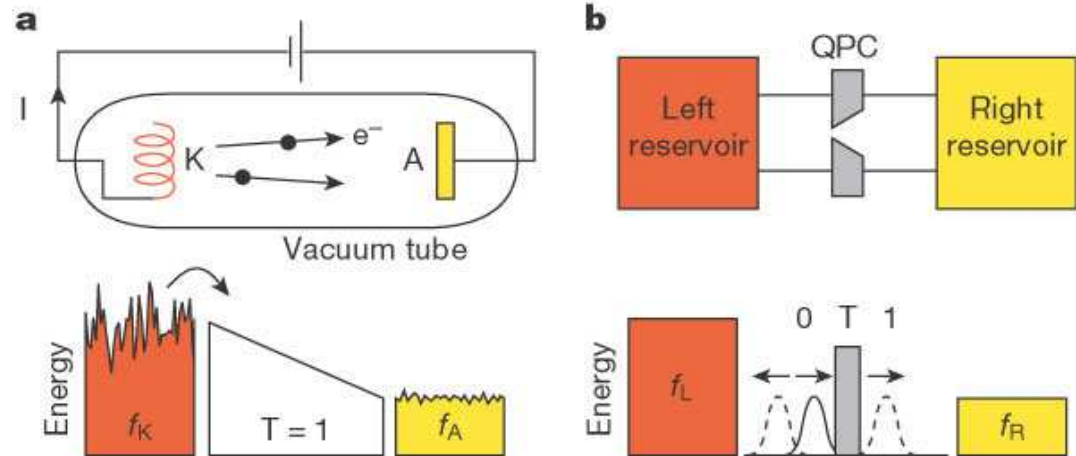
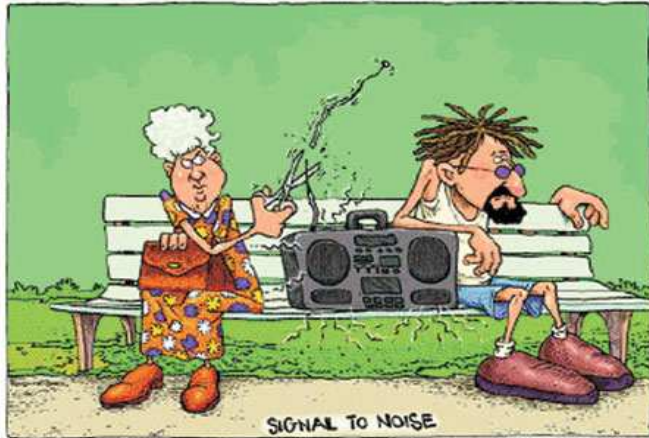


• Weak antilocalization in Rashba spin-split 2DEG (cond-mat/0408127)



• Metal-Insulator transition in 2DEG with SO interaction (cond-mat/0404630)

# Beyond Conductance: "Noise (Shot) is Signal"



Origin of noise. **A)** In a classical vacuum tube, the randomness that gives rise to shot noise stems solely from fluctuations in the reservoirs. The transmission from cathode (K) to anode (A) through the vacuum is noiseless, because it occurs with unit probability. **B)** A quantum point contact as a coherent conductor: at zero temperature randomness is due to scattering within the conductor only. Although this scattering can generally be classical or quantum-mechanical, only quantum scattering generates noise.

• Noise is quantified by Fourier transform of current autocorrelation function:

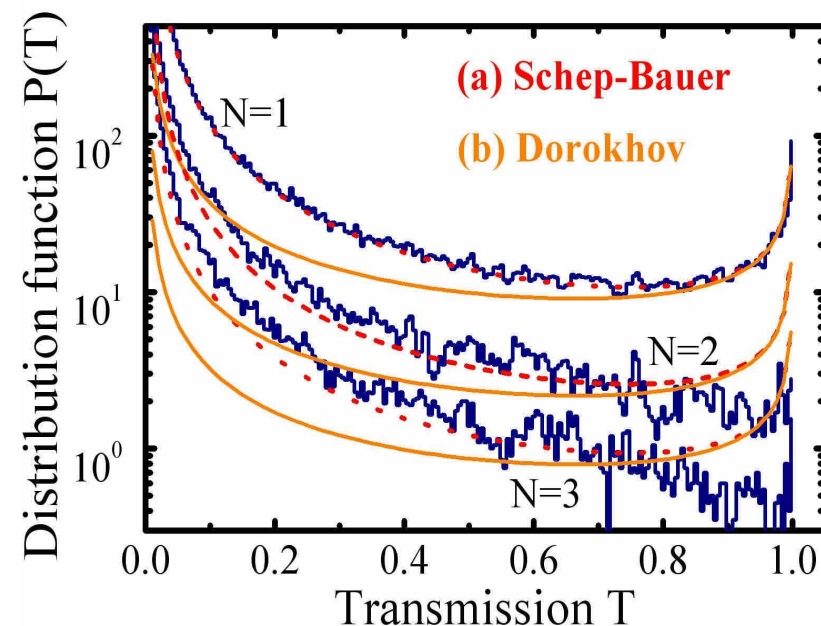
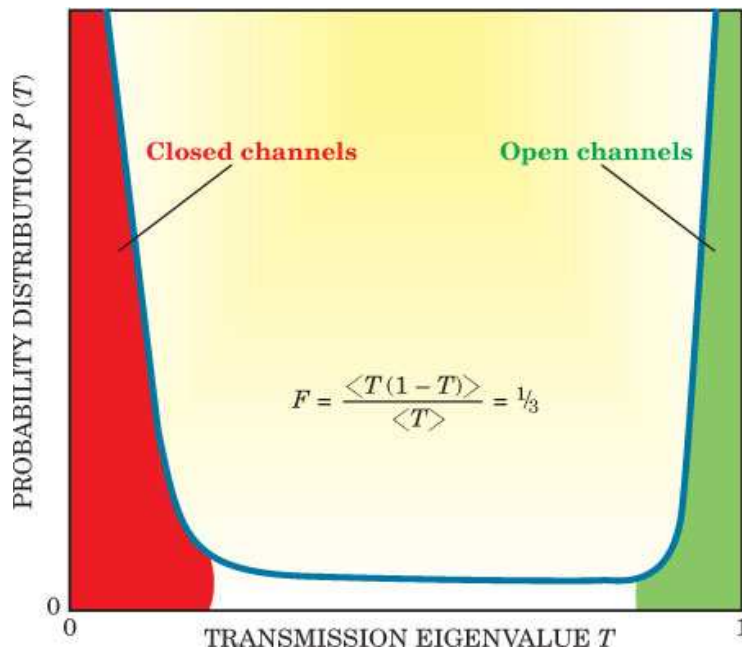
$$S(\omega) = 2 \int_{-\infty}^{+\infty} \frac{d(t-t')}{2\pi} e^{i(\omega-\omega')} [\overline{I(t)I(t')} - \overline{I(t)}\overline{I(t')}]$$

# Landauer Formula: Distribution of Transmission Eigenvalues

- Shot Noise Power at zero frequency:  $S(0) \sim \int_0^1 dT P(T) T(1-T) = 2Fe\bar{I}$
- Other linear statistics:  $A = \sum_{n=1}^M a(T_n)$

$$G_{NS} \sim \int_0^1 dt P(T) T$$

$$G_{NS} \sim \int_0^1 dt P(T) T / (2 - T)^2$$



# Shot Noise in Spintronics

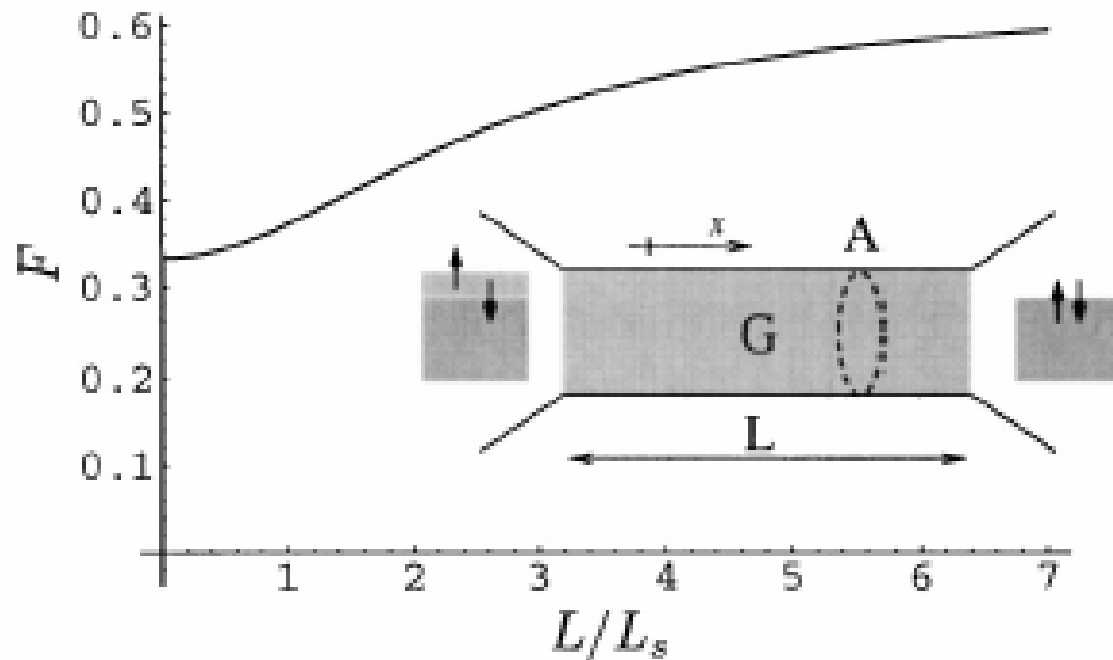


FIG. 2. Fano factor (defined by  $S = FeI$ ) for a diffusive wire passing a spin-polarized current, as a function of spin-relaxation length  $L_s$ . Inset: experimental geometry.

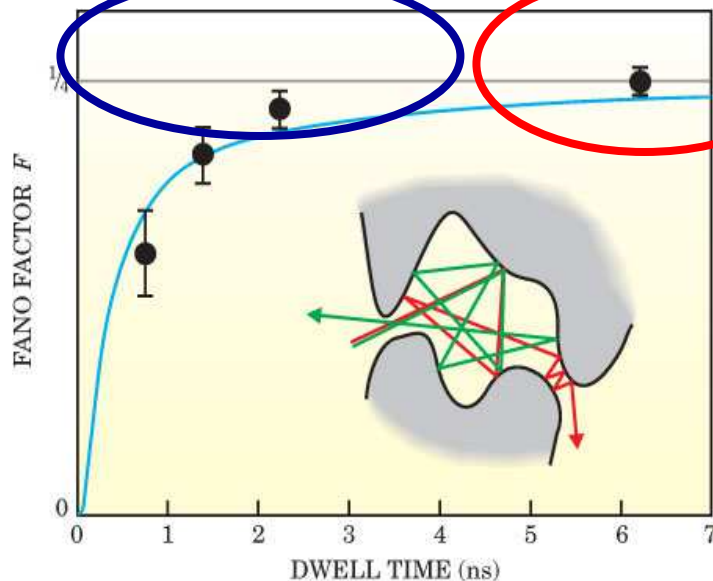
Phys. Rev. B **69**, 081301 (2004)

# Quantum Chaos in Quantum Transport

- Quantum transport through ballistic chaotic nanostructures: Fingerprints of classical chaos ("hard" or "soft") appear in kinetic properties.

Not enough quantum effects ("diffraction")

$$\tau_{dwell} \gg \tau_E$$



$$F = \frac{1}{4} \exp(-\tau_{dwell}/\tau_E)$$

$\tau_E \simeq |\ln \hbar|/\lambda$ —Ehrenfest time

$\rho(t) = \rho(0)e^{\lambda t}$ —Lyapunov  $\lambda$