

Name (first and last):

Mini test

## Introduction to Solid State Physics

**Problem 1.** What is the average number of particles in the single particle state of momentum  $\mathbf{p}$ , spin  $\sigma$ , and energy  $\varepsilon_{\mathbf{p},\sigma} = \mathbf{p}^2/2m$  for an ideal gas of

- bosons,  $n_{\mathbf{p},\sigma} =$
- fermions,  $n_{\mathbf{p},\sigma} =$
- “boltzmannons”,  $n_{\mathbf{p},\sigma} =$

if its chemical potential is  $\mu$  and temperature is  $T$ ?

**Problem 2.** An element  $\sigma_v$  of the group  $\mathcal{C}_{3v}$  is represented by a matrix

$$D(\sigma_v) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

What is the character of this representation  $\chi(\sigma_v) =$

**Problem 3.** If  $|\alpha\rangle$  is a state vector of a quantum system, then what is the value of the following expression  $\text{Tr}(|\alpha\rangle\langle\alpha|) =$

**Problem 4.** An electron enclosed in a box of size  $L^3$  is described by a wave function  $\phi_{\mathbf{k}}(\mathbf{r}) = L^{-3/2}e^{i\mathbf{k}\cdot\mathbf{r}}$  (where components of  $\mathbf{k}$  are integer multiples of  $2\pi/L$  to satisfy the periodic boundary conditions). If another electron with wave vector  $\mathbf{k}'$  is added to the box, write down their two-particle wave function when:

- the sum of the electron spins is zero (“spin singlet”),  
 $\Psi(\mathbf{r}_1, \mathbf{r}_2) =$
- the sum of the electro spins is one (“spin triplet”),  
 $\Psi(\mathbf{r}_1, \mathbf{r}_2) =$