Homework Set 7.

Problem 1. Consider a one-dimensional array of atoms, in which every atom has mass M, but the springs alternate in strength on every other site between values K_1 and K_2. Find the vibrational frequencies of this array as a function of wave number q.

Problem 2. Consider a two-dimensional square lattice with one atom of mass m per lattice point interacting with only nearest neighbors with force constant K. Take the phonon dispersion curve to be $\omega(q) = \sqrt{4K/m}\sin(qa/2)$.

(a) In the long-wavelength limit $q \to 0$, obtain the density of phonon states $D(\omega) = dN/d\omega$, i.e., the number of lattice-vibrations modes per frequency interval $d\omega$.

(b) At high temperature ($k_B T \gg \hbar\omega$), find the mean square displacement of an atom from its equilibrium position, and comment on the stability of two-dimensional crystals (i.e., on the possibility to break continuous translational symmetry in two dimensions).