Homework Set 6.

Problem 1. The idea of screening comes originally from electrolytic solutions. Imagine placing a charged ion into such solution. At first the potential due to the added ion extends its influence to the far reaches of the system, dying slowly off as $1/r$. However, mobile ions nearby rapidly react to the intruder, and the motions they make in response have the effect of almost completely canceling out its electric field, except within a characteristic distance called the *screening length*. Because the phenomenon occurs generally for any assembly of charged particles, it can be studied in the context of Thomas-Fermi theory which yields the following equation

$$\frac{\hbar^2}{2m}\left(3\pi^2\right)^{2/3} n^{2/3}(\vec{r}) + U(\vec{r}) + \int d\vec{r}' \frac{e^2 n(\vec{r}')}{|\vec{r}-\vec{r}'|} - \left(\frac{3}{\pi}\right)^{1/3} e^2 n^{1/3}(\vec{r}) = \mu$$

where $n(\vec{r})$ is the density of particles and μ is the chemical potential characterizing the equilibrium state.

(a) Consider this Thomas-Fermi equation where the last term on the left-hand side (resulting from the exchange interaction) is omitted for simplicity. Suppose that n_0 is the solution of this equation when the potential $U(\vec{r})$ vanishes $U(\vec{r}) \to 0$. If now a small potential $U(\vec{r})$ is added, find the equation governing deviations $\delta n(\vec{r})$ of the density from perfect uniformity to first order in U.

(b) Consider adding one extra electron to the uniform electron gas and therefore specializing to the case $U(\vec{r}) = e^2/r$. Solve the linearized equation for $\delta n(\vec{r})$ by use of Fourier *transform*. The answer should be of the form $\delta n \sim \frac{e^{-r/\xi}}{r}$. Identify the screening length ξ and express it in terms of the Bohr radius and the average volume per particle of the original uniform electron gas.

(c) Estimate the screening length for aluminum and copper.

Problem 2. Two-dimensional Electron Gas: The dimensionality of a system can be reduced by confining the electrons in certain directions. Consider an electron gas in an external potential $V = 0$ for $z < d/2$ and $V = V_0$ for $|z| > d/2$. What is the density of states as a function of energy for $V_0 \to \infty$ (discuss what happens at low and high energies)? Assume $d = 100$Å. Up to what temperatures can we consider the electrons to be two-dimensional? If we can produce a potential of 100 meV and reach a temperature of 20 mK, what is the range of thicknesses feasible for the study of such two-dimensional electron gas? (NOTE: A two-dimensional electron gas is produced in semiconductor heterostructures and is used for the investigation of the quantum Hall effect as well as other phenomena).

Problem 3. Conductance of a perfect 2D wire: Determine the resistance in Ohms of a perfect two-dimensional wire in the form of a strip of width $D = 1.75\lambda_F$, where λ_F is de Broglie wavelength of an electron at the Fermi energy. Assume the conduction electrons in the wire can be described by the free-particle Schrödinger equation with Dirichlet [i.e., $\Psi(\mathbf{r}) = 0$] boundary conditions along the lateral edges of the strip.

Problem 4. Sharvin point contact conductance: The Sharvin formula for the electrical conductance of an extremely short contact area A between two pieces of metal is

$$G = \frac{2e^2}{h}\frac{k_F^2 A}{4\pi}$$

Derive the Sharvin formula by considering the total current flowing through a hole of area A in a thin insulating barrier separating two free electron gases with different Fermi energies—the gas on the left has Fermi energy $\varepsilon_F^0 + eV$, while the gas on the right has Fermi energy ε_F^0, where V is the applied voltage bias. Use purely macroscopic arguments. Hint: In a free electron gas, the number of electrons with energies between E and $E + dE$ traveling at an angle between θ and $\theta + d\theta$ with respect to a given axis is

$$\frac{\partial^2 n}{\partial E \partial \theta} dE d\theta = \frac{D(E)}{2} \sin\theta d\theta dE,$$

where $D(E)$ is the density of states in three dimensions.

Problem 5. Consider a two-dimensional hexagonal lattice of lattice spacing $a = 3$ Å, and one electron per unit cell. If the electrons are considered free within the two-dimensional plane, what is the Fermi energy E_F? (Provide a numerical answer in eV).