Homework Set 5.

Problem 1. Consider a free Fermi gas with N electrons. Find the energy of the ground state (ground state means at zero temperature $T = 0$) of the gas as N varies from 1 through 15 (HINT: Note that for small number of particles one cannot convert sums over $\mathbf{k}$ states into integrals, but must instead use explicit enumeration of states in the box with periodic boundary conditions, whose wave vector is $\mathbf{k} = \frac{2\pi}{L}(n_x, n_y, n_z)$ and eigenenergy is $\epsilon_{\mathbf{k}} = \hbar\mathbf{k}^2/2m$).

Problem 2. Find the pressure of the *ideal* Fermi gas in three dimensions at zero temperature (HINT: Use the fact that grand canonical thermodynamic potential for this system is $\Phi = -k_B T \sum_{\mathbf{k}} \ln[1 + e^{\beta(\mu - \epsilon_{\mathbf{k}})}]$).

Problem 3. Consider a thin layer of silver (density of particles $n = 5.86 \cdot 10^{22}$ electrons/cm^3), 10^6 Å wide and 10^6 Å long along the x and y axis, respectively.

(a) Take the layer to be 4.1 Å thick along the z-axis. Treat the layer as a free Fermi gas, demanding that the wave function vanish at the boundaries along the z-axis. Find the difference between the energies of the lowest and highest occupied single-particle states, and compare the difference to the bulk Fermi energy.

(b) Repeat the problem (a) with a layer 8.2 Å thick along the z-axis.