Homework Set 1.

Problem 1. Estimate a pressure needed to compress a solid to $\alpha = 10$ times its normal density. Use the fact that to compress isothermally an elastic solid of volume V by dV, the pressure increment dp required is given by

$$K = -V\left(\frac{\partial p}{\partial V}\right)_T$$

where isothermal compressibility K of a typical solid is $K \sim 10^{11}$ Pa. Express your result in both Pascals (Pa) and atmospheres.

Problem 2. Consider a line of $2N$ ions of alternating charges $\pm q$ with a repulsive potential A/R^n between nearest neighbors in addition to the usual Coulomb potential.

(a) Find the equilibrium separation R_0 between ions in such one-dimensional solid and evaluate the equilibrium energy $U(R_0)$.

(b) Let the crystal be compressed so that $R_0 \to R_0(1-\delta)$. Calculate the work done in compressing a unit length of the crystal (the total length of the crystal is approximately $2NR_0$) to order δ^2.

Hint: Neglecting surface effects, the lattice energy of this ionic system is the sum of two usual Coulomb and repulsive potential terms

$$U(R) = N\left(-\frac{aq^2}{R} + \frac{A}{R^n}\right),$$

where $a = 2\ln 2$ is the so-called Madelung constant for a one-dimensional chain.

Problem 3. Suppose a sudden electric field pulse is applied to a material, $\vec{E} = \vec{E}_0\delta(t)$. Sketch qualitatively, as a function of time, the current $j(t)$ that would develop in

(a) metal

(b) insulator

(c) superconductor

Label the time scales in your sketch, both in qualitative and quantitative terms.