

THE EXTENT AND CAUSE OF THE PRE-WHITE DWARF INSTABILITY STRIP

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ABSTRACT

One of the least understood aspects of white dwarf evolution is the process by which they are formed. The initial stages of white dwarf evolution are characterized by high luminosity, high effective temperature, and high surface gravity, making it difficult to constrain their properties through traditional spectroscopic observations. We are aided, however, by the fact that many H- and He-deficient pre-white dwarfs (PWDs) are multiperiodic g -mode pulsators. These stars fall into two classes: the variable planetary nebula nuclei (PNNV) and the “naked” GW Vir stars. Pulsations in PWDs provide a unique opportunity to probe their interiors, which are otherwise inaccessible to direct observation. Until now, however, the nature of the pulsation mechanism, the precise boundaries of the instability strip, and the mass distribution of the PWDs were complete mysteries. These problems must be addressed before we can apply knowledge of pulsating PWDs to improve understanding of white dwarf formation. This paper lays the groundwork for future theoretical investigations of these stars. In recent years, Whole Earth Telescope observations led to determination of mass and luminosity for the majority of the GW Vir pulsators. With these observations, we identify the common properties and trends PWDs exhibit as a class. We find that pulsators of low mass have higher luminosity, suggesting the range of instability is highly mass dependent. The observed trend of decreasing periods with decreasing luminosity matches a decrease in the maximum (standing-wave) g -mode period across the instability strip. We show that the red edge can be caused by the lengthening of the driving timescale beyond the maximum sustainable period. This result is general for ionization-based driving mechanisms, and it explains the mass dependence of the red edge. The observed form of the mass dependence provides a vital starting point for future theoretical investigations of the driving mechanism. We also show that the blue edge probably remains undetected because of selection effects arising from rapid evolution.

Subject headings: stars: interiors — stars: oscillations — stars: variables: other (GW Virginis) — white dwarfs

1. INTRODUCTION

Until about 20 years ago, the placement of stars in the transition region between the planetary nebula nucleus track and the upper end of the white dwarf cooling sequence was problematic. This was due not only to the rapidity with which stars must make this transition (making observational examples hard to come by) but also to the difficulty of specifying $\log g$ and T_{eff} for such objects. Determining these quantities from spectra requires that we construct a reasonable model of the star's atmosphere. This is very difficult for compact stars with T_{eff} in excess of 50,000 K. The assumption of local thermal equilibrium (LTE), so useful in modeling the spectra of cooler stars, breaks down severely at such high temperatures and gravities.

Fortunately, the known sample of stars that occupy this phase of the evolutionary picture has grown over the last two decades. The most important discovery was that of a new spectral class called the PG 1159 stars. They are defined by the appearance of lines of He II, C IV, and O VI (and sometimes N V) in their spectra. Over two dozen are known, ranging in T_{eff} from over 170,000 K down to 80,000 K. About half are central stars of planetary nebulae. The most evolved PG 1159 stars merge with the $\log g$ and T_{eff} of the hottest normal white dwarfs. This class thus forms a complete evolutionary sequence from planetary nebula nucleus (PNN) to the white dwarf cooling track (Werner 1995; Dreizler & Huber 1998).

About half of the PG 1159 stars are pulsating variable stars, spread over the entire range of $\log g$ and T_{eff} occupied

by members of the spectral class. This represents the widest instability “strip” (temperature-wise) in the H-R diagram. Central star variables are usually denoted as planetary nebula nucleus variable (PNNV) stars. Variable PG 1159 stars with no nebula make up the GW Virginis (or simply GW Vir) stars. PG 1159 thus serves as the prototype for both a spectroscopic class and a class of variable stars. Farther down the white dwarf sequence, we find two additional instability strips. At surface temperatures between about 22,000 and 28,000 K (Beauchamp et al. 1999), we find the DBV (variable DB) stars. Even cooler are the ZZ Ceti (variable DA) stars, with T_{eff} between 11,300 and 12,500 K (Bergeron et al. 1995). Variability in all three strips results from g -mode pulsation (for the ZZ Ceti stars, see Warner & Robinson 1972; for the DBVs, see Winget et al. 1982; for the PG 1159 variables, see Starrfield et al. 1983). The pulsation periods provide a rich mine for probing the structure of white dwarf and pre-white dwarf (PWD) stars.

Despite the wealth of pulsational data available to us in studying the variable PWDs, they have so far resisted coherent generalizations of their group properties. Such a classification is required for understanding possible driving mechanisms or explaining the observed period distribution. Until now, the error bars associated with spectroscopic determination of mass, luminosity, and T_{eff} for a given variable star spanned a significant fraction of the instability strip.

Even given the limited information provided by spectroscopic determinations of $\log g$ and T_{eff} , many attempts have

been made to form a coherent theory of their pulsations. Starrfield et al. (1984) proposed that GW Vir and PNNV stars are driven by cyclical ionization of carbon (C) and oxygen (O). However, their proposal suffers from the deficiency that a helium (He) mass fraction of only 10% in the driving zone will “poison” C/O driving. Later, they managed to create models unstable to (C-driven) pulsation with surface He abundance as high as 50%, but only at much lower T_{eff} than most GW Vir stars (Stanghellini, Cox, & Starrfield 1991; see also the review by Cox 1993). Another problem is the existence of nonpulsating stars within the strip with nearly identical spectra to the pulsators (Werner 1995). More precise observations can detect subtle differences between the pulsators and nonpulsators. For instance, Dreizler (1998) finds N v in the spectra of all the pulsators but only some of the nonpulsators. It remains to be seen if these differences are important.

More recently, Bradley & Dziembowski (1996) studied the effects of using the newer OPAL opacities in creating unstable stellar models. Their models pulsate via O driving, and—in exact opposition to Starrfield, Stanghellini, and Cox—they require *no* C (or He) in the driving region to obtain unstable modes that match the observed periods. Their models also require radii up to 30% larger than those derived from prior asteroseismological analyses of GW Vir stars, in order to match the observed range of periods.

Finally, Saio (1996) and Gautschy (1997) proposed driving by a “metallicity bump,” where the opacity derivative changes sign as a result of the presence of metals such as iron in the envelope. This κ mechanism is similar to those currently thought to drive pulsation in the β Cephei variables (see, for instance, Moskalik & Dziembowski 1992) and in the recently discovered subdwarf B variables (Charpinet et al. 1996). Unfortunately, their simplified models are inconsistent with the evolutionary status of PG 1159 stars. More importantly, their period structures do not match published WET observations of real pre-white dwarfs (Winget et al. 1991; Kawaler et al. 1995; Bond et al. 1996; O’Brien et al. 1998).

With so many different explanations for pulsational driving, stricter constraints on the observable conditions of the pulsators and nonpulsators are badly needed. The most effective way to thin the ranks of competing theories (and perhaps point the way to explanations previously unthought of) is to obtain better knowledge of when the pulsations begin and end for PWD stars of a given mass.

Of course, with only a few stars available to study, even complete asteroseismological solutions for all of them might not prove significantly illuminating. Even if their properties follow recognizable patterns, it is difficult to show this compellingly given only a few cases. However, with asteroseismological analyses now published for the majority of the GW Vir stars, we can finally attempt to investigate their behavior as a class of related objects. In the next section, we outline the analytic theory of PWD pulsation. In § 3, we use the observed properties of the variable PWDs to show that they do follow compelling trends, spanning their entire range of stellar parameters, to which any model of PWD pulsation must conform. Next we introduce a new set of numerical models developed to help interpret this behavior. In § 5, we show how the evolution (and eventual cessation) of pulsation in PWDs can be governed by the changing relationship of the driving timescale to the maximum sustainable g -mode period. These results suggest

several fruitful directions for future work, both theoretical and observational, which we discuss in the concluding section.

2. THEORY

The set of periods excited to detectable limits in white dwarf stars is determined by the interplay of several processes. The first is the driving of pulsation by some—as yet unspecified—mechanism (no matter how melodious the bell, it must be struck to be heard). Second is the response of the star to the driving taking place somewhere in its interior. A pulsating PWD star is essentially a (spherically symmetric) resonant cavity, capable of sustained vibration at a characteristic set of frequencies. Those frequencies are determined by the structure of the star, its mass and luminosity, as well as the thickness of its surface layers. Finally, the actual periods we see are affected by the mechanism through which internal motions are translated into observable luminosity variations. This is the so-called “transfer function,” and clues to its nature are to be found in the observed variations as well.

2.1. Asymptotic Relations

If we wish to make the most of the observed periods, we must understand the process of driving, and response to driving, in as much detail as possible. However, we can learn a great deal by simply comparing the periods of observed light variations to the normal-mode periods of model stars.

The normal mode oscillations of white dwarf and PWD stars are most compactly described using the basis set of spherical harmonic functions, Y_m^ℓ , coupled with an appropriate set of radial wave functions R_n . Here n is the number of nodal surfaces in the radial direction, ℓ denotes the number of nodal planes orthogonal to such surfaces, and m is the number of these planes that include the pulsational axis of the star.

The periods of g -modes of a given ℓ are expected to increase monotonically as the number of radial nodes n increases. The reason is that the buoyant restoring force is proportional to the total mass displaced, and this mass gets smaller as the number of radial nodes increases. A weaker restoring force implies a longer period (see, e.g., Cox 1980). In the “asymptotic limit” that $n \gg \ell$, the periods of consecutive modes should obey the approximate relation

$$\Pi_n \cong \frac{\Pi_0}{[\ell(\ell + 1)]^{1/2}} (n + \epsilon) \quad n \gg \ell, \quad (1)$$

where Π_n is the g -mode period for a given value of n , and Π_0 is a constant that depends on the overall structure of the star (see, e.g., Tassoul 1980; Kawaler 1986).¹

Equation (1) implies that modes of a given ℓ should form a sequence based upon a fundamental period spacing $\Delta\Pi = \Pi_0/[\ell(\ell + 1)]^{1/2}$. This is the overall pattern identified by various investigators in the light curves of most of the GW Vir stars. Once a period spacing is identified, we can

¹ The additional constant ϵ is assumed to be small, though its exact value depends on the boundary conditions. Since the actual boundary conditions depend on the period, ϵ probably does, too.

compare this spacing to those computed in models to decipher the star's structure. Kawaler (1986) found that the parameter Π_0 in static PWD models is dependent primarily on the overall stellar mass, with a weak dependence on luminosity. Kawaler & Bradley (1994) present the approximate relation

$$\Pi_0 \cong 15.5 \left(\frac{M}{M_\odot} \right)^{-1.3} \left(\frac{L}{100 L_\odot} \right)^{-0.035} \left(\frac{q_y}{10^{-3}} \right)^{-0.00012}, \quad (2)$$

where q_y is the fraction by mass of He at the surface.²

Other questions, however, can only be answered from knowledge of the cause of the light variations we measure. In the case of the PWDs, this is chief among the mysteries we would like to solve. The most telling clue will be the extent and location of the region of instability in the H-R diagram—derived in part from pulsational analysis of the structure of stars that bracket this region. They will now provide some of the background needed to attack these issues.

2.2. Pulsation in Pre-White Dwarf Stars

In a star, energy generally flows down the temperature gradient from the central regions to the surface in a smooth, relatively unimpeded fashion. Of course, small, random perturbations to this smooth flow constantly arise. The situation is stable as long as such perturbations quickly damp out, restoring equilibrium. For instance, equilibrium is restored by the forces of buoyancy and pressure; these forces define the nature of g - and p -modes. They resist mass motions and local compression or expansion of material away from equilibrium conditions.

2.2.1. Driving and Damping

Thermodynamics and opacity affect local equilibrium. In general, if a parcel of material in a star is compressed, its temperature goes up while its opacity decreases. The higher temperature causes more radiation to flow out to the surrounding material, while lower opacity decreases the efficiency with which radiation is absorbed. From the first law of thermodynamics, an increasing temperature accompanied by net heat loss implies that work is being done on the parcel by its surroundings. Similar arguments show that work is done on the parcel during expansion, also. Thus, any initial perturbation will be quickly damped out—each parcel demands work to do otherwise. The requirement that a region lose heat when compressing and gain heat when expanding is the fundamental criterion for stability. When the opposite is true, and work is done by mass elements on their surroundings during compression and expansion, microscopic perturbations can grow to become the observed variations in pulsating stars.

Under certain circumstances, the sign of the opacity derivative changes compared to that described above. If the opacity, κ , increases upon compression, then heat flowing through a mass element is trapped there more efficiently. Within regions where this is true, work is done on the surrounding material. Thus, these regions can help destabilize the star, if this driving is not overcome elsewhere in the star. However, other regions, where work is required to com-

press and expand material, tend to damp such pulsation out. Global instability arises only when the work performed by the driving regions outweighs the work done on the damping regions over a pulsation cycle. In this case, the flow of thermal energy can do mechanical work, and this work is converted into the pulsations we observe.

This method of driving pulsation is called the κ mechanism. A region within a star will drive pulsation via this mechanism if the opacity derivatives satisfy the condition (see, for instance, Cox 1980)

$$\frac{d}{dr} \left(\kappa_T + \frac{\kappa_\rho}{\Gamma_3 - 1} \right) > 0, \quad (3)$$

where

$$\kappa_T \equiv \left(\frac{\partial \ln \kappa}{\partial \ln T} \right)_\rho, \quad \kappa_\rho \equiv \left(\frac{\partial \ln \kappa}{\partial \ln \rho} \right)_T, \quad \text{and} \quad \Gamma_3 - 1 \equiv \left(\frac{\partial \ln T}{\partial \ln \rho} \right)_S.$$

Here S represents the specific entropy, κ is the opacity in $\text{cm}^2 \text{g}^{-1}$, and the variables r , ρ , and T all have their usual meanings.

Equation (3) is satisfied most commonly when some species within a star is partially ionized. In particular, κ_T usually increases in the hotter (inner) portion of a partial ionization zone and decreases in the cooler (outer) portion. Thus the inner part of an ionization zone may drive while the outer part damps pulsation. The adiabatic exponent, $\Gamma_3 - 1$, is always positive but usually reaches a minimum when material is partially ionized. This enhancement of the κ mechanism is called the γ mechanism. Physically, the γ mechanism represents the conversion of some of the work of compression into further ionization of the species in question. This tends to compress the parcel more, aiding the instability. Release of this ionization energy during expansion likewise increases the perturbation. Since they usually occur together, instabilities caused by both the opacity and ionization effects are known as the κ - γ mechanism.

The pulsations of Cepheid and RR Lyrae variables, for instance, are driven via the κ - γ mechanism operating within a region where He I and He II have similar abundances. This same partial ionization zone is apparently the source of instability for the DBV white dwarfs. The variations observed in ZZ Ceti white dwarfs have long been attributed to partial ionization of H. However, Goldreich & Wu (1999a, 1999b, 1999c) have recently shown the ZZ Ceti pulsations can be driven through a different mechanism in efficient surface convection.³ Great efforts have been expended by theorists attempting to explain GW Vir and PNNV pulsations in terms of some combination of C and O partial ionization (Starrfield et al. 1983; Stanghellini, Cox, & Starrfield 1990; Bradley & Dziembowski 1996). A primary difficulty arises from the damping effects of He (and C in the O-driving scheme proposed by Bradley & Dziembowski 1996) in the driving zone, which can “poison” the driving. More recently, Saio (1996) and Gautschy (1997) attempted to explain driving in terms of an “opacity bump” in the models, without partial ionization—in other words, using the κ mechanism alone. A fundamental problem has been

² Note that the sign of the exponent in the L term is in error in Kawaler & Bradley (1994).

³ Such convection zones often accompany regions of partial ionization associated with the κ - γ mechanism. However, the driving proposed by Goldreich & Wu is not directly related to ionization. It is possible this theory might eventually be expanded to account for DBV pulsation as well. It is unlikely to find application in PWDs, though, since models of PG 1159 stars generally do not support convection.

the lack of information on the exact extent of the instability strip and the structure of the pulsating stars themselves. The initial goal in this paper, therefore, is to define as precisely as possible the observational attributes that any proposed driving source must reproduce. Whether or not the mechanism we later identify is correct, we hope to lay the groundwork for future studies that will eventually provide a definitive answer to this question.

If a star pulsates, we wish to know what relationship the driving has to the periods we observe. In general, no star will respond to driving at just any arbitrary period; the pulsation period range is determined by the driving mechanism. Cox (1980) showed that the approximate pulsation period is determined by the thermal timescale of the driving zone:

$$\Pi \sim \tau_{\text{th}} = \frac{c_v T m_{\text{dz}}}{L}. \quad (4)$$

Here τ_{th} is the thermal timescale, c_v is the heat capacity, and m_{dz} is the mass above the driving zone. This equation gives the approximate time it takes the star to radiate away—via its normal luminosity, L —the energy contained in the layers above the region in question. Though Cox derived this relationship for radial modes, Winget (1981) showed that it applies equally well to nonradial g -mode pulsation.

The basic idea behind equation (4) is that energy must be modulated at approximately the same rate at which it can be dammed up and released by the driving zone. Consider the question of whether a given zone can drive pulsation near a certain period. If the driving zone is too shallow, then the thermal timescale is shorter than the pulsation period. Any excess heat is radiated away before the compression increases significantly. Thus, it cannot do work to create mechanical oscillations on the timescale in question. If the driving zone is too deep ($\tau_{\text{th}} > \Pi$), then excess heat built up during contraction is not radiated away quickly enough during expansion; it then works against the next contraction cycle. Of course, this relationship is only approximate, and other factors might intervene to limit pulsation to periods far from those implied by equation (4).

2.2.2. Period Limits

One such factor is the range of pulsation modes a star can sustain in the form of standing waves. Hansen, Winget, & Kawaler (1985) showed that there is a maximum period for white dwarf and PWD stars above which oscillations will propagate only as running waves that quickly damp out. They attempt to calculate this maximum g -mode period, Π_{max} , to explain the observed trend of ZZ Ceti periods with T_{eff} . We can recast their equations (6)–(8) as

$$\Pi_{\text{max}} \approx 940 \text{s} \left[\frac{\mu}{\ell(\ell+1)} \right]^{0.5} \left(\frac{R}{0.02 R_{\odot}} \right) \left(\frac{T_{\text{eff}}}{10^5 \text{K}} \right)^{-0.5}, \quad (5)$$

or, using the relation $R^2 = L/4\pi\sigma T_{\text{eff}}^4$,

$$\Pi_{\text{max}} \approx 940 \text{s} \left[\frac{\mu}{\ell(\ell+1)} \right]^{0.5} \left(\frac{L}{35 L_{\odot}} \right)^{0.5} \left(\frac{T_{\text{eff}}}{10^5 \text{K}} \right)^{-2.5}, \quad (6)$$

where R is the stellar radius, μ represents the mean molecular weight, and ℓ is the pulsation index introduced earlier. For a complete derivation of these equations, see the Appendix.

For cool white dwarfs of a given mass, the radius is roughly constant with time, and Π_{max} is expected to

increase as stars evolve to lower T_{eff} and the driving zone sinks deeper. Cooler ZZ Ceti and DBV white dwarf stars should in general have longer periods; for the ZZ Ceti at least, this is indeed the case.

However, in PWDs degeneracy is not yet sufficient to have halted contraction, and the R dependence is still a factor in determining how Π_{max} varies through the instability strip. We cannot say a priori which dominates: the shrinking radius, which tends to decrease Π_{max} , or the falling T_{eff} , which has the opposite effect. We cannot even predict in advance whether Π_{max} is a factor in determining the pulsation periods at all. In § 5, we answer these questions in view of the observed properties of both the PNNV and GW Vir stars.

In summary, we can measure individual white dwarf mass and luminosity through identification of the period spacing. The resulting determinations of GW Vir structure, summarized in the next section, will tell us the precise boundaries of the instability strip, in other words, when the pulsations begin and end in the evolution of PWD stars of a given mass. This knowledge, coupled with the timescales of driving and the maximum g -mode period in models matched to the observations, will provide strict constraints on the allowable form of the driving mechanism. This information is absolutely necessary to any research program designed to discover why PWDs pulsate in the first place.

3. OBSERVED TEMPERATURE TRENDS OF THE PG 1159 PULSATORS

3.1. Mass Distribution

White dwarfs exhibit a very narrow mass distribution, centered at about 0.56 – $0.59 M_{\odot}$ (Bergeron, Saffer & Liebert 1992; Weidemann & Koester 1984). If, for a given T_{eff} , pulsating PWD masses conform to the same distribution expected of nonvariables, then we are led to certain expectations concerning the pulsations seen in the former. Based on equations (1) and (2), period spacing (proportional to Π_0) should increase monotonically as the luminosity and T_{eff} of a given star decrease.

Four PWD stars have so far yielded to asteroseismological scrutiny: the GW Vir stars PG 1159 (Winget et al. 1991), PG 2131 (Kawaler et al. 1995), and PG 0122 (O'Brien et al. 1998), and the central star of the planetary nebula NGC 1501 (Bond et al. 1996). We summarize the parameters of these four stars in Table 1. All show patterns of equal period spacing very close to 21 s. This is a remarkable trend, or more accurately, a remarkable lack of a trend! If these stars follow the narrow mass distribution observed in white dwarf stars, then the period spacing is expected to increase with decreasing T_{eff} as they evolve from the blue to the red edge of the instability strip. Observationally, this is not the case.

For instance PG 1159, with a T_{eff} of 140,000 K, should see its period spacing increase by about 20%, from 21 to 26 s, by the time it reaches the effective temperature of PG 0122—80,000 K. In other words, the farther PG 0122's period spacing is from 26 s, the farther is its mass from that of PG 1159. In fact, these two objects, representing the high and low T_{eff} extremes of the GW Vir stars, have almost exactly the *same* period spacing despite enormous differences in luminosity and temperature. For PG 0122, its low T_{eff} pushes it toward longer $\Delta\Pi$; this must be offset by a higher mass. With such a significant T_{eff} difference, the mass

TABLE 1
SUMMARY OF VARIABLE PWD PARAMETERS

Name	$\log g$	T_{eff} (kK)	$\log (L/L_{\odot})$	M/M_{\odot}	$\Delta\Pi$ (s)
NGC 1501	~ 6.0	80 ± 10	3.3 ± 0.3	0.55 ± 0.03	~ 22.3
PG 1159	7.22 ± 0.06	14 ± 10	2.3 ± 0.2	0.57 ± 0.01	21.5 ± 0.1
PG 2131	7.67 ± 0.12	95 ± 10	1.0 ± 0.2	0.61 ± 0.02	21.6 ± 0.4
PG 0122	7.97 ± 0.15	80 ± 10	0.7 ± 0.2	0.68 ± 0.04	21.1 ± 0.4

difference between PG 0122 and PG 1159 must be significant also, and it is: $0.69 M_{\odot}$ versus $0.58 M_{\odot}$.

Two stars with a coincident period spacing—despite widely different mass and luminosity—might simply be curious. In fact, all four PWDs with relatively certain period spacing determinations have the same spacing to within 2 s, or 10%. This includes the central star of NGC 1501, which has a luminosity over 3 orders of magnitude larger than that of PG 0122. Hence, NGC 1501 must have an even lower mass than PG 1159 by comparison. Figure 1 shows the mass versus luminosity values for the known GW Vir stars plus NGC 1501, based on the values of $\Delta\Pi$ from Table 1.

Figure 2 shows the implications of the common 21–22 s spacing for the instability “strip” in the $\log g$ – $\log T_{\text{eff}}$ plane. The observational region of instability has shrunk significantly. It exhibits such a striking slope in the figure that, unlike most other instability regions in the H-R diagram, it can no longer be referred to accurately as an instability strip (in temperature) at all. Nevertheless, we will continue to refer to the region of instability pictured in Figure 2 as the GW Vir “instability strip” with the understanding that the effective temperatures of the red and blue edges are highly dependent on $\log g$ (or L).

Why should the PWD instability strip apparently straddle a line of approximately constant period spacing? Normally, theorists search for explanations for the observed boundaries (the red and blue edges) of an instability strip based on the behavior of a proposed pulsation mechanism. That behavior is determined by the thermal properties of a PWD star, while its period spacing is determined by its

mechanical structure. In degenerate or nearly degenerate stars, the thermal properties are determined by the ions, while the mechanical structure is determined by the degenerate electrons, and normally the two are safely treated as separate, isolated systems. If the 21–22 s period spacing is somehow a *prerequisite* for pulsation, then this implies an intimate connection between the mechanical oscillations and the thermal pulsation mechanism.

The alternative is that the mass-luminosity relationship along the instability strip is caused by some other process—or combination of processes—which approximately coincides with the relationship that governs the period spacing. In this case, some mechanism must shut off observable pulsation in low-mass PWDs before they reach low temperature and delay observable pulsation in higher mass PWDs until they reach low temperature.⁴ We will explore mechanisms that meet these criteria in § 5.

3.2. Period Distribution

The PWD pulsators exhibit another clear observational trend: their periods decrease with decreasing luminosity (increasing surface gravity). Figure 3 shows the luminosity versus dominant period (that is, the period of the largest amplitude mode) for the same stars from Figure 1. The trend apparent in the figure is in marked contrast to the one seen in the ZZ Ceti stars, which show longer periods with lower T_{eff} . The ZZ Ceti period trend is generally attributed to the changing thermal timescale in the driving zone, which

⁴ We use the phrase “observable pulsation” to indicate that possible solutions might reside in some combination of observational selection effects as well the intrinsic behavior of a driving mechanism.

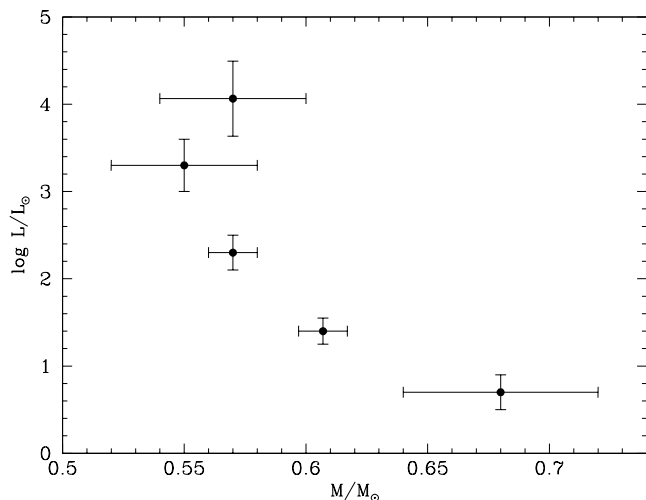


FIG. 1.—Luminosity vs. mass determined from pulsation data for the GW Vir stars PG 0122, PG 2131, and PG 1159, and for two “evolved” PNNV stars, NGC 1501 and RXJ 2117.

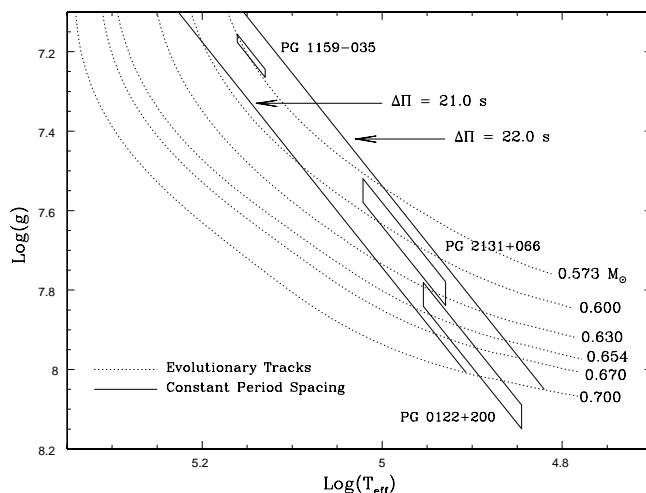


FIG. 2.—Portion of the $\log g$ – $\log T_{\text{eff}}$ plane showing PWD evolutionary tracks based on ISUEVO. The error boxes for each star derive from period spacing combined with spectroscopically determined T_{eff} .

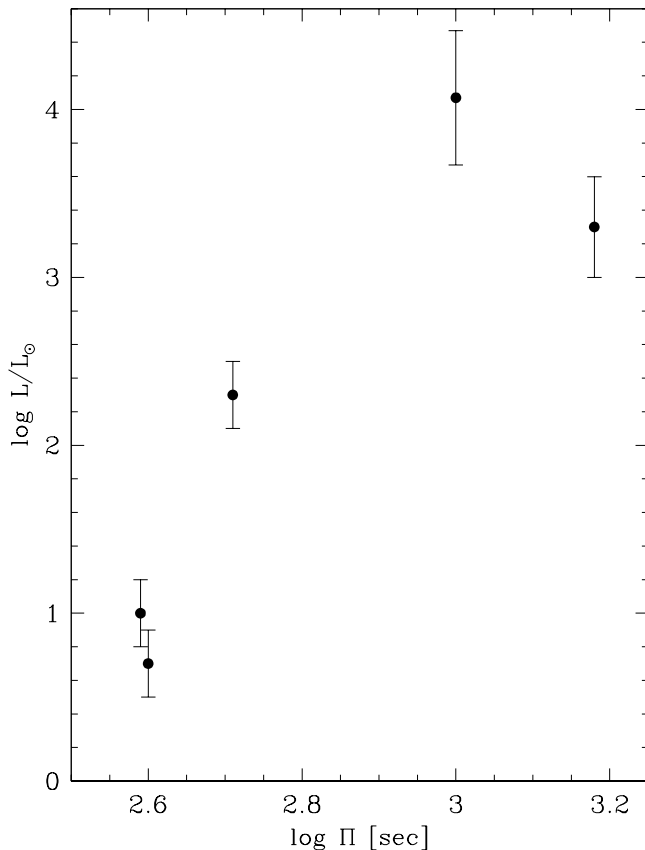


FIG. 3.—Luminosity (determined using the pulsation data) vs. dominant period for the same PWD stars as in Fig. 1.

sinks deeper (to longer timescales) as the star cools. If the same effect determines the periods in GW Vir and PNNV stars, then Figure 3 might be taken to indicate that the driving zone becomes more shallow with decreasing T_{eff} . We will show that this is not the case in PWD models. We conclude that some other mechanism must be responsible for setting the dominant period in PWD variables.

Are the trends seen in Figures 1–3 related? To explore this question in detail, we developed a new set of PWD evolutionary models, which we summarize in the next section. In § 5 we analyze the behavior of driving zones in PWD models in light of—and to seek an explanation for—the trends just discussed.

4. A NEW SET OF PRE-WHITE DWARF EVOLUTIONARY MODELS

To understand the various trends uncovered in the hot pulsating PWDs, we appeal to stellar models. Models have been essential for exploiting the seismological observation of individual stars. For this work, though, we needed models over the entire range of the GW Vir stellar parameters of mass and luminosity. Our principal computational tool is the stellar evolution program ISUEVO, which is described in some detail by Dehner (1996; see also Dehner & Kawaler 1995). ISUEVO is a “standard” stellar evolution code that is optimized for the construction of models of PWDs and white dwarfs.

The seed model for the models used in this section was generated with ISUEVO by evolution of a $3 M_{\odot}$ model from the zero-age main sequence through the thermally

pulsing AGB phase. After reaching a stable thermally pulsing stage (about 15 thermal pulses), mass loss was invoked until the model evolved to high temperatures. This model (representing a PNN) had a final mass of $0.573 M_{\odot}$ and a helium-rich outer layer.

To obtain self-consistent models within a small range of masses, we used the $0.573 M_{\odot}$ model and scaled the mass up or down. For example, to obtain a model at $0.60 M_{\odot}$, we scaled all parameters by the factor $0.60/0.573$ for an initial model. Relaxation to the new conditions was accomplished by taking many very short time steps with ISUEVO. Following this relaxation, the evolution of the new model proceeded as before. In this way, we produced models that were as similar as possible, with mass being the only difference.

Comparison of our evolutionary tracks and trends with the earlier model grids of Dehner (1996) shows very close agreement, given the different evolutionary histories. Dehner’s initial models were derived from a single model with a simplified initial composition profile, while our models are rooted in a self-consistent evolutionary sequence. We note that the work by Dehner (1996) included elemental diffusion (principally by gravitational settling), while the models we use here did not include diffusion. Within the temperature range of the GW Vir stars, however, observations of their surface abundances indicate that the effects of diffusion have only a small influence.

5. SELECTION EFFECTS, DRIVING, AND THE BLUE AND RED EDGES

5.1. The Observed “Blue” and “Red” Edges

In explaining the observed distribution of pulsating stars with respect to stellar parameters, we must distinguish observational selection effects from causes intrinsic to the objects under study. Usually, understanding selection effects is an important part of decoding the shape of the distribution in terms of physical effects. In this case, the blue and red edges exhibit a similar slope in the $\log g$ – $\log T_{\text{eff}}$ plane, but we must still separate out selection effects from the intrinsic shape of one or both of them.

The more rapid the evolution through a particular region of the H-R diagram, the less likely it is that stars will be found there. Also, the relative sample volume is larger for stars with higher luminosity, since they are detectable at greater distances. Some combination of these effects will determine the odds of finding stars of a particular mass at a particular point in their evolution. One of the most common ways to explore these combined effects is to construct a luminosity function, which is simply a plot of the expected number density of stars per unit luminosity, based on how bright they are and how fast they are evolving at different times. Figure 4 shows schematic luminosity functions for PWD stars of two different masses, based on the models described in the previous section and normalized according to the white dwarf mass distribution (see, for instance, Wood 1992). One important result of this figure is that higher mass models always achieve a given number density per unit luminosity later (at lower L and T_{eff}) than lower mass models. Also, the number density per unit luminosity increases for all models as they evolve to lower T_{eff} , as a result of the increasing amount of time spent in a luminosity bin.

These effects together imply that, if PWDs of all mass pulsate all the way down to 80,000 K, then the distribution

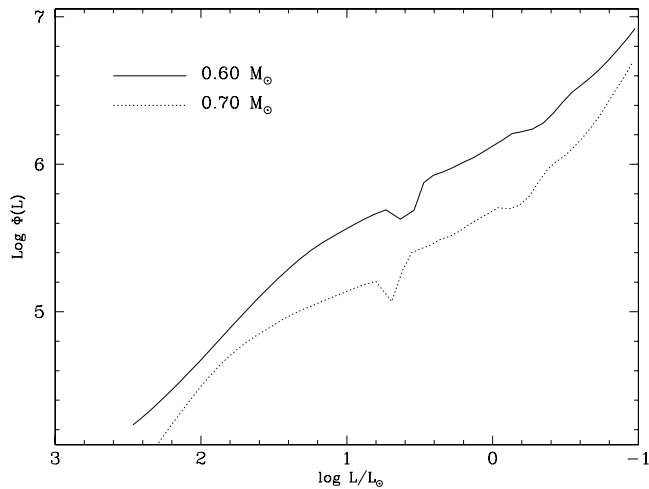


FIG. 4.—Theoretical luminosity functions, $\log \Phi(L)$, for two different single-mass populations of PWD stars.

of known pulsators should be skewed heavily toward those with both low T_{eff} and low mass. We do not see such stars; thus the red edge must exclude the low-mass, low- T_{eff} stars from the distribution.

While the observed red edge actually marks the disappearance of pulsations from PWD stars, selection effects change our chances of finding high-mass, high- T_{eff} pulsators. In this case, we expect that the likelihood of finding stars of a given mass within the instability strip will increase the closer those stars get to the red edge. This would tend to render the theoretical blue edge (as defined by the onset of pulsation in models of a given mass) undetectable in real PWD stars—given the small number of known variables. In other words, stars “bunch up” against the red edge owing to their continually slowing rate of evolution, causing the *apparent* blue edge to shadow the slope of the red edge in the $\log g$ – $\log T_{\text{eff}}$ plane. This could explain the approximately linear locus of pulsating stars found within that plane implied by their tight distribution of $\Delta\Pi$.

We are left to explain the observed red edge in terms of the intrinsic properties of the stars themselves, which we defer to § 5.3. First, however, we will discuss the effects of the observed mass distribution along the strip on the period trend seen in Figure 3.

5.2. The Π versus T_{eff} Trend

As mentioned previously (and as we show in § 5.3, below), the depth of an ionization-based driving zone increases—moves to larger thermal timescales—with decreasing T_{eff} for PWD stars of a given mass. This implies a period trend opposite to that observed in Figure 3. Bradley & Dziembowski (1996) discuss this very problem, since their models predict that the maximum period of *unstable* modes should increase as T_{eff} decreases. They suggest that the composition of the driving zone might somehow change with time, or that the stellar radii shrink much more quickly than is currently thought (or some combination of the two), in such a way as to make the maximum unstable period decrease with decreasing T_{eff} . However, no one has yet calculated how (and whether) these suggestions could reasonably account for the observed trend. It is clear, however, that something other than the depth of the driving zone alone determines the observed period range.

What other mechanisms might affect the observed periods? One such mechanism is the changing value of the maximum sustainable g -mode period, Π_{max} . For the ZZ Ceti stars, Π_{max} probably does not influence the period distribution much, since from equations (5) and (6) it increases through the ZZ Ceti instability strip. In PWDs, however, the R dependence in equation (5) must be taken into account; we cannot be certain that the trend implied by a lengthening driving timescale will not find itself at odds with a decreasing Π_{max} .

Figure 5 shows the (arbitrarily normalized) run of Π_{max} versus T_{eff} for three PWD model sequences of different mass. Clearly, Π_{max} decreases significantly as models evolve along all three sequences, with high-mass stars exhibiting a smaller Π_{max} than low-mass stars at all T_{eff} .⁵ These two effects, when combined with the PWD mass distribution, imply that Π_{max} should plummet precipitously with increasing $\log g$ in the models shown in Figure 2. For example, the ratio of Π_{max} for PG 1159 to that of PG 0122 is expected to be $\sim 1.42:1$, while the ratio of their observed dominant periods is $540:400 = 1.35:1$. The period distribution seen in Figure 2 is thus consistent with the idea that the value of Π_{max} determines the dominant period in GW Vir stars. As we will see in the next section, Π_{max} probably also plays an important part in determining the more fundamental question of *when* a given star is likely to pulsate.

5.3. Driving Zone Depth, Π_{max} , and the Red Edge

In § 2.2, we discussed the relationship between the depth of the driving zone and the period of g -mode oscillations. Equation (4) implies that the dominant period should increase in response to the deepening driving zone, as long as other amplitude-limiting effects do not intervene. Figure 5 shows how one particular effect—the decreasing maximum period—might reverse the trend connected to driving zone depth, and the observed periods of GW Vir stars supports the suggestion that Π_{max} is the key factor in setting the dominant period. While Π_{max} limits the range of periods that can respond to driving, τ_{th} in the driving zone

⁵ If this trend continued all the way to the cooler white dwarf instability strips, then ZZ Ceti stars could only pulsate at very short periods, but T_{eff} begins to dominate below around 60,000–70,000 K, pushing Π_{max} back to longer and longer periods once the stars approach their minimum radius at the top of the white dwarf cooling track.

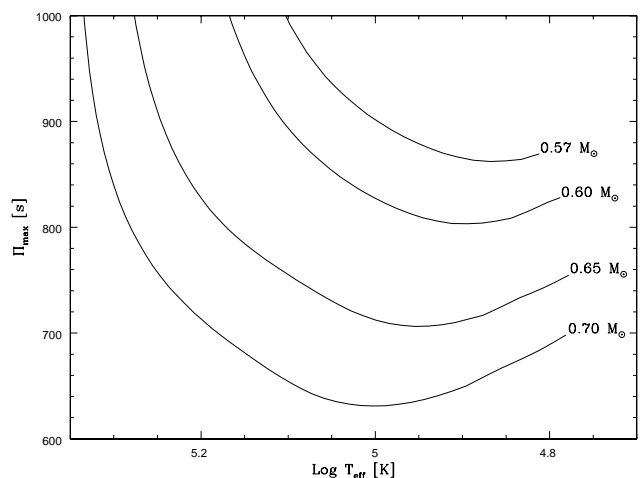


FIG. 5.—Maximum theoretical g -mode period, Π_{max} , independent of the driving period, vs. T_{eff} for four PWD evolution sequences of different mass.

limits the periods that can be driven. However, τ_{th} increases steadily for all GW Vir stars, and Π_{max} decreases steadily. Eventually, therefore, every pulsator will reach a state where $\tau_{\text{th}} > \Pi_{\text{max}}$ over the entire extent of the driving zone. In such a state, the star can no longer respond to driving at all, and pulsation will cease. If Π_{max} remains the most important amplitude-limiting factor for stars approaching the red edge, then the red edge itself could be caused by the situation just described.

We can test this idea by asking if it leads to the kind of mass-dependent red edge we see. To answer this question, we need to know how the depth of the driving zone (as measured by τ_{th}) changes with respect to Π_{max} for stars of various mass. Figure 6 depicts the driving regions for three different model sequences ($M = 0.57, 0.60,$ and $0.63 M_{\odot}$) at three different effective temperatures (144,000, 100,000, and 78,000 K). The driving strength, dk/dr , is determined from equation (3), where k represents the expression inside parentheses. The vertical axis has not been normalized and is the same scale in all three panels. The surface of each model is on the left, at $\tau_{\text{th}} = 0$. The vertical lines in the figure represent Π_{max} , for each model, normalized to 1000 s in the $0.57 M_{\odot}$ model at 144,000 K. We have made no attempt to calculate the actual value of Π_{max} (however, see the Appendix); the important thing is its changing relationship to the depth of the driving zone with changing mass and T_{eff} .

A couple of important trends are clear in the figure. First, the driving zone in models of a given mass sinks to longer τ_{th} , and gets larger and stronger, with decreasing T_{eff} . In the absence of other effects, this trend would lead to ever

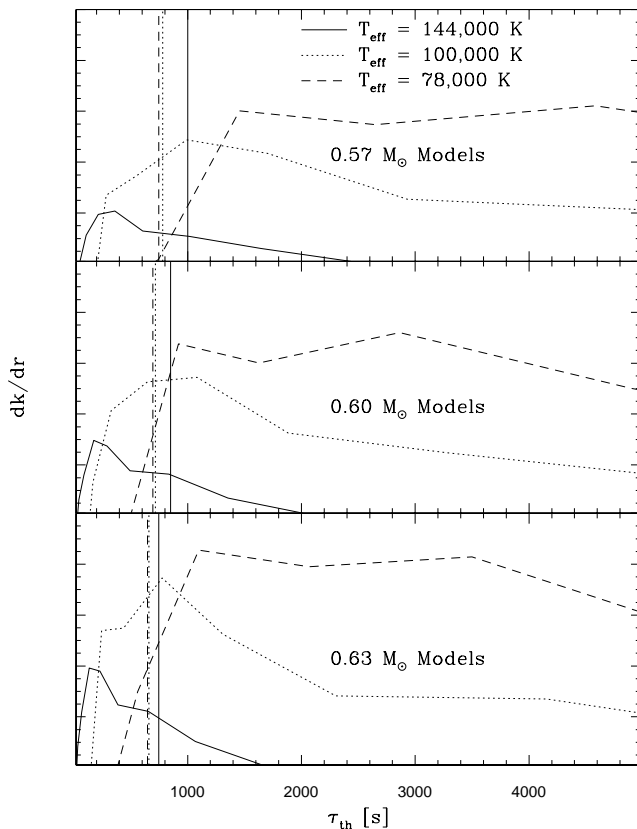


FIG. 6.—Driving strength, $dk/dr > 0$, for three different model sequences at three different effective temperatures. Vertical lines represent the maximum g -mode period at which the star can respond to driving.

increasing periods of larger and larger amplitude as T_{eff} decreases. Meanwhile, Π_{max} changes more moderately, moving toward slightly shorter timescales with decreasing T_{eff} and increasing mass. If we make the somewhat crude assumption that the effective driving zone consists only of those parts of the full driving zone with $\tau_{\text{th}} < \Pi_{\text{max}}$, then a picture of the red edge emerges. At $T_{\text{eff}} = 144,000$ K, the driving zone is relatively unaffected in all three model sequences. As T_{eff} decreases, and the driving zone sinks to longer τ_{th} , the Π_{max} -imposed limit encroaches on the driving zone more and more for every mass. Thus the longer periods, while driven, are eliminated, moving the locus of power to shorter periods than would be seen if Π_{max} was not a factor. Eventually, *all* the driven periods are longer than the maximum period at which a star can respond, and pulsation ceases altogether. This is the red edge.

Pulsations will not shut down at the same T_{eff} in stars of every mass. High-mass models retain more of their effective driving zones than low-mass models at a given T_{eff} . This occurs because at a given T_{eff} , the top of the driving zone moves toward the surface (to smaller τ_{th}) with increasing mass. This effect of decreasing τ_{th} at the top of the driving zone outstrips the trend to lower Π_{max} with increasing mass. The result is that, at 78,000 K, the driving zone in the $0.57 M_{\odot}$ model (Fig. 6, upper panel) has moved to timescales entirely above the Π_{max} limit, while the $0.63 M_{\odot}$ model (appropriate for PG 0122: see the bottom panel in Figure 6) still produces significant driving at thermal timescales below Π_{max} .

Recall from Table 1 and Figure 2 that GW Vir stars of lower T_{eff} have higher mass. Why? We can now give an answer: at low T_{eff} , the low-mass stars have stopped pulsating because they only drive periods longer than those at which they can respond to pulsation! Higher mass stars have shallower ionization zones (at a given T_{eff}) that still drive periods shorter than the maximum allowed g -mode period, even at low T_{eff} . This causes the red edge to move to higher mass with decreasing luminosity and T_{eff} , *the same trend followed by lines of constant period spacing*. The interplay between driving zone depth and Π_{max} enforces the strikingly small range of period spacings ($\Delta\Pi \sim 21.5$ s) seen in Table 1 and Figure 2.

These calculations are not an attempt to predict the exact location of the red edge at a given mass. We are interested at this point only in demonstrating how the position of the red edge is expected to vary with mass, given an ionization-based driving zone with an upper limit placed on it by Π_{max} . In the particular models shown in Figure 6, the top of the driving zone corresponds to the ionization temperature of O VI, but the behavior of the red edge should be the same no matter what species causes the driving. Its absolute location, though, would probably be different given driving by different species. In order to use the location of the red edge for stars of different mass to identify the exact species that accomplishes driving, we would need to calculate Π_{max} precisely for all the models in Figure 6 and better understand exactly how the value of Π_{max} affects the amplitudes of modes nearby in period. Such a calculation is beyond the scope of this paper, and we leave it to future investigations to attempt one.

Alternatively, the discovery of more GW Vir stars would help us better understand these processes by defining the red edge with greater observational precision. The simplest test of our theories would be to find low- T_{eff} GW Vir pulsa-

tors of low mass. If they exist, then Π_{\max} is probably not a factor in determining their periods, and variation should be sought in their light curves at longer periods than in the GW Vir stars found so far; Figure 6 suggests their dominant periods should be in the thousands of seconds. It is possible that these stars could be quite numerous (and *should* be quite numerous if they exist at all, given their slower rate of evolution) and still escape detection, since standard time-series aperture photometry is not generally effective at these timescales. Most current CCD photometers are quite capable of searching for such variability, however. Based on these results, we encourage future studies to determine whether or not low-mass, low- T_{eff} GW Vir pulsators do indeed exist.

6. SUMMARY AND CONCLUSIONS

Our purpose is to understand how and why PWD stars pulsate, so that astronomers can confidently apply knowledge of PWD structure—gained via asteroseismology—to study how white dwarfs form and evolve and better understand the physics that governs these processes. We have pursued PWD structure via the pulsation periods and the additional constraints they provide for our models. A surprising similarity emerged among them: their patterns of average period spacing span a very small range of 21–23 s. Including the PNNV star NGC 1501, this uniformity extends over 3 orders of magnitude in PWD luminosity. Since the average period spacing increases with decreasing luminosity—and decreases with increasing mass—this result implies a trend toward significantly higher mass with decreasing luminosity through the instability strip. This trend has several important implications for our understanding of PWD pulsations.

The instability is severely sloped toward lower T_{eff} with stellar mass increasing down the instability strip. To understand this sloped instability strip, we needed information inherent in the other fundamental observed trend: the dominant period in PWD pulsators decreases with decreasing luminosity. We found that the observed dominant period is correlated with the theoretical maximum g -mode period, Π_{\max} . If Π_{\max} is a factor in determining the range of periods observed in pulsating PWD stars, then it should play a role in determining when pulsation ceases because the driving zone tends to drive longer and longer periods in models of given mass as the models evolve to lower T_{eff} . At low enough temperatures, the driving zone is capable of driving only periods longer than Π_{\max} , and pulsations should then cease. Since the top of the driving zone moves to shorter timescales with increasing mass, and does so faster than Π_{\max} decreases with increasing mass, higher mass models should pulsate at lower T_{eff} than lower mass models. This behavior is compatible with the observed slope of the red edge in the $\log g$ – $\log T_{\text{eff}}$ plane.

This mechanism does not account for the lack of observed high-mass pulsators at high T_{eff} , however. Theoretical luminosity functions for the PWD stars indicate the observed blue edge is probably significantly affected by selection effects due to rapid evolution at high T_{eff} . Since higher mass models are both less numerous and less luminous at a given T_{eff} than models of lower mass, we are more likely to detect low-mass than high-mass PWDs at a given temperature. This will cause stars to “bunch up” against the red edge in the observed instability strip, and the apparent blue edge will thus “artificially” resemble the shape of

the red edge—no matter what the true shape might be. This selection effect, in isolation, would imply that low-mass, low- T_{eff} GW Vir pulsators should be most numerous of all. That we in fact find *none* strengthens our contention that the mass dependence of the red edge is a real effect—we find no low-mass, low- T_{eff} GW Vir stars because they do not exist.

The boundaries of the PWD instability strip derived from our pulsation studies are far smaller than those based on spectroscopic measurements alone. Though the actual range of $\log g$ and T_{eff} spanned by the known pulsators is no smaller than before, the newly discovered mass-luminosity relationship implies that the width of the strip in T_{eff} at a given $\log g$ is quite small, and vice versa. We can therefore no longer say for certain that *any* nonpulsators occupy this newly diminished instability strip, since the uncertainties in $\log g$ and T_{eff} as determined from spectroscopy are larger now than the observed width of the strip itself at a given $\log g$ or T_{eff} .

This highlights the importance of finding additional pulsators with which to refine our knowledge of the instability strip boundaries further. In particular, we still have no observations with which to constrain theories of the blue edge, since there is no reason that the *intrinsic* blue edge lies near the *observed* blue edge at any effective temperature.

If the trend we have discovered continues down to effective temperatures below the coolest known pulsating PWDs, then high-mass ($\sim 1 M_{\odot}$ or greater) white dwarfs might pulsate at temperatures as low as 50,000–60,000 K (and $\log g \sim 8$). Their dominant periods (again, assuming they follow the trends outlined in §§ 3 and 5) would be shorter than any known PWDs, perhaps as low as 200–300 s. Such stars would not be PWDs at all but rather white dwarfs proper. Their discovery would complete a “chain” of variable stars from PNN stars to “naked” PWDs to hot white dwarfs, and as such they would represent an incalculable boon to astronomers who study the late stages of stellar evolution.

On the theory side, we need to understand how PWD stars react to driving near the maximum g -mode period. Studies should be undertaken to determine the actual maximum period in PWD models. PWD evolution sequences should be constructed that contain all the elements thought to exist in PWD stars. We have observational information to test all these calculations, and with the observational program proposed above, we will gain more. There is much to do.

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APPENDIX A

THE MAXIMUM PERIOD

A variable pre-white dwarf star is a resonant cavity for nonradial g -mode oscillations. This spherically symmetric cavity is bounded by the stellar center (or the outer edge of the degenerate core in ZZ Ceti and DBV white dwarfs) and surface. At sufficiently long periods, however, the surface layers no longer reflect internal waves. The pulsation energy then leaks out through the surface, damping the pulsation. This idea was first applied to white dwarf pulsations by Hansen, Winget, & Kawaler (1985), who attempted to calculate the approximate critical frequencies to explain both the red edge and maximum observed periods in ZZ Ceti stars. Assuming an Eddington gray atmosphere, they derive the following expression for the dimensionless critical g -mode frequency:

$$\omega_g^2 \approx \frac{\ell(\ell + 1)}{V_g}, \quad (\text{A1})$$

where

$$V_g = \frac{3g\mu R}{5N_a k T_{\text{eff}}}. \quad (\text{A2})$$

Here g and R represent the photospheric surface gravity and radius, and N_a , k , and μ have their usual meanings.

The dimensionless frequency, ω , is related to the pulsation frequency, σ , according to

$$\omega^2 = \frac{\sigma^2 R}{g}. \quad (\text{A3})$$

The pulsation period is $\Pi = 2\pi/\sigma$, so combining equations (A1)–(A3) we arrive at the maximum g -mode period:

$$\Pi_{\text{max}} \approx 940 \text{ s} \left[\frac{\mu}{\ell(\ell + 1)} \right]^{0.5} \left(\frac{R}{0.02 R_\odot} \right) \left(\frac{T_{\text{eff}}}{10^5 \text{ K}} \right)^{-0.5}, \quad (\text{A4})$$

or, using the relation $R^2 = L/4\pi\sigma T_{\text{eff}}^4$,

$$\Pi_{\text{max}} \approx 940 \text{ s} \left[\frac{\mu}{\ell(\ell + 1)} \right]^{0.5} \left(\frac{L}{35 L_\odot} \right)^{0.5} \left(\frac{T_{\text{eff}}}{10^5 \text{ K}} \right)^{-2.5}. \quad (\text{A5})$$

For PG 1159, with $L = 200 L_\odot$ and $T_{\text{eff}} = 140,000 \text{ K}$, equation (A5) predicts $\Pi_{\text{max}} = 850 \text{ s}$ for $\ell = 1$ modes and 492 s for $\ell = 2$ modes.⁶ PG 0122, with $L = 5.6 L_\odot$ and $T_{\text{eff}} = 80,000 \text{ K}$, would have $\Pi_{\text{max}} = 580 \text{ s}$ for $\ell = 1$ and 340 s for $\ell = 2$. Because of the simplicity of the gray atmosphere assumption, these numbers are more useful in comparison to each other than as quantitative diagnostics of the maximum period. For instance, Hansen, Winget, & Kawaler (1985) find that the values of Π_{max} derived from this analysis are “within a factor of 2” of those based on more rigorous calculations. We are more interested here in the run of Π_{max} with respect to global stellar quantities such as L and T_{eff} .

If Π_{max} determines the long-period cutoff in GW Vir pulsators, then the longest period $\ell = 1$ modes should be (approximately) a factor of 1.73 times longer than the longest period $\ell = 2$ modes. PG 1159 is the only GW Vir star with positively identified $\ell = 2$ modes. In the period list of Winget et al. (1991), the longest period (positively identified) $\ell = 2$ mode has a period of 425 s, while the longest period $\ell = 1$ mode has a period of 840 s. The ratio of these two periods is 1.98, close to the predicted ratio. The periods themselves are also surprisingly close to the calculated values.

The predicted ratios between the longest period modes also hold for intercomparison of different stars. The longest period mode so far identified for PG 0122 is 611 s, 1.37 times smaller than the longest period in PG 1159, again very close to the predicted ratio of 1.43 from equations (A4) and (A5). The agreement between Π_{max} from our rough calculations and the observed maximum periods in GW Vir stars is impressive enough to warrant further study. In particular, more rigorous theoretical calculations of Π_{max} should be undertaken to corroborate or refute these results.

⁶ Assuming a mix of C:O:He = 0.4:0.3:0.3 by mass, implying $\mu = 1.59$.

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