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A DETECTION OF THE EVOLUTIONARY TIME SCALE OF THE DA WHITE DWARF G117−B15A WITH THE WHOLE EARTH TELESCOPE

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ABSTRACT

We have detected the time rate of change for the main pulsation period of the 13,000 K DA white dwarf G117−B15A, using the Whole Earth Telescope (WET). The observed rate of period change, $\dot{P} = (12.0 \pm 3.5) \times 10^{-15}$ s s^{-1}, is somewhat larger than the published theoretical calculations of the rate of period change due to cooling, based on carbon core white dwarf models. We discuss other effects that could contribute to the observed rate of period change.

Subject headings: stars: evolution — stars: variables — stars: white dwarfs

1. INTRODUCTION

G117−B15A (≡RY LMi) is one of the hottest of the pulsating DA (hydrogen atmosphere) white dwarf (WD) stars, the DAV or ZZ Ceti stars (McGraw 1979). It was found to be a variable by McGraw & Robinson (1976), and its light curve was deciphered by Kepler et al. (1982). Its effective temperature is $T_{\rm eff} \approx 13{,}200$ K (Wesemael, Lamontagne, & Fontaine 1986; Daou et al. 1990). Because the DAVs are normal stars except for their variability (Robinson 1979), and because most DA WDs are photometric variables in the temperature range $13{,}200 \gtrsim T_{\rm eff} \gtrsim 11{,}500$ K (Fontaine et al. 1982; Greenstein 1982), it is likely that the DAVs are broadly representative of *all* DA WDs.

Because the time rate of change of the pulsation period is directly related to the evolutionary time scale of that star, we have been working since 1975 to obtain a measurement of the rate of period change with time ($\equiv \dot{P}$) for the $P = 215$ s periodicity in G117−B15A, most recently obtaining a limit of $\dot{P} = (8.3 \pm 5.0) \times 10^{-15}$ s s^{-1} using all the high-speed photometric data obtained from 1975 to 1989 (Kepler et al. 1990). Two other DAV stars have been analyzed for measurement of the rate of period change: R548 (Stover et al. 1980; Tomaney 1987), and L19-2 (O'Donoghue & Warner 1987). The best limit on R548 is for the 213 s pair, $\dot{P} = (0.8 \pm 19.2) \times 10^{-15}$ s s^{-1} (Tomaney 1987), where this value is twice that quoted in his paper because his $\dot{P}$ definition was nonstandard (see § 3). For L19-2, the best limit is for the 113 s pair, $\dot{P} = (1.5 \pm 2.0) \times 10^{-14}$ s s^{-1}.

Kepler et al. (1990) estimated that it would take 4 years or more to obtain a *detection* of $\dot{P}$, if they continued to obtain data with the sparse, single-site coverage that was the norm up to then—an estimate based on the assumption that the uncertainty in the measurement of $\dot{P}$ decreases in proportion to the square of the time spanned by the observations. However, because the accuracy of an individual timing is also proportional to the square root of the number of photons observed, a substantial improvement in the photon statistics also decreases the time span needed to detect a significant change in the period.

Considering the detection of an evolutionary time scale for a DAV to be extremely important, we decided to include G117−B15A in the target list for the Whole Earth Telescope (WET) observations scheduled for 1990 March (Nather et al. 1990). In practice, the high signal-to-noise ratio data obtained at the 3.6 m CFHT proved to be crucial to our project, and the new data improved the photon statistics enough to allow the detection we report here.

The evolutionary time scale of a WD star is extremely sensitive to the core composition, and because the rate of period change of a *g*-mode nonradial pulsation reflects directly the evolutionary time scale of that star, an observed value of $\dot{P}$ can be interpreted as a measure of the mean core composition (see Robinson & Kepler 1980; Kepler et al. 1990), if cooling is the sole cause of the observed rate of change.

This is the first reported detection of an evolutionary time scale for a cool WD. The only other similar measurement is for the 140,000 K pre–WD PG 1159−035 (Werner, Heber, & Hunger 1991) by Winget et al. (1985, 1991). The measurement of the evolutionary time scales for WDs is an essential cali-

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bration of the method of using the WD luminosity function to derive the age of the disk of the Galaxy (Winget et al. 1987; Wood 1990a, b).

2. OBSERVATIONS

We obtained 86.4 hr of high-speed photometry in 1990 March, using either two-star (Nather 1973) or three-star (Vauclair, Chevreton, & Dolez 1987; Nather et al. 1990) photometers on telescopes with sizes from 0.6 to 3.60 m in diameter, distributed in global longitude to obtain as nearly continuous observations as possible. The journal of observations for the new data is given in Table 1. All the observations were obtained with unfiltered light, first because G117−B15A is faint ($V = 15.52$; Eggen & Greenstein 1965), and most important because the light variations are nonradial g-mode pulsations, and therefore in phase at all wavelengths (Kepler 1984).

We reduced and analyzed the data as described in Kepler et al. (1982). We computed the Fourier transforms of each individual light curve, verifying that the main pulsation at 215 s dominated each data set and was stable in amplitude with time. We next constructed a Fourier transform of the *total* data set. There was no contamination of the main pulsation by any other periodicity to the detection limit of ~1 mmag.

3. OBSERVATIONAL RESULTS

After concluding that the 215 s pulsation mode has remained stable in both frequency and amplitude since our first observations in 1975, we calculated the time of maximum for each new run separately by fitting a single sine curve with a period of 215.19738 s to the light curve, and included the new measurements in the $(O-C)$ diagram with the data sets reported in Kepler et al. (1990). The rate of period change for the pulsation, as well as a correction to the period and epoch, was then obtained by fitting a parabola to the $(O-C)$ diagram; i.e., we assume a periodic variation of the form $T_{\max} = E_0 + PE$, where P is the period at E_0, and E is the epoch (cycle number from E_0). See Figure 1.

TABLE 1

JOURNAL OF OBSERVATIONS ON G117−B15A

Observatory and Telescope (m)	Run	BJDD$_{\max}$ −2,440,000	$\sigma_{\max}$ (s)	Amplitude (mmag)	$\sigma_{\rm amp}$ (mmag)
McDonald 2.1	sjk54	7856.832697	1.6	17.9	0.8
McDonald 1.0	sjk72	7918.644630	2.9	16.3	1.2
McDonald 1.0	sjk74	7920.619811	3.1	19.3	1.6
McDonald 1.0	jcc138	7952.622834	2.7	24.8	1.8
McDonald 2.1	pab22	7972.620899	6.0	17.8	3.1
McDonald 2.1	pab25	7973.709340	2.4	11.3	0.8
McDonald 2.1	pab26	7973.741682	1.0	19.6	0.6
OHP 1.9	gv63	7977.403038	2.1	23.0	1.4
OHP 1.9	gv65	7978.327055	3.1	20.7	1.9
McDonald 2.1	pab30	7978.601069	2.3	19.4	1.3
Mauna Kea 0.6	a214	7978.770467	1.9	19.3	1.0
Wise Observatory 1.0	ren76	7979.281057	2.7	19.4	1.6
OHP 1.9	gv67a	7979.358189	3.2	21.8	2.0
OHP 1.9	gv67b	7979.358145	4.8	17.4	2.4
Mauna Kea 0.6	a216	7979.781717	2.9	20.0	1.7
Wise Observatory 1.0	ren78	7980.224899	2.7	17.6	1.4
OHP 1.9	gv69	7980.319627	1.2	20.5	0.7
McDonald 2.1	pab34	7980.621017	3.3	15.8	1.5
Mauna Kea 0.6	a219	7980.782929	2.1	19.8	1.2
OHP 1.9	gv73	7981.325918	0.97	20.2	0.6
McDonald 2.1	pab35	7981.592393	0.92	20.8	0.6
CFHT 3.6	fbvl	7981.779185	0.47	21.1	0.3
OHP 1.9	gv77	7982.329663	1.5	22.6	1.0
CFHT 3.6	fbv3	7982.743093	0.70	20.2	0.4
CFHT 3.6	fbv5	7983.734400	0.64	20.9	0.4
CFHT 3.6	fbv7	7984.735678	0.47	22.0	0.3

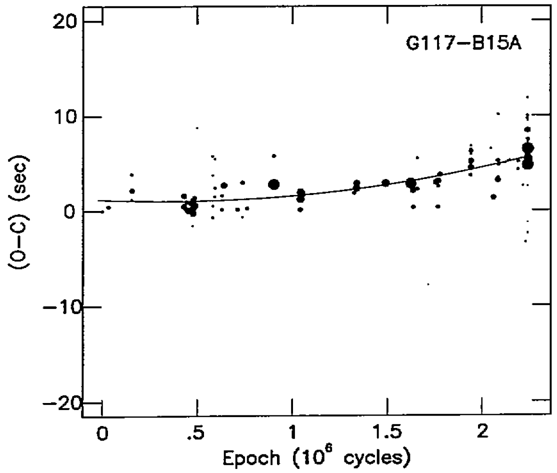

FIG. 1.—The $(O-C)$ diagram for the 215 s pulsation of G117−B15A. The observed times of maxima are listed in Table 1, and the calculated times of maxima were derived using a linear ephemeris starting on BJED 2,442,397.917520. The size of a point is inversely proportional to the error of that point, so the larger the point, the higher its weight. For the largest points, their error bars are smaller than the size of the point itself. The line is the best-fit parabola, with $\dot{P} = 12 \times 10^{-15}$ s s^{-1}.

If the period is changing slowly with time, we may expand $T_{\max}$ in a Taylor series, discarding terms beyond the quadratic:

$$T_{\max} = T_{\max}\Big|_{E_0} + \frac{dT_{\max}}{dE}\Big|_{E_0}(E - E_0) + \frac{1}{2}\,\frac{d^2T_{\max}}{dE^2}\Big|_{E_0}(E - E_0)^2 \,. \tag{1}$$

Writing

$$\frac{d^2T_{\max}}{dE^2} = \frac{dP}{dE} = \frac{dt}{dE}\,\frac{dP}{dt} = P\,\frac{dP}{dt}\,, \tag{2}$$

expanding the quadratic term and assuming $2E_0 \ll E$, we get

$$T_{\max} = T_{\max}^0 + PE + \tfrac{1}{2}P\dot{P}E^2 \,. \tag{3}$$

If we next define: $O \equiv T_{\max}^{\rm obs} = T_{\max}$, and $C \equiv T_{\max}^1 + P_1E$, we get

$$(O-C) = \Delta E_0 + \Delta PE + \tfrac{1}{2}P\dot{P}E^2 \,, \tag{4}$$

where $\Delta E_0 = (T_{\max}^0 - T_{\max}^1)$, and $\Delta P = (P - P_1)$.

By fitting equation (4) to the $(O-C)$, we obtained a new value for the epoch of maximum, $T_{\max}^0 = 2{,}442{,}397.917521$ BJDD $\pm$ 0.6 s, a new value for the period, $P = 215.197387 \pm 0.000001$ s, and most importantly, a rate of period change of

$$\dot{P} = (12.0 \pm 3.5) \times 10^{-15}\ {\rm s\ s^{-1}} \,. \tag{5}$$

The observed rate of period change implies a *time scale* of period change of

$$\tau = P/\dot{P} = (5.7 \pm 1.7) \times 10^8\ {\rm yr} \,. \tag{6}$$

To make sure the parabola is significant, we compared the variance of the parabolic fit with the variance obtained by fitting only a straight line to the $O-C$, i.e., by assuming the period is *not* changing with time. The variance of the fit

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without the quadratic term is 4.2 times larger, which means there is one chance in 10^{33} that the quadratic term is not necessary (Pringle 1975). This procedure, of course, assumes that the deviations of the data points from the parabola are randomly distributed, which might not be the case here; the accuracy of measurement has increased more slowly, in practice, than we would expect from equation (4).

4. THEORETICAL CONSIDERATIONS

From the theory of stellar pulsations, we know that the period of a given mode changes in response to changes in the structure of the star caused by its evolution. The theory of nonradial g-mode pulsations in WDs shows that the time scale of period change is related to changes in the radius and temperature of the star, $\dot{P}/P \approx -a(\dot{T}/T) + b(\dot{R}/R)$, where the constants a and b are of the order of unity, and where T is the temperature *in the region of period formation* (Winget, Hansen, & Van Horn 1983). For the DAV stars, which cool at nearly constant radius, $\dot{R}/R \ll \dot{T}/T$; the time scale of period change is essentially a measurement of the evolutionary time scale of the physical properties of the WD in the region of period formation—dP/dt depends directly upon the global cooling time scale.

We compare the observed value of $\dot{P}$ with the range of theoretical values derived from models with carbon cores, subject to g-mode pulsations in the temperature range of G117−B15A. More realistic models with C/O cores are not yet available. The adiabatic pulsation calculations of Winget (1981), Wood & Winget (1988), Bradley et al. (1989), Bradley & Winget (1991), and Brassard et al. (1991), which allow for mode trapping, give $\dot{P} \simeq 2\text{–}5 \times 10^{-15}$ s s^{-1} for the low-l, low-k oscillations observed—considerably smaller than the observed $\dot{P}$.

The mass of G117−B15A can be obtained by comparing the surface gravity derived from the observed optical spectra (Daou et al. 1990) and the evolutionary models of Wood (1990a). The observed $\log g = 7.81 \pm 0.06$ implies a mass of $M = 0.49 \pm 0.03\ M_\odot$.

For a given mass and internal temperature distribution, theoretical models show that the rate of period change increases if the mean atomic weight of the core is increased, for models which have not yet crystalized in their interiors. Effects not yet included in the available models (e.g., mode trapping feedback, or nonlinear mode interactions) might also have a significant effect, but we are not yet in a position to describe them quantitatively. Finally, the presence of an orbital companion could contribute to the period change we have detected.

4.1. *Core Composition*

Since the heavier the particles that compose the nucleus of the WD, the faster it cools, the large observed value of $\dot{P}$ seems to imply a composition heavier than the pure carbon core models which have been calculated. The best estimate of mean atomic weight A of the core comes from the comparison of the observed $\dot{P}$ with values from an evolutionary sequence of WD models. Brassard et al. (1991) computed the rates of period changes for 800 evolutionary models with various masses, all with carbon cores but differing He/H surface layer masses, obtaining values similar to those of Winget (1981), Wood & Winget (1988), Bradley et al. (1989), and Bradley & Winget (1991). The average value of $\dot{P}$ for all $l = 1$, 2 and 3 modes with periods around 215 s in models with an effective temperature around 13,000 K, and a mass of 0.5 $M_\odot$, is $\dot{P}(\text{C core}) = (4.3 \pm 0.5) \times 10^{-15}$ s s^{-1}. Using a Mestel-like cooling law (Mestel 1952; Kawaler et al. 1986), i.e., $\dot{T} \propto A$, where A is the mean atomic weight in the core, we can write

$$\dot{P}(A) = 4.3 \times 10^{-15}\,\frac{A}{12}\ \mathrm{s\ s^{-1}}\,, \tag{7}$$

which, when compared to the observed $\dot{P}$, implies $A_{\text{core}} = 33 \pm 11$.

As the evolutionary model cools, its nucleus crystallizes due to Coulomb interactions between the ions (Lamb & Van Horn 1975), and crystallization slows down the cooling by the release of latent heat. We must be sure our assumption of a Mestel-like cooling law is still valid for heavier core compositions. From the WD evolutionary model sequences of Wood (1990a), we can show that a 0.5 $M_\odot$ white dwarf with even a core composition of pure ^{32}S would not begin to crystallize until after it had cooled through the DAV instability strip. We conclude that the Mestel-like cooling law assumed in the derivation of equation (8) is valid.

It is important to note that the observed value of $\dot{P}$ is consistent, within 2 σ, with the $\dot{P}$ derived from models with pure carbon cores.

4.2. *Reflex Motion*

When a star has an orbital companion, the variation of its line-of-sight position with time produces a variation in the time of arrival of the pulsation maxima, by changing the light travel time between the star and the observer. Because G117−B15A has a proper motion companion 15″ away (G117−B15B), it is important to calculate the effect of this companion under the assumption that they make a physical pair.

The Doppler shift $\Delta P_{\text{pul}} = (P_{\text{pul}} - P_{\text{obs}})$ on the pulsation period will be $\Delta P_{\text{pul}}/P_{\text{pul}} = v_r/c$, where c_r is the radial velocity of the pulsating star, and c is the speed of light. Taking the derivative with respect to time,

$$\dot{P}_{\text{pul}} = \frac{dP_{\text{pul}}}{dt} = \frac{P_{\text{pul}}}{c}\,\frac{dv_r}{dt}\,. \tag{8}$$

Defining i as the angle between the plane of the sky and the plane of the orbit (inclination angle), and θ as the position angle in the orbit, measured from the line-of-sight direction, the observed radial velocity is then $v_r = v_{\text{orb}} \cos\theta \times \sin i$. The maximum value of the radial velocity variation occurs for an edge-on system ($i = 90°$), and for $\theta = 0$ (tangential velocity = 0)

$$\left(\frac{dv_r}{dt}\right)_{\max} = v_{\text{orb}}\,\frac{2\pi}{P_{\text{orb}}} = \frac{2\pi r_1}{P_{\text{orb}}}\,\frac{2\pi}{P_{\text{orb}}}\,, \tag{9}$$

where r_1 is the distance between the pulsating star and the system center of mass, and P_{orb} is the orbital period. Using Kepler's third law: $GM_T = a^3(2\pi/P_{\text{orb}})^2$, where a is the separation between the two stars, M_T is the total mass of the system, $r_1 = (M_2/M_T)a_T$, and M_2 is the mass of the companion star, we obtain

$$\dot{P} = \frac{P_{\text{pul}}}{c}\,\frac{GM_2}{a_T^2} = 1.97 \times 10^{-11} P_{\text{pul}}\,\frac{M_2/M_\odot}{(a_T/AU)^2}\ \mathrm{s\ s^{-1}}\,. \tag{10}$$

According to Eggen & Greenstein (1965), the star G117−B15B is a dM2 dwarf, with $V = 16.10$, and $(B-V) = 1.60$. The distance to G117−B15A is (60 ± 17) pc, using an average distance derived from parallax and absolute magnitudes obtained from the observed *UBV*, *uvby*, and MCSP colors given in McCook & Sion (1987), which implies a total separation between the stars G117−B15A and G117−B15B of $a = 925$ AU. Considering that a dM2 star has a mass of $M_2 \sim 0.39\ M_\odot$ (Allen 1973), we obtain a contribution to $\dot{P}$ caused by reflex orbital motion of $\dot{P} \lesssim 1.9 \times 10^{-15}$ s s^{-1}, small enough to be lost in the errors of measurement. Even if the orbit is highly eccentric, and G117−B15A is near periastron, the orbital velocity could not be higher than twice that derived above, or it would exceed escape velocity. In the above derivation we have assumed the orbit to be nearly edge on to give the largest effect possible. Therefore $\dot{P}_{\rm orb} \lesssim 3.8 \times 10^{-15}$ s s^{-1}. We conclude that even if the pair is related physically, the contribution of an orbital reflex motion to the observed $\dot{P}$ is *not* dominant. This value is also not enough, if added to the carbon core theoretical $\dot{P}$, to reach the observed rate of period change within 1 σ, but it could be responsible for a significant part of the observed $\dot{P}$. The observed rate of period change *could*, however, be explained, for example, by a planet of Jupiter's mass orbiting the WD at a distance of 32.6 AU, which corresponds to an orbital period of 263 yr, or a smaller planet on a closer orbit. Note that reflex motion produces *sinusoidal* variations on the $(O-C)$, which are distinguishable from parabolic variations after a significant portion of the orbit has been covered. The presence of a planet around the WD is unlikely, however, since it would have had to survive while the current white dwarf was in its red giant phase.

5. CONCLUSION

We have observed a rate of period change with time for the 215 s pulsation of the DAV star G117−B15A of $\dot{P} = (12.0 \pm 3.5) \times 10^{-15}$ s s^{-1}, the first such measurement for a cool (13,000 K) WD. The observed value is of the same order of magnitude as that predicted by the theoretical calculations of the cooling of the white dwarf.

Assuming that the observed $\dot{P}$ is entirely due to cooling of the WD, and extrapolating from the theoretical $\dot{P}$ values of the most realistic theoretical models available implies a core composition with a mean atomic weight $A = 33 \pm 11$, which is within 2 σ of the carbon core white dwarf models.

These results suggest a number of additional avenues of research. First, it is *crucial* that we continue to monitor G117−B15A with large telescopes during the next few observing seasons, to better determine the magnitude of $\dot{P}$ and the errors of measurement. Second, the possible nonlinear (feedback) effects on $\dot{P}$ associated with mode trapping and mode interactions should be studied in detail, to see if they can contribute significantly to the rate of period change we have detected. Third, we must obtain $\dot{P}$ measurements for other DAV stars; currently, the most promising candidates are ZZ Ceti (R548) itself, and L19-2. Fourth, we should determine if and under what circumstances the progenitor of a 0.5 $M_\odot$ WD can evolve to give a core composition heavier than C/O. Finally, these results indicate that evolutionary models with core compositions heavier than carbon should be computed.

Now that we seem on the verge of a significant measurement of $\dot{P}$ for G117−B15A, we must consider whether the $\dot{P}$ we measure is due entirely to cooling, which was the original motivation for doing the measurements, or whether other effects may contribute. Only when we have eliminated significant contributions from all known effects can we be comfortable in assigning the observed change to cooling (and hence deduce the core composition from its value). We must also take into account the apparent non-Gaussian nature of the noise in assigning an error to the measurement.

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