1. Consider scattering of a particle of mass $m$ and momentum $k$ on the Yukawa potential, $V(r) = ge^{-\mu r}/r$, in the Born approximation. Show that

$$
\sigma_{\text{Yukawa}} = 16\pi r_0^2 \left( \frac{gm r_0}{\hbar^2} \right)^2 \frac{1}{1 + 4k^2 r_0^2}
$$

where $r_0 = 1/\mu_0$ is the range. Compare $\sigma$ to the geometrical cross-section associated with this range.

2. Consider scattering of a particle of mass $m$ and momentum $k$ on the potential $V(r) = -V_0 \Theta(r_0 - r)$, where $\Theta$ is the Heaviside function. Use the first Born approximation.

(a) Show that

$$
\frac{d\sigma}{d\Omega} = 4r_0^2 \left( \frac{V_0 mr_0^2}{\hbar^2} \right)^2 \frac{2 (\sin(q r_0) - q r_0 \cos(q r_0))^2}{q^6 r_0^6}
$$

where $hq$ is the momentum transfer.

(b) Show that as $k r_0 \to 0$, the scattering becomes isotropic and

$$
\sigma \approx \frac{16}{9} \pi r_0^2 \left( \frac{V_0 mr_0^2}{\hbar^2} \right)^2
$$

3. Compute the $s$-wave phase shift for the elastic scattering of a spinless particle on a repulsive spherical $\delta$-shell potential

$$
V(r) = \frac{\hbar^2}{2\mu} \lambda \delta(r - a), \quad \lambda > 0.
$$

4. Consider a hard sphere represented by the potential $V(r) = \infty$ for $r < r_0$ and zero otherwise. Find the expression for the total scattering cross section. Show that at small energies the $l = 0$ partial scattering cross section dominates.

*Hint:* The asymptotic behavior of the spherical Bessel and Neumann functions is as follows:

$$
\begin{align*}
\hat{j}_l(\rho) &\xrightarrow{\rho \to 0} \frac{\rho^l}{(2l + 1)!!} & \hat{n}_l(\rho) &\xrightarrow{\rho \to 0} - \frac{(2l - 1)!!}{\rho^{l+1}} \\
\hat{j}_l(\rho) &\xrightarrow{\rho \to \infty} \frac{1}{\rho} \sin(\rho - l \pi/2) & \hat{n}_l(\rho) &\xrightarrow{\rho \to \infty} - \frac{1}{\rho} \cos(\rho - l \pi/2)
\end{align*}
$$