1. Use perturbation theory to show that the interaction between two hydrogen atoms in their ground (unperturbed) states is attractive at large internuclear separations and proportional to $R^{-6}$, where $R$ is the distance between the atomic nuclei.

2. A hydrogen atom is in the ground state at $t = -\infty$. An electric field $\mathbf{E}(t) = E \hat{\mathbf{z}} e^{-t^2/\tau^2}$ is applied until $t = \infty$. Show that the probability that the atom ends up in any of the $n = 2$ states is, to first order,

$$P(n = 2) = \frac{e^2 E^2 \tau^2}{3 \hbar^2} e^{-\omega^2 \tau^2/2}$$

where $\omega = (E_{2\ell m} - E_{100})/\hbar$. Does the answer depend on whether or not we incorporate spin in the picture?

3. Consider the $\beta$ decay of the tritium atom $^3\text{H}$ (with nucleus consisting of two neutrons and one proton) into the helium ion $^3\text{He}^+$ (with nucleus of this particular isotope consisting of two protons and one neutron). Assume that the electron emitted by the $^3\text{H}$ nucleus has a kinetic energy of 16 keV. Argue that the sudden approximation may be used to describe the response of an electron that is initially in the $1s$ state of $^3\text{H}$. Show that the amplitude for it to end up in the ground state of $^3\text{He}^+$ is $16\sqrt{2}/27$. What is the probability for it to be in the state $|n = 16, l = 3, m = 0\rangle$ of $^3\text{He}^+$?

4. Consider a system subject to a perturbation $H^{(1)}(t) = \tilde{H}^{(1)}(t)$, where $\tilde{H}^{(1)}$ is an operator independent of time. Show that if at $t = 0^-$ the system is in the state $|i(0)\rangle$, the amplitude to be in a state $|f(0)\rangle$ at $t = 0^+$ is, to first order,

$$d_f = \frac{1}{i\hbar} \langle f(0) | \tilde{H}^{(1)} | i(0) \rangle, \quad f \neq i.$$ 

Notice that (1) the state of the system does change instantaneously; (2) Even though the perturbation is “infinite” at $t = 0$, we can still use first-order perturbation theory if the “area under it” is small enough.