1. (a) Define a conservative force field in terms of the properties of the work integral between two arbitrary points \( r_1 \) and \( r_2 \).

(b) Show that if a force field is expressible as a minus gradient of a scalar function, \( F = -\nabla U(r) \), then the work integral over a closed path is zero: \( \oint F \cdot dr = 0 \).

(c) Show that the theorem formulated in (b) implies that if \( F = -\nabla U(r) \), then \( F \) is conservative.

2. Consider a rigid body rotating with an angular velocity \( \omega \) about a fixed axis. Take the axis of rotation to be the \( z \) axis and use cylindrical coordinate system.

(a) Show that the linear velocity of a point within the body is \( \rho \alpha \omega \) in the direction of the unit vector \( \hat{\phi} \), where \( \rho \alpha \) is the cylindrical radius of this point.

(b) Now connect a small mass \( dm_\alpha \) with this point. Show that the \( z \) component of the angular momentum \( L_\alpha \) of this mass is \( (l_\alpha)_z = dm_\alpha \rho_\alpha^2 \omega \).

Hint: In cylindrical coordinates \( r = \rho \hat{\rho} + z \hat{z} \).

(c) Show that the \( z \) component of the total angular momentum of the body can be written as \( L_z = I \omega \), where \( I \) is the moment of inertia of the body about the \( z \) axis: \( I = \int \rho^2 dm \) (\( \rho \) here is still the cylindrical radius, i.e., the distance of \( dm \) from \( z \)).

3. A simple pendulum consists of a mass \( m \) suspended from a fixed point in Earth’s gravitational field (acceleration \( g \)) by a weightless rigid rod of length \( l \). The pendulum moves in a viscous medium with retarding force \( -2m \sqrt{g/l} v \), where \( v \) is the velocity.

(a) Write down the equation of motion and then an approximation to it in the limit of small oscillations (\( \text{Hint: } \sin x \approx x \) for small \( x \)).

(b) Solve the approximate equation of motion.

(c) What kind of motion will one observe for this particular combination of the natural oscillator frequency and the given retarding force?