Numerical Modeling of Plasmas: Magnetic Reconnection Magnetic Explosions

Michael Shay
University of Maryland
http://www.glue.umd.edu/~shay/presentations
Overview

• What is Reconnection?
• How do you simulate it?
Part I: What is Reconnection?
What is a Plasma?

<table>
<thead>
<tr>
<th>Solid</th>
<th>Liquid</th>
<th>Gas</th>
<th>Plasma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>Example</td>
<td>Example</td>
<td>Example</td>
</tr>
<tr>
<td>Ice H₂O</td>
<td>Water H₂O</td>
<td>Steam H₂O</td>
<td>Ionized Gas</td>
</tr>
<tr>
<td>Cold T&lt;0°C</td>
<td>Warm 0&lt;T&lt;100°C</td>
<td>Hot T&gt;100°C</td>
<td>Hotter T&gt;100,000°C</td>
</tr>
<tr>
<td>Molecules Fixed in Lattice</td>
<td>Molecules Free to Move</td>
<td>Molecules Free to Move, Large Spacing</td>
<td>Ions and Electrons Move Independently, Large Spacing</td>
</tr>
</tbody>
</table>

H₂ → H⁺ + H⁺ + + 2e⁻
The Sun is a Big Ball of Plasma

Solar Flare
1971 October 10

Big Bear Solar Observatory

http://science.msfc.nasa.gov/ssl/pad/solar/flares.htm
Space Weather

- Plasma streams away from the sun and hits the Earth.
  - Astronaut safety.
  - Satellite disruptions.
  - Communication disruptions.
Unlimited Clean Energy: Fusion

- Hydrogen gas must have:
  - Very high temperature and density.
- Plasma
Fusion 1: Tokamaks

- Compress and heat the plasma using magnetic fields.
Fusion 2: Laser Fusion

- Compress and Heat the plasma with multiple lasers
Outside the Solar System

- Clumps of matter gradually compress due to gravity and heat.
  - Star formation.

Eagle Nebula
Accretion Disks

- When matter collects onto an object, it tends to form a disk.
- Difficult for matter to accrete:
  - Plasma Turbulence is key.
The Wide Range of Plasmas
A Normal Gas (non-plasma)

- All dynamics is controlled through sound wave physics (Slinky Example).
Plasmas are More Complicated
Magnetic Fields

• Wave a magnet around with a plasma in it and you will created wind!
• In fact, in the simplest type of plasmas, magnetic fields play an extremely important role.
Frozen-in Condition

- In a simple form of plasma, the plasma moves so that the magnetic flux through any surface is preserved.
Magnetic Field Waves

- Magnetic field waves have tension and pressure.
  - Think of them as rubber tubes.
- Magnetic fields can store a lot of energy!

\[ \beta = \frac{\text{Sound Wave Energy}}{\text{Magnetic Energy}} \]

- \( \beta_{\text{magnetosphere}} \geq 0.003 \)  \( \beta_{\text{surface of Earth}} \approx 3 \cdot 10^7 \)
- \( \beta_{\text{sun}} \geq 0.01 \)
Magnetic Fields: Rubber Tubes

- Disparate scales: $w \ll R \ll L$
- Incompressible: $Lw \sim R^2$
- Conservation of Magnetic Flux: $B_f \sim (w/R) B_i$
- Change in Magnetic Energy:
  \[
  B \text{ energy density } \sim B^2/8\pi \\
  E_f \sim (w/L) E_i \ll E_f
  \]
Magnetic Field Lines Can’t Break
Everything Breaks Eventually
Approximations

• Magnetic fields acting like rubber tubes assumes the slow plasma response.
  – Good for slow motions
  – Large scales
• Slinky
• It will break:
  – Fast Timescales/motions
  – Small lengths.
Field Lines Breaking: Reconnection

Process breaking the frozen-in constraint determines the width of the dissipation region, $\delta$. 

\[ \delta \]

\[ \downarrow V_{in} \]

\[ C_A \]
Field Lines Breaking: Reconnection

$J_z$ and Magnetic Field Lines

t = 0.00000

$X$

$Y$

-20  -40  -60  -80

-0.250330  0.000966638
What “Reconnection” Isn’t
Application – Solar Flares

Reconnection
Reconnection in Solar Flares

- X-class flare: $\tau \sim 100 \text{ sec.}$
- $B \sim 100 \text{ G}, n \sim 10^{10} \text{ cm}^{-3}, L \sim 10^9 \text{ cm}$
- $\tau_A \sim \frac{L}{c_A} \sim 10 \text{ sec.}$

F. Shu, 1992
Application - Magnetospheric Physics

To Sun

Dayside Reconnection

Magnetotail Reconnection
Part II: Simulating Reconnection
Reconnection is Hard

• Remember slinky?
• Now global (important) answers are strongly dependent on very fast/small timescales.
• If you have to worry about very small timescales, it makes the problem very hard.
Currently, Two Choices

• Macro Simulations:
  – Treat reconnection in a non-physical way.
  – Simulate Large Systems.

• Micro Simulations
  – Treat reconnection physically.
  – Simulate small idealized systems.
Our General Simulations

• Initial Value Problems
  – You give me the system initially, and I’ll tell you how it will behave in the future.
A “Real” Plasma

- Individual charge particles (on board)
- Simply Calculate forces between each particle.
  - Problem: N total particles.
  - For each N particle, have to calculate force from (N-1) particles.
  - Calculations per time step: N^2. Prohibitively expensive.
One Simplification: The Fluid Approximation
Fluid Approximation

• Break up plasma into infinitesimal cells.
• Define average properties of each cell (fluid element)
  – density, velocity, temperature, etc.
  – Okay as long as sufficient particles per cell.
The Simplest Plasma Fluid: MHD

- Magnetohydrodynamics (MHD):
  - Describes the slow, large scale behavior of plasmas.
- Now, very straightforward to solve numerically.

\[ m_i n \frac{d}{dt} \mathbf{V} = \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} - \nabla \left( nT + \frac{\mathbf{B}^2}{8\pi} \right) \]

\[ \frac{\partial}{\partial t} \mathbf{B} = -c \nabla \times \mathbf{E} \]

\[ \frac{\partial}{\partial t} n = -\nabla \cdot n \mathbf{V} \]

\[ \mathbf{E} = -\frac{\mathbf{V} \times \mathbf{B}}{c} \]
Simulating Fluid Plasmas

- Define Fluid quantities on a grid cell.
- Dynamical equations tell how to step forward fluid quantities.

- Problem with Numerical MHD:
  - No reconnection in equations.
  - Reconnection at grid scale.
MHD Macro Simulations

• Courtesy of the University of Michigan group:
  – Remember that reconnection occurs only at grid scale.
Non-MHD Micro Fluid Simulations

- Include smaller scale physics but still treat the system as a fluid.
Effective Gyration Radius

- Frozen-in constraint broken when scales of variation of $\mathbf{B}$ are the same size as the gyro-radius.
  
  **Electron gyroradius** $<<$ **Ion gyroradius**

$\Rightarrow$ Dissipation region develops a 2-scale structure.
Removing this Physics

$m_e/m_i = 1/25$

Out of Plane Current

Hall Term

No Hall Term

Reconnected Magnetic Flux

0 1 2 3

0 10 20 30 40
time

Hall Term

No Hall Term

$\text{X}$

$\text{Y}$

$\text{Z}$
Simulating Particles

• Still have $N^2$ problem. How do we do it?
• Forces due to electric and magnetic fields.
  – Fields exist on grids $\Rightarrow$ Fluid
  – Extrapolate to each particles location.

• Particles can be thought of as a Monte-Carlo simulation.
Simulating Kinetic Reconnection

- **Finite Difference**
  - Fluid quantities exist at grid points.
- **E, B treated as fluids always**
  - Maxwell’s equations
- **Two-Fluid**
  - E, B, ions, electrons are fluid
- **Kinetic Particle in Cell**
  - E, B fluids
  - Ions and electrons are particles.
  - Stepping fluids: particle quantities averaged to grid.
  - Stepping particles: Fluids interpolated to particle position.
3-D Magnetic Reconnection: with guide field

- Particle simulation with 670 million particles
- $B_z = 5.0 \, B_x$, $m_i/m_e = 100$, $T_e = T_i = 0.04$, $n_i = n_e = 1.0$
- Development of current layer with high electron parallel drift
  - Buneman instability evolves into electron holes
Formation of Electron holes

- Intense electron beam generates Buneman instability
  - nonlinear evolution into “electron holes”
    - localized regions of intense positive potential and associated anti-parallel electric field

\[ E_z \]
Electron Holes

• Localized region of positive potential in three space dimensions
  – ion and electron dynamics essential
    • different from structures studied by Omura, et al. 1996 and Goldman, et al. 1999 in which the ions played no role
  – scale size $V_d/\omega_{pe}$ in all directions
  – drift speed $\sim V_d/3$
  – dynamic structures (spontaneously form, grow and die)
Electron drag due to scattering by parallel electric fields

- Drag $D_z$ has complex spatial and temporal structure with positive and negative values – quasilinear ideas fail badly
- $D_z$ extends along separatrices at late time
- $D_z$ fluctuates both positive and negative in time.
The End