Physics 207: Lecture 27

Announcements

• Make-up labs are this week
• Final hwk assigned this week, final quiz next week
• Review session on Thursday May 19, 2:30 – 4:00pm, Here

Today's Agenda

• Statics recap
  ⇐Beam & Strings
    » What if a string breaks?
• Hinged Beams
• Angular Momentum:
  ⇐Definitions & Derivations
  ⇐What does it mean?
• Rotation about a fixed axis
  ⇐\( L = I \omega \)
  ⇐Example: Two disks
  ⇐Student on rotating stool

Exam 3 Results

• Average score ~ 54.4%; Median ~ 53%

A 16 percentage point curve will be applied to this exam
Statics Review:

- In general, we can use the two equations
  \[ \sum F = 0 \quad \sum \tau = 0 \]
  to solve any statics problem.

- When choosing axes about which to calculate torque, we can be clever and make the problem easy!!

Lecture 27, Act 1
Statics

- In which of the static cases shown below is the tension in the supporting wire bigger? In both cases \( M \) is the same, and the blue strut is massless.

  (a) case 1  (b) case 2  (c) same

![Diagram](image-url)
Lecture 27, Act 1
Solution

- Consider the torque about the hinge between the strut & the wall:

\[ \tau_{\text{total}} = MgL - T_1 \sin(30^\circ) L = 0 \]

due to gravity
due to wire

\[ MgL = \frac{T_1}{2} \]

\[ T_1 = 2Mg \]

does not depend on length of massless beam!

Lecture 27, Act 1
Solution

- The tension is the same in both cases.
Review: Beam and Strings

- Previously we solved for the tensions in the strings in the following problem:

![Diagram of a beam with tensions T1 and T2, masses M1 and M2, and a string breaking]

\[
T_1 = \frac{1}{3} Mg \\
T_2 = \frac{2}{3} Mg
\]

- But what if a string breaks...

Beam and Strings...

- If the left string breaks, what is the initial acceleration of the CM?
  🔄 The beam will rotate about the axis at A.

- Using \( F = ma \) in the y direction:
  🔄 \( Mg - T = Ma \)

- Figure out \( I \) about A:
  🔄 \( I = I_{CM} + Md^2 \).

- Recall \( I_{CM} = \frac{1}{12} ML^2 \)
  🔄 \( I = M\left(\frac{L^2}{12} + d^2\right) \)
Beam and Strings...

- Figure out $\tau$ about $A$:
  $$\tau = Mgd$$
- Now use $\tau = I\alpha$ and $a = \alpha d$

$$Mgd = M \left( \frac{L^2}{12} + d^2 \right) \frac{a}{d}$$

Hinged Beams:

- Consider a structure made from two beams, attached to each other and the wall with hinges:
What we want to find:
\((A_x, A_y)\) and \((B_x, B_y)\).

What we know:
Any forces present at C will act in pairs, and will therefore cancel if we consider the entire structure.

First use \(F_{\text{NET}} = ma\) in \(x\) and \(y\) directions:

\[
\begin{align*}
  x \quad A_x + B_x &= 0 \quad \iff \quad A_x &= -B_x \\
  y \quad A_y + B_y &= (m_1 + m_2)g
\end{align*}
\]

That's two equations, but we have four unknowns...
Hinged Beams...

- Now use some torque relationships.
  - First, consider the torque on the whole structure about an axis though the hinge at $B$.

$$\frac{L}{2}m_1g + \frac{L}{2}m_2g - L\tan \phi A_x = 0$$

$$\frac{L}{2}g(m_1 + m_2) = L\tan \phi A_x$$

$$A_x = \frac{g(m_1 + m_2)}{2\tan \phi} \quad (c)$$

(No torques from forces at $C$)

Hinged Beams...

- If we knew something about $A_y$ or $B_y$ we would be done!
  - Do the simplest thing we can think of!
  - Consider the torque on the bottom beam about an axis through $C$:

$$LB_y - \frac{L}{2}m_1g = 0$$

$$B_y = \frac{m_1g}{2} \quad (d)$$
Hinged Beams...

- So we have the following equations:

(a) \( A_x = -B_x \)

(b) \( A_y + B_y = (m_1 + m_2)g \)

(c) \( A_x = \frac{g(m_1 + m_2)}{2\tan \phi} \)

(d) \( B_y = \frac{m_1 g}{2} \)

And we solve these to get:

\[
\begin{align*}
A_x &= \frac{g(m_1 + m_2)}{2\tan \phi} \\
A_y &= \left(\frac{1}{2} m_1 + m_2\right)g \\
B_x &= \frac{g(m_1 + m_2)}{2\tan \phi} \\
B_y &= \frac{m_1 g}{2}
\end{align*}
\]
More on Stability:

- Consider a truck moving a refrigerator of mass $M$. The CM of the fridge is a height $h$ above the bed of the truck and the width of the fridge is $2w$. If the truck is on a horizontal road, what is the maximum acceleration $a_M$ that the truck can have without tipping the fridge? (Assume the fridge does not slip).

![Diagram of truck and refrigerator]

Fridge...

- Suppose the truck’s acceleration $a$ is such that the fridge is just starting to tip. In this case the weight of the fridge is supported by a normal force acting only at the back corner.
- There must also be a frictional force acting to keep the fridge accelerating:
Fridge...

- Since the fridge is not rotating, the sum of all the torques about an axis through the CM must be zero.

\[ \sum \text{torques} = 0 \]

\[ Mgw = Mah \]

\[ a = g \frac{w}{h} \]

Since the torque due to the normal force can't be any bigger, if we increased the acceleration, the net torque would be non-zero, and the fridge would flip.

\[ F = Ma_m \]

This must be the maximum allowable acceleration!
Lecture 27, Act 2
Rotations

- A girl is riding on the outside edge of a merry-go-round turning with constant $\omega$. She holds a ball at rest in her hand and releases it. Viewed from above, which of the paths shown below will the ball follow after she lets it go?

Lecture 27, Act 2
Solution

- Just before release, the velocity of the ball is tangent to the circle it is moving in.
Lecture 27, Act 2
Solution

- After release it keeps going in the same direction since there are no forces acting on it to change this direction.

Angular Momentum: Definitions & Derivations

- We have shown that for a system of particles

\[ F_{\text{EXT}} = \frac{dp}{dt} \]

Momentum is conserved if

\[ F_{\text{EXT}} = 0 \]

- What is the rotational version of this??

- The rotational analogue of force \( F \) is torque \( \tau = r \times F \)

- Define the rotational analogue of momentum \( p \) to be angular momentum \( L = r \times p \)
Definitions & Derivations...

- First consider the rate of change of $L$:
  \[ \frac{dL}{dt} = \frac{d}{dt}(r \times p) \]

  \[ \frac{d}{dt}(r \times p) = \left( \frac{dr}{dt} \times p \right) + \left( r \times \frac{dp}{dt} \right) \]

  \[ = (v \times mv) \]

  \[ = 0 \]

  So \[ \frac{dL}{dt} = r \times \frac{dp}{dt} \] (so what...?)

Definitions & Derivations...

- Recall that \[ F_{EXT} = \frac{dp}{dt} \quad \Rightarrow \quad \frac{dL}{dt} = r \times F_{EXT} \]

- Which finally gives us: \[ \tau_{EXT} = \frac{dL}{dt} \]

- Analogue of \[ F_{EXT} = \frac{dp}{dt} \]!!
What does it mean?

- \( \tau_{\text{EXT}} = \frac{dL}{dt} \) where \( L = r \times p \) and \( \tau_{\text{EXT}} = r \times F_{\text{EXT}} \)

- In the absence of external torques \( \tau_{\text{EXT}} = \frac{dL}{dt} = 0 \)

Total angular momentum is conserved

Angular momentum of a rigid body about a fixed axis:

- Consider a rigid distribution of point particles rotating in the x-y plane around the z axis, as shown below. The total angular momentum around the origin is the sum of the angular momenta of each particle:

\[
L = \sum_i r_i \times p_i = \sum_i m_i r_i \times v_i = \sum_i m_i r_i v_i \hat{k} \quad \text{(since } r_i \text{ and } v_i \text{ are perpendicular)}
\]

We see that \( L \) is in the z direction.

Using \( v_i = \omega r_i \), we get

\[
L = \sum_i m_i r_i^2 \omega \hat{k}
\]

\[
L = I \omega \quad \text{Analogue of } p = mv!!
\]