Phys 207
Announcements

• Hwk3 submission deadline = 1st thing this morning
  include DSC number on homework

Today’s Agenda
• Ch. 4 problems on board
• More discussion of dynamics
  ➞ Recap
  ➞ The tools we have for making & solving problems:
    » Ropes & Pulleys (tension)
    » Hooke’s Law (springs)
  ➞ Problems: Accelerometer, inclined plane, motion in a circle

Review

• Discussion of dynamics.
  ➞ Newton’s 3 Laws
  ➞ The Free Body Diagram
  ➞ The tools we have for making & solving problems:
    » Ropes & Pulleys (tension)
    » Hooke’s Law (springs)
Review: Pegs & Pulleys

- Used to change the direction of forces
  - An ideal massless pulley or ideal smooth peg will change the direction of an applied force without altering the magnitude: *The tension is the same on both sides!*

\[ F_1 = -T \hat{i} \]
\[ |F_1| = |F_2| \]
\[ F_2 = T \hat{j} \]

Springs

- **Hooke’s Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

\[ F_x = -kx \]

Where \( x \) is the displacement from the relaxed position and \( k \) is the constant of proportionality.
Springs...

- **Hooke’s Law**: The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

\[ F_x = -kx \]

Where \( x \) is the displacement from the relaxed position and \( k \) is the constant of proportionality.

\[ F_x = -kx > 0 \quad \text{for} \quad x < 0 \]

\[ F_x = -kx < 0 \quad \text{for} \quad x > 0 \]
Scales:

- Springs can be calibrated to tell us the applied force.
  ➕ We can calibrate scales to read Newtons, or...
  ➕ Fishing scales usually read weight in kg or lbs.

1 lb = 4.45 N

Lecture 9, Act 1
Springs

- A spring with spring constant 40 N/m has a relaxed length of 1 m. When the spring is stretched so that it is 1.5 m long, what force is exerted on a block attached to the end of the spring?

(a) -20 N  (b) 60 N  (c) -60 N
Lecture 9, Act 1
Solution

- Recall Hooke's law:
  \[ F_x = -kx \quad \text{Where } x \text{ is the displacement from equilibrium.} \]

  \[ F_x = -(40)(.5) \]
  \[ F_x = -20 \text{ N} \]

(a) -20 N  (b) 60 N  (c) -60 N

Problem: Accelerometer

- A weight of mass \( m \) is hung from the ceiling of a car with a massless string. The car travels on a horizontal road, and has an acceleration \( a \) in the \( x \) direction. The string makes an angle \( \theta \) with respect to the vertical (\( y \)) axis. Solve for \( \theta \) in terms of \( a \) and \( g \).
Accelerometer...

- Draw a **free body diagram** for the mass:
  - What are all of the forces acting?

![Diagram showing a free body diagram with forces](image)

\[ T \text{ (string tension)} \]

\[ m \text{ (gravitational force)} \]

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Accelerometer...

- Using components *(recommended):*

\[ i: F_x = T_x = T \sin \theta = ma \]

\[ j: F_y = T_y - mg = T \cos \theta - mg = 0 \]
Accelerometer...

- **Using components**:
  
  \[ i: \quad T \sin \theta = ma \]
  
  \[ j: \quad T \cos \theta - mg = 0 \]

- Eliminate \( T \):
  
  \[
  \begin{align*}
  T \sin \theta &= ma \\
  T \cos \theta &= mg \\
  \tan \theta &= \frac{a}{g}
  \end{align*}
  \]

Accelerometer...

- **Alternative solution using vectors** (elegant but not as systematic):

- Find the total vector force \( \mathbf{F}_{\text{NET}} \):

\[
\mathbf{F}_{\text{TOT}} \quad \theta
\]

\[
\mathbf{T} \quad \theta
\]

\[
m \quad \theta
\]

\[
mg \quad \text{(gravitational force)}
\]

\[
T \quad \text{(string tension)}
\]
Accelerometer...

- Alternative solution using vectors (elegant but not as systematic):
- Find the total vector force $F_{\text{NET}}$:
- Recall that $F_{\text{NET}} = ma$:

\[ \begin{align*}
mg & \quad \theta \\
ma & \quad \text{(gravitational force)}
\end{align*} \]

- So

\[ \tan \theta = \frac{ma}{mg} = \frac{a}{g} \]

Let's put in some numbers:

- Say the car goes from 0 to 60 mph in 10 seconds:
  - $60 \text{ mph} = 60 \times 0.45 \text{ m/s} = 27 \text{ m/s}$.
  - Acceleration $a = \frac{\Delta v}{\Delta t} = 2.7 \text{ m/s}^2$.
  - So $a/g = 2.7 / 9.8 = 0.28$.

\[ \theta = \arctan \left( \frac{a}{g} \right) = 15.6 \text{ deg} \]
Problem: Inclined plane

- A block of mass $m$ slides down a frictionless ramp that makes angle $\theta$ with respect to the horizontal. What is its acceleration $a$?

Inclined plane...

- Define convenient axes parallel and perpendicular to plane:
  - Acceleration $a$ is in $x$ direction only.
Inclined plane...

- Consider \( x \) and \( y \) components separately:
- \( i: \ mg \sin \theta = ma. \Rightarrow a = g \sin \theta \)
- \( j: \ N - mg \cos \theta = 0. \Rightarrow N = mg \cos \theta \)

Alternative solution using vectors:

\[ a = g \sin \theta \ i \]
\[ N = mg \cos \theta \ j \]
Angles of an Inclined plane

The triangles are similar, so the angles are the same!

\[ ma = mg \sin \theta \]

Lecture 9, Act 2
Forces and Motion

- A block of mass \( M = 5.1 \, \text{kg} \) is supported on a frictionless ramp by a spring having constant \( k = 125 \, \text{N/m} \). When the ramp is horizontal the equilibrium position of the mass is at \( x = 0 \). When the angle of the ramp is changed to \( 30^\circ \) what is the new equilibrium position of the block \( x_1 \)?

  (a) \( x_1 = 20\, \text{cm} \)  \hspace{1cm} (b) \( x_1 = 25\, \text{cm} \)  \hspace{1cm} (c) \( x_1 = 30\, \text{cm} \)
Lecture 9, Act 2
Solution

- Choose the x-axis to be along downward direction of ramp.
- FBD: The total force on the block is zero since it's at rest.
- Consider x-direction: 
  Force of gravity on block is \( F_{x,g} = Mg \sin \theta \)
  Force of spring on block is \( F_{x,s} = -kx_1 \)

\[ \begin{align*}
    F_{x,g} &= Mg \sin \theta \\
    F_{x,s} &= -kx_1
\end{align*} \]

Since the total force in the x-direction must be 0:

\[ \begin{align*}
    Mg \sin \theta - kx_1 &= 0 \\
    x_1 &= \frac{Mg \sin \theta}{k} \\
    x_1 &= \frac{5.1 \text{kg} \cdot 9.81 \text{m/s}^2 \cdot 0.5}{125 \text{N/m}} = 0.2 \text{m}
\end{align*} \]
Problem: Two Blocks

- Two blocks of masses \( m_1 \) and \( m_2 \) are placed in contact on a horizontal frictionless surface. If a force of magnitude \( F \) is applied to the box of mass \( m_1 \), what is the force on the block of mass \( m_2 \)?

\[
F = \frac{m_1 + m_2}{m_1} a
\]

- Realize that \( F = (m_1 + m_2) a \):

- Draw FBD of block \( m_2 \) and apply \( F_{\text{NET}} = ma \):

\[
F_{1,2} = m_2 a
\]

- Substitute for \( a \):

\[
F_{1,2} = m_2 \left( \frac{F}{(m_1 + m_2)} \right)
\]

\[
F_{1,2} = \frac{m_2}{(m_1 + m_2)} F
\]
Problem: Tension and Angles

- A box is suspended from the ceiling by two ropes making an angle $\theta$ with the horizontal. What is the tension in each rope?

\[ m \]

\[ \theta \]

\[ \theta \]

\[ T_1 \sin \theta \]

\[ T_2 \sin \theta \]

\[ T_1 \cos \theta \]

\[ T_2 \cos \theta \]

\[ mg \]

\[ i \]

\[ j \]

Problem: Tension and Angles

- Draw a FBD:

\[ F_{x,NET} = 0 \text{ and } F_{y,NET} = 0 \]

\[ F_{x,NET} = T_1 \cos \theta - T_2 \cos \theta = 0 \]

\[ \Rightarrow T_1 = T_2 \]

\[ F_{y,NET} = T_1 \sin \theta + T_2 \sin \theta - mg = 0 \]

\[ \Rightarrow T_1 = T_2 = \frac{mg}{2 \sin \theta} \]
Problem: Motion in a Circle

- A boy ties a rock of mass $m$ to the end of a string and twirls it in the vertical plane. The distance from his hand to the rock is $R$. The speed of the rock at the top of its trajectory is $v$.

What is the tension $T$ in the string at the top of the rock’s trajectory?

Motion in a Circle...

- Draw a Free Body Diagram (pick y-direction to be down):
- We will use $F_{\text{NET}} = ma$ (surprise)
- First find $F_{\text{NET}}$ in y direction:

$$F_{\text{NET}} = mg + T$$
Motion in a Circle...

\[ F_{NET} = mg + T \]

- Acceleration in y direction:
  \[ ma = \frac{mv^2}{R} \]
  \[ mg + T = \frac{mv^2}{R} \]
  \[ T = \frac{mv^2}{R} - mg \]

What is the minimum speed of the mass at the top of the trajectory such that the string does not go limp? i.e. find \( v \) such that \( T = 0 \).

\[ \frac{mv^2}{R} = mg + T \]

\[ \frac{v^2}{R} = g \]

\[ v = \sqrt{Rg} \]

- Notice that this does not depend on \( m \).
A skier of mass $m$ goes over a mogul having a radius of curvature $R$. How fast can she go without leaving the ground?

\[ v = \sqrt{\frac{mg}{R}} \]

\[ v = \sqrt{\frac{Rg}{m}} \]

\[ v = \sqrt{Rg} \]

\[ \frac{mv^2}{R} = mg - N \]

For $N = 0$:

\[ v = \sqrt{Rg} \]