Phys 207

Announcements

- Hwk2 submission deadline = 1st thing Monday morning
- TA office hours
  - R: Philip Castro / 7:00 – 9:00 pm / 227 SHL
  - F: Dan Pajerowski / 1:30 – 3:30 pm / 131B SHL
  - F: Xing Chen / 5:00 – 7:00 pm / 225 SHL (Commons Room)

Today’s Agenda

- Uniform Circular Motion

Phys 207

Uniform Circular Motion
(UCM)

- What does it mean?
- How do we describe it?
- What can we learn about it?
What is UCM?

- Motion in a circle with:
  - Constant Radius $R$
  - Constant Speed $v = |v|$

How can we describe UCM?

- In general, one coordinate system is as good as any other:
  - Cartesian:
    - $(x,y)$ [position]
    - $(v_x, v_y)$ [velocity]
  - Polar:
    - $(R, \theta)$ [position]
    - $(v_R, \omega)$ [velocity]

- In UCM:
  - $R$ is constant (hence $v_R = 0$).
  - $\omega$ (angular velocity) is constant.
  - Polar coordinates are a natural way to describe UCM!
Polar Coordinates:

- The arc length $s$ (distance along the circumference) is related to the angle in a simple way:
  
  \[ s = R\theta, \text{ where } \theta \text{ is the angular displacement.} \]

- Units of $\theta$ are called radians.

- For one complete revolution:
  
  \[ 2\pi R = R\theta_c \]
  
  \[ \theta_c = 2\pi \]

  $\theta$ has period $2\pi$.

1 revolution = $2\pi$ radians

\[ x = R \cos \theta \]
\[ y = R \sin \theta \]
Polar Coordinates...

- In Cartesian coordinates, we say velocity \( dx/dt = v \).
  \( x = vt \)
- In polar coordinates, angular velocity \( d\theta/dt = \omega \).
  \( \theta = \omega t \)
  \( \omega \) has units of radians/second.
- Displacement \( s = vt \).
  but \( s = R\theta = R\omega t \), so:
  \[ v = \omega R \]

Period and Frequency

- Recall that 1 revolution = \( 2\pi \) radians
  \( \text{frequency} (f) = \text{revolutions / second} \) \( (a) \)
  \( \text{angular velocity} (\omega) = \text{radians / second} \) \( (b) \)
- By combining (a) and (b)
  \( \omega = 2\pi f \)
- Realize that:
  \( \text{period} (T) = \text{seconds / revolution} \)
  \( \text{So} \ T = 1 / f = 2\pi/\omega \)
  \[ \omega = 2\pi / T = 2\pi f \]
Recap:

\[ x = R \cos(\theta) = R \cos(\omega t) \]
\[ y = R \sin(\theta) = R \sin(\omega t) \]
\[ \theta = \arctan \left( \frac{y}{x} \right) \]

- \( \theta = \omega t \)
- \( s = v t \)
- \( s = R\theta = R\omega t \)
- \( v = \omega R \)

Aside: Polar Unit Vectors

- We are familiar with the Cartesian unit vectors: \( i \ j \ k \)
- Now introduce “polar unit-vectors” \( \hat{r} \) and \( \hat{\theta} \):
  - \( \hat{r} \) points in radial direction
  - \( \hat{\theta} \) points in tangential direction
  (counter clockwise)
Acceleration in UCM:

- Even though the speed is constant, velocity is not constant since the direction is changing: must be some acceleration!
  - Consider average acceleration in time $\Delta t$ \( a_{av} = \Delta v / \Delta t \)

Let’s take a look – link to Active Figure 4-18
Acceleration in UCM:

- This is called Centripetal Acceleration.
- Now let’s calculate the magnitude:

\[
\frac{\Delta v}{v} = \frac{\Delta R}{R}
\]

But \(\Delta R = v\Delta t\) for small \(\Delta t\)

So:
\[
\frac{\Delta v}{v} = \frac{v\Delta t}{R} \Rightarrow \frac{\Delta v}{\Delta t} = \frac{v^2}{R}
\]

\[
a = \frac{v^2}{R}
\]

Centripetal Acceleration

- UCM results in acceleration:
  - **Magnitude:** \(a = \frac{v^2}{R}\)
  - **Direction:** \(-\hat{r}\) (toward center of circle)
Useful Equivalent:

We know that \( a = \frac{v^2}{R} \) and \( v = \omega R \)

Substituting for \( v \) we find that:

\[
a = \frac{(\omega R)^2}{R}
\]

\[
a = \omega^2 R
\]

Lecture 5, Act 1
Uniform Circular Motion

- A fighter pilot flying in a circular turn will pass out if the centripetal acceleration he experiences is more than about 9 times the acceleration of gravity \( g \). If his F18 is moving with a speed of 300 m/s, what is the approximate diameter of the tightest turn this pilot can make and survive to tell about it?

(a) 500 m
(b) 1000 m
(c) 2000 m
Lecture 5, Act 1
Solution

\[ a = \frac{v^2}{R} = 9g \]

\[ R = \frac{v^2}{9g} = \frac{9000 \frac{m^2}{s^2}}{9 \times 9.81 \frac{m}{s^2}} \]

\[ R = \frac{10000}{9.81} \text{ m} \approx 1000 \text{ m} \]

Example: Propeller Tip

- The propeller on a stunt plane spins with frequency \( f = 3500 \text{ rpm} \). The length of each propeller blade is \( L = 80\text{ cm} \). What centripetal acceleration does a point at the tip of a propeller blade feel?

what is \( a \) here?
Example:

- First calculate the angular velocity of the propeller:
  \[ 1 \text{ rpm} = \frac{1 \text{ rot}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times 2\pi \frac{\text{rad}}{\text{rot}} = 0.105 \frac{\text{rad}}{\text{s}} = 0.105 \text{ s}^{-1} \]
  \( \Rightarrow \) so 3500 rpm means \( \omega = 367 \text{ s}^{-1} \)

- Now calculate the acceleration.
  \( \Rightarrow a = \omega^2 R = (367 \text{ s}^{-1})^2 \times (0.8 \text{ m}) = 1.1 \times 10^5 \text{ m/s}^2 \)
  \( = 11,000 \text{ g} \)

  \( \Rightarrow \) direction of \( a \) points at the propeller hub (-\( \hat{r} \)).

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Example: Newton & the Moon

- What is the acceleration of the Moon due to its motion around the Earth?
- What we know (Newton knew this also):
  \( \Rightarrow T = 27.3 \text{ days} = 2.36 \times 10^6 \text{ s} \) (period ~ 1 month)
  \( \Rightarrow R = 3.84 \times 10^8 \text{ m} \) (distance to moon)
  \( \Rightarrow R_E = 6.35 \times 10^6 \text{ m} \) (radius of earth)
Moon...

- Calculate angular velocity:

\[
\omega = \frac{1 \text{ rot}}{27.3 \text{ day}} \times \frac{1 \text{ day}}{86400 \text{ s}} \times 2\pi \frac{\text{rad}}{\text{rot}} = 2.66 \times 10^{-6} \text{ s}^{-1}
\]

- So \(\omega = 2.66 \times 10^{-6} \text{ s}^{-1}\).

- Now calculate the acceleration.
  \(\mathbf{a} = \mathbf{a} = \omega^2 \mathbf{R} = 0.00272 \text{ m/s}^2 = 0.000278 \text{ g}\)
  - Direction of \(\mathbf{a}\) points at the center of the Earth (\(\uparrow\)).

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Moon...

- So we find that \(a_{\text{moon}} / g = 0.000278\)
- Newton noticed that \(R_E^2 / R^2 = 0.000273\)

- This inspired him to propose that \(F_{\text{MM}} \propto 1 / R^2\)
  - (more on gravity later)
Lecture 5, Act 2
Centripetal Acceleration

- The Space Shuttle is in Low Earth Orbit (LEO) about 300 km above the surface. The period of the orbit is about 91 min. What is the acceleration of an astronaut in the Shuttle in the reference frame of the Earth? (The radius of the Earth is $6.4 \times 10^6$ m.)

(a) $0 \text{ m/s}^2$
(b) $8.9 \text{ m/s}^2$
(c) $9.8 \text{ m/s}^2$

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First calculate the angular frequency $\omega$:

$$\omega = \frac{1 \text{ rot}}{91 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times 2\pi \frac{\text{rad}}{\text{rot}} = 0.00115 \text{ s}^{-1}$$

- Realize that:

$$R_O = R_E + 300 \text{ km}$$
$$= 6.4 \times 10^6 \text{ m} + 0.3 \times 10^6 \text{ m}$$
$$= 6.7 \times 10^6 \text{ m}$$
Lecture 5, Act 2
Centripetal Acceleration

- Now calculate the acceleration:

\[ a = \omega^2 R \]

\[ a = (0.00115 \text{ s}^{-1})^2 \times 6.7 \times 10^6 \text{ m} \]

\[ a = 8.9 \text{ m/s}^2 \]

Recap of Lectures 4 and 5

- 2-D Kinematics (Text: 4-1 thru 4-3)
  - Independence of \( x \) and \( y \) components
  - Projectile motion
  - Baseball problem
  - Shoot the monkey (example 4.4)

- Uniform circular motion (Text: 4.4)

Reminder
Homework 2 is posted on web
Submission deadline is Monday, Feb. 21 at the BEGINNING of lecture