Physics 207: Lecture 3

Reminders

- Discussion and Lab sections start TODAY

Homework

Submission deadline = Today, BEFORE lecture begins

WRITE YOUR DSC # ON YOUR HOMEWORK

Agenda for Today

- 1-D Kinematics (review, cont'd)
- Vectors (review)

Recap

- If the position $x$ is known as a function of time, then we can find both velocity $v$ and acceleration $a$ as a function of time!

\[
x = x(t) \\
v = \frac{dx}{dt} \\
a = \frac{dv}{dt} = \frac{d^2x}{dt^2}
\]
1-D Motion with constant acceleration

- High-school calculus: \[ \int t^n \, dt = \frac{1}{n+1} t^{n+1} + \text{const} \]
- Also recall that \[ a = \frac{dv}{dt} \]
- Since \( a \) is constant, we can integrate this using the above rule to find:
  \[ v = \int a \, dt = a \int dt = at + v_0 \]
- Similarly, since \( v = \frac{dx}{dt} \) we can integrate again to get:
  \[ x = \int v \, dt = \int (at + v_0) \, dt = \frac{1}{2} at^2 + v_0 t + x_0 \]

Recap

- So for constant acceleration we find:
  \[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]
  \[ v = v_0 + at \]
  \[ a = \text{const} \]
Lecture 3, Act 1
Motion in One Dimension

- When throwing a ball straight up, which of the following is true about its velocity $v$ and its acceleration $a$ at the highest point in its path?

(a) Both $v = 0$ and $a = 0$.

(b) $v \neq 0$, but $a = 0$.

(c) $v = 0$, but $a \neq 0$.

Lecture 3, Act 1
Solution

- Going up the ball has positive velocity, while coming down it has negative velocity. At the top the velocity is momentarily zero.

- Since the velocity is continually changing there must be some acceleration.
  - In fact the acceleration is caused by gravity ($g = 9.81 \text{ m/s}^2$).
  - (more on gravity in a few lectures)

- The answer is (c) $v = 0$, but $a \neq 0$. 
Useful Formula

\[ v = v_0 + at \quad x = x_0 + v_0 t + \frac{1}{2} at^2 \]

- Solving for \( t \):
  \[ t = \frac{v - v_0}{a} \]

- Plugging in for \( t \):
  \[ x = x_0 + v_0 \left( \frac{v - v_0}{a} \right) + \frac{1}{2} a \left( \frac{v - v_0}{a} \right)^2 \]

\[ v^2 - v_0^2 = 2a(x - x_0) \]

Alternate (Calculus-based) Derivation

\[ a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} \quad \text{(chain rule)} \]

\[ a = v \cdot \frac{dv}{dx} \quad \Rightarrow \quad a \cdot dx = v \cdot dv \]

\[ \int_{x_0}^{x} a \, dx = \int_{x_0}^{x} dv \cdot \int_{v_0}^{v} \, dv \quad (a = \text{constant}) \]

\[ \Rightarrow \quad a(x - x_0) = \frac{1}{2} (v^2 - v_0^2) \]

\[ v^2 - v_0^2 = 2a(x - x_0) \]
Recap:

- For constant acceleration:

\[
x = x_0 + v_0 t + \frac{1}{2} a t^2
\]
\[
v = v_0 + at
\]
\[
a = \text{const}
\]

- From which we know:

\[
v^2 - v_0^2 = 2a(x - x_0)
\]
\[
v_{av} = \frac{1}{2}(v_0 + v)
\]

Example 1

- A car is traveling with an initial velocity \(v_0\). At \(t = 0\), the driver puts on the brakes, which slows the car at a rate of \(a_b\).
Example 1...

- A car is traveling with an initial velocity $v_0$. At $t = 0$, the driver puts on the brakes, which slows the car at a rate of $a_b$. At what time $t_f$ does the car stop, and how much farther $x_f$ does it travel?

- Earlier, we derived: $v = v_0 + at$
- Realize that $a = -a_b$
- Also realizing that $v = 0$ at $t = t_f$:
  find $0 = v_0 - a_b t_f$ or
  \[ t_f = \frac{v_0}{a_b} \]
Example 1...

- To find stopping distance we use:
  \[ v^2 - v_0^2 = 2a(x - x_0) \]

- In this case \( v = v_f = 0, \ x_0 = 0 \) and \( x = x_f \)

\[ -v_0^2 = 2(-a_b)x_f \]

\[ x_f = \frac{v_0^2}{2a_b} \]

Example 1...

- So we found that \( t_f = \frac{v_0}{a_b}, \ x_f = \frac{1}{2} \frac{v_0^2}{a_b} \)

- Suppose that \( v_o = 65 \text{ mi/hr} = 29 \text{ m/s} \)
- Suppose also that \( a_b = g = 9.81 \text{ m/s}^2 \)

\[ \Leftrightarrow \text{ Find that } t_f = 3 \text{ s and } x_f = 43 \text{ m} \]
Tips:

- **Read!**
  - Before you start work on a problem, read the problem statement thoroughly. Make sure you understand what information is given, what is asked for, and the meaning of all the terms used in stating the problem.

- **Watch your units!**
  - Always check the units of your answer, and carry the units along with your numbers during the calculation.

- **Understand the limits!**
  - Many equations we use are special cases of more general laws. Understanding how they are derived will help you recognize their limitations (for example, constant acceleration).

1-D Free-Fall

- This is a nice example of constant acceleration (gravity):
- In this case, acceleration is caused by the force of gravity:
  - Usually pick y-axis “upward”
  - Acceleration of gravity is “down”:

\[
\begin{align*}
  a_y &= -g \\
  v_y &= v_{0y} - gt \\
  y &= y_0 + v_{0y} t - \frac{1}{2} g t^2
\end{align*}
\]
Gravity facts:

- $g$ does not depend on the nature of the material!
  - Galileo (1564-1642) figured this out without fancy clocks & rulers!

- demo - feather & penny in vacuum

- Nominally, $g = 9.81 \text{ m/s}^2$
  - At the equator $g = 9.78 \text{ m/s}^2$
  - At the North pole $g = 9.83 \text{ m/s}^2$

- More on gravity in a few lectures!

Problem:

- The pilot of a hovering helicopter drops a lead brick from a height of 1000 m. How long does it take to reach the ground and how fast is it moving when it gets there? (neglect air resistance)
Problem:
- First choose coordinate system.
  - Origin and $y$-direction.
- Next write down position equation:
  \[ y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \]
- Realize that $v_{0y} = 0$.
  \[ y = y_0 - \frac{1}{2}gt^2 \]

Problem:
- Solve for time $t$ when $y = 0$ given that $y_0 = 1000 \text{ m}$.
  \[ t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2 \times 1000 \text{ m}}{9.81 \text{ m/s}^2}} = 14.3 \text{ s} \]
- Recall:
  \[ v_y^2 - v_{0y}^2 = 2a(y - y_0) \]
- Solve for $v_y$:
  \[ v_y = \pm \sqrt{2gy_0} = -140 \text{ m/s} \]
Lecture 3, Act 2
1D free fall

- Alice and Bill are standing at the top of a cliff of height $H$. Both throw a ball with initial speed $v_0$. Alice straight down and Bill straight up. The speed of the balls when they hit the ground are $v_A$ and $v_B$ respectively. Which of the following is true:

(a) $v_A < v_B$
(b) $v_A = v_B$
(c) $v_A > v_B$

Since the motion up and back down is symmetric, intuition should tell you that $v = v_0$.

We can prove that your intuition is correct:

Equation: $v^2 - v_0^2 = 2(-g)(H - H) = 0$

This looks just like Bill threw the ball down with speed $v_0$, so the speed at the bottom should be the same as Alice’s ball.
Lecture 3, Act 2  
1D Free fall

- We can also just use the equation directly:

\[ v^2 - v_0^2 = 2(-g)(0-H) \]

**Alice:** \[ v^2 - v_0^2 = 2(-g)(0-H) \]

**Bill:** \[ v^2 - v_0^2 = 2(-g)(0-H) \]

Vectors (review):

- In 1 dimension, we could specify direction with a + or - sign. For example, in the previous problem \( a_y = -g \) etc.

- In 2 or 3 dimensions, we need more than a sign to specify the direction of something:

- To illustrate this, consider the position vector \( \mathbf{r} \) in 2 dimensions.

Example: Where is Philadelphia?

- Choose origin at Newark
- Choose coordinates of distance (miles), and direction (N,S,E,W)
- In this case \( \mathbf{r} \) is a vector that points 40 miles northeast.
Vectors...

- There are two common ways of indicating that something is a vector quantity:

  ✶ Boldface notation: \( \mathbf{A} \)

  \[ \mathbf{A} = \vec{A} \]

  ✶ “Arrow” notation: \( \vec{A} \)

- The components of \( \mathbf{r} \) are its \((x,y,z)\) coordinates

  \[ \mathbf{r} = (r_x, r_y, r_z) = (x,y,z) \]

- Consider this in 2-D (since it’s easier to draw):

  ✶ \( r_x = x = r \cos \theta \)

  ✶ \( r_y = y = r \sin \theta \)

  where \( r = |\mathbf{r}| \)

\[ \theta = \arctan\left( \frac{y}{x} \right) \]
Vectors...

- The magnitude (length) of \( r \) is found using the Pythagorean theorem:
  \[
  r = \sqrt{x^2 + y^2}
  \]

- The length of a vector clearly does not depend on its direction.

Unit Vectors:

- A Unit Vector is a vector having length 1 and no units
- It is used to specify a direction
- Unit vector \( u \) points in the direction of \( U \)
  \( \Rightarrow \) Often denoted with a “hat”: \( u = \hat{u} \)

- Useful examples are the Cartesian unit vectors \( [i, j, k] \)
  \( \Rightarrow \) point in the direction of the \( x, y \) and \( z \) axes
Vector addition:

- Consider the vectors $A$ and $B$. Find $A + B$.

$A \rightarrow B$

- We can arrange the vectors as we want, as long as we maintain their length and direction!!

Vector addition using components:

- Consider $C = A + B$.

(a) $C = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$

(b) $C = (C_x \hat{i} + C_y \hat{j})$

- Comparing components of (a) and (b):

$C_x = A_x + B_x$

$C_y = A_y + B_y$
Lecture 3, Act 3
Vectors

- Vector A = (0,2,1)
- Vector B = (3,0,2)
- Vector C = (1,-4,2)

What is the resultant vector, D, from adding A + B + C?

(a) (3,5,-1)  (b) (4,-2,5)  (c) (5,-2,4)

Lecture 2, Act 3
Solution

\[D = (A_x i + A_y j + A_z k) + (B_x i + B_y j + B_z k) + (C_x i + C_y j + C_z k)\]

\[= (A_x + B_x + C_x) i + (A_y + B_y + C_y) j + (A_z + B_z + C_z) k\]

\[= (0 + 3 + 1) i + (2 + 0 + 4) j + (1 + 2 + 2) k\]

\[= (4,-2,5)\]
Recap of Lecture 3

- Recap of 1-D motion with constant acceleration. (Text: 2-6)
- 1-D Free-Fall (Text: 2-6)  
  example
- Review of Vectors (Text: 3-1, 3-2, & 3-4)
- For Wed., look at textbook sections  
  Chapter 4-1 thru 4-4