This and the next two questions are about the following situation:

1. (*) A 10.0-kg block is released from a height of 3.0 m as indicated by point A in the figure below. Assume the entire track is frictionless. The block travels down the track, hits a spring of force constant 2000 N/m, and compresses it. The amount that the spring is compressed from its equilibrium position before the block comes to rest momentarily is:

   \[ \Delta E = 0 \quad \Delta KE = -\Delta U = -\left( \frac{1}{2} k x^2 - \phi \right) \]

   (a) 0.44 m  
   (b) 0.49 m  
   (c) 0.54 m  
   (d) 0.60 m  
   (e) 0.66 m

2. (**) Now suppose that the track is frictionless except for the portion between points B and C, which has a length of 6.0 m. The block travels down the track and this time compresses the spring 0.500 m from its equilibrium position before coming to rest momentarily. The coefficient of kinetic friction between the block and the rough surface between B and C is:

   \[ W_{nc} = \Delta E \]
   \[ W_{nc} = -fd = -\mu mg x \]

   \[ \Delta E = E_f - E_i = \frac{1}{2} k (0.5)^2 - mg (3) \]

   \[ x = \frac{\sqrt{2 (10)(9.8)(3)}}{2000} = 0.54 m \]

   \[ \mu = \frac{44}{98(6)} = 0.075 \]

   \[ 0.29 \]
3. (***) A small block of mass $m_1$ is released from rest at the top of a curve-shaped frictionless wedge of mass $m_2$, which sits on a frictionless horizontal surface as shown in the adjacent figure. When the block leaves the wedge, its velocity is measured to be $v_f$ to the right. What is the height $h$ of the wedge?

   (a) $\frac{v_f^2}{2g} \left( 1 + \frac{m_1}{m_2} \right)$
   (b) $\frac{v_f^2}{2g} \left( 1 - \frac{m_2}{m_1} \right)$
   (b) $v_f^2/2g$
   (c) $\frac{v_i^2}{2g} \left( \frac{m_1}{m_2} \right)$
   (d) $\frac{v_i^2}{2g} \left( \frac{m_2}{m_1} \right)$

   $p_x$ conserved $\Delta p_x = 0$

   $p_x = 0 = p_f = m_1 v_1 - m_2 v_2$

   $v_2 = \frac{m_2}{m_1} v_1$

   $\Delta E = 0$ due to conservative forces

   $\Delta KE = -\Delta U = m_1 gh$

   $\Delta KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left( \frac{m_2}{m_1} \right)^2 v_1^2$

   $\Rightarrow m_1 gh = \frac{1}{2} m_1 v_1^2 \left( 1 + \frac{m_1}{m_2} \right)$

4. (*) Three objects of uniform density – a solid sphere, a solid cylinder, and a hollow cylinder – are placed at the top of an incline as shown in the figure below. They are all released from rest at the same elevation and roll without slipping. In what order do the objects reach the bottom (from 1st to last one down)? Note: the result is independent of the masses and radii of the objects.

   (a) Hollow cylinder, solid cylinder, sphere
   (b) Solid cylinder, sphere, hollow cylinder
   (c) Sphere, solid cylinder, hollow cylinder

   $I_{\text{sphere}} < I_{\text{cylinder}} < I_{\text{hollow cylinder}}$
This and the next question are about the following situation:

5. (*) What is the net torque on the wheel shown in the figure below about the axle through \( O \)? Look carefully.

\[
\begin{align*}
\text{(a)} & \quad [ -aF_i + b(F_2 + F_3) ] \hat{k} \\
\text{(b)} & \quad [ aF_i - b(F_2 + F_3) ] \hat{k} \\
\text{(c)} & \quad [ aF_i \cos 30^\circ - b(F_2 + F_3) ] \hat{k}
\end{align*}
\]

\[30^\circ \text{ is unnecessary information}\]

6. (**) The moment of inertia of a solid disc of radius \( R \) and mass \( M \) about an axis perpendicular to the surface and passing through the edge of the disk is:

\[
\begin{align*}
\text{(a)} & \quad I = \frac{1}{2} MR^2 \\
\text{(b)} & \quad I = MR^2 \\
\text{(c)} & \quad I = 3MR^2/2
\end{align*}
\]

\[
I_{cm} = \frac{1}{2} MR^2
\]

\[
I_{total} = I_{cm} + MR^2 = \frac{3}{2} MR^2
\]

\[
\frac{2}{5} MR^2 + \frac{4}{5} MR^L
\]
This and the next question are about the following situation:

7. (*) A spherical object of mass $M$, initially at rest, explodes into 3 pieces with masses, $M/2$, $M/4$ and $M/4$. After the explosion, the pieces move in the $x$-$y$ plane.

Suppose the final velocity of the large piece is $v = v_j$. In this case, after the explosion, the total momentum of the two smaller pieces, $\vec{P}_{\text{small}}$, is

(a) $\vec{P}_{\text{small}} = 0$
(b) $\vec{P}_{\text{small}} = \left(Mv/2\right) \hat{j}$
(c) $\vec{P}_{\text{small}} = -\left(Mv/2\right) \hat{j}$

$\vec{P}$ conserved  \hspace{1cm} $\Delta \vec{P} = \vec{0}$
$P_{x_i} = 0$
$P_{y_i} = 0$
$P_{\text{f}y} = \frac{M}{2} v + \vec{P}_{\text{small}}$
$\vec{P}_{\text{small}} = -\frac{M}{2} v$

$MV + \vec{P}_{\text{small}}$

8. (***): Suppose in a different situation that the magnitudes of the final momenta of all three pieces were the same – all equal to $P$. If the final momentum of the larger piece is again in the $j$ direction, what is the component, $p_x$, of the final momentum of one of the smaller fragments in the $i$ direction?

(a) $|p_x| = 0.71P$
(b) $|p_x| = 0.5P$
(c) $|p_x| = 0.87P$
(d) $|p_x| = 0.78P$
(e) $|p_x| = P$

$P_x$ components must cancel
$P_y$ components must cancel

$\vec{P}_b - \vec{P}_a \cos \Theta - \vec{P}_c \cos \Theta = \vec{0} = P - P(2 \cos \Theta) = 0$
$\vec{P}(1 - 2 \cos \Theta) = 0$
$P_x = \frac{P \sin 60^\circ}{0.87P}$
$\cos \Theta = \frac{1}{2}$
$\Theta = 60^\circ$
This and the next question are about the following situation:

9. (***) A hoop of mass $m$ and radius $r$ rolls without slipping across a horizontal floor with speed $v$, down an incline, and then onto a horizontal floor again as shown below. Take $I = \frac{1}{2} mr^2$

After rolling without slipping a distance $d$ down the incline, what is the final speed of the hoop, $v_f$?

- (a) $v_f = \sqrt{v^2 + gd \sin \theta}$
- (b) $v_f = \sqrt{v^2 + 2gd}$
- (c) $v_f = \sqrt{v^2 + gr/2}$
- (d) $v_f = \sqrt{v^2 + 2gr \sin \theta}$
- (e) $v_f = \frac{r}{d} (v^2 + gd \sin \theta)$

\[ \Delta E = \phi \]
\[ \Delta KE = -\Delta U = mgd \sin \Theta \]
\[ \Delta KE = \frac{1}{2} I \omega_f^2 + \frac{1}{2} MV_{cmf}^2 - \frac{1}{2} I \omega_i^2 - \frac{1}{2} MV_{cmi}^2 \]
\[ = \frac{1}{2} \frac{MR^2}{R^2} V_{cmf}^2 + \frac{1}{2} MV_{cmf}^2 - \frac{1}{2} \frac{MR^2}{R^2} V_{cmi}^2 - \frac{1}{2} MV_{cmi}^2 \]
\[ = MV_{cmf}^2 - MV_{cmi}^2 = mgd \sin \Theta \]
\[ v_f^2 = mg \sin \Theta + mv_i^2 \]

10. (*) If instead of rolling without slipping down the incline, the hoop slides down a frictionless incline, it:

(a) would take longer to reach the bottom.
(b) would take the same amount of time to reach the bottom.
(c) would take less time to reach the bottom.
This and the next two question are about the following situation:
A string is wrapped around a disk-shaped pulley of radius \( r \), mass \( m \), and moment of interia \( I = mr^2/2 \).
The string has a block of mass \( M \) attached to one end of it as shown in the figure below. The mass is released from rest at a height \( h \) above the ground; the string does not slip on the pulley. Assume the pulley rotates without friction and there is no air drag.

11. (*) The behavior of the angular displacement of the pulley as a function of time, \( \theta(t) \), is best represented by:

\[
\alpha = \text{constant} \quad \quad \quad \quad \quad \Theta = \frac{1}{2} \alpha t^2 
\]

12. (*) What is the net work done on the system consisting of the block and pulley for the period during which the block drops a distance \( h \) to the ground?

(a) \( Mgh/3 \)  \hspace{1cm} (b) \( 2Mgh/3 \)  \hspace{1cm} (c) \( Mgh \)

\[
E = \text{constant} \\
-\Delta U = \Delta KE = w_{\text{net}} = Mgh \\
pulley + block
\]
13. (***) What is the angular speed of the pulley when the mass hits the ground? Be very careful with the notation for the masses of the pulley and block.

(a) \( \omega = \frac{\sqrt{4gh}}{3r^2} \)

(b) \( \omega = \frac{2Mgh}{\sqrt{r^2(M + m/2)}} \)

(c) \( \omega = \frac{2Mgh}{3\sqrt{mr^2}} \)

(d) \( \omega = \frac{2mgh}{\sqrt{3r^2(M + m/2)}} \)

\[ T - Mg = Ma_{cm} \]
\[ T = \frac{1}{2} mr^2 \dot{\alpha} = \frac{1}{2} mr^2 \left( \frac{a_{cm}}{r} \right) \]
\[ T = -\frac{1}{2} ma_{cm} \]
\[ -\frac{1}{2} ma_{cm} - Mg = Ma_{cm} \]
\[ -\frac{1}{2} ma_{cm} - Mg = Ma_{cm} \]
\[ a_{cm} = -g \frac{M}{M + \frac{m}{2}} \]

\[ -h = \frac{1}{2} a_{cm} t^2 \]

\[ t = \frac{-2h}{a_{cm}} = \frac{2h}{-g} \frac{M + \frac{m}{2}}{M} \]

Too complicated

\[ \omega_f = \omega_0 + \alpha t \]

\[ \omega_f^2 = \omega_0^2 + 2\alpha \Delta \theta \]

\[ \Delta \theta = \frac{-h}{r} \]

\[ \omega_f^2 = 2a_{cm} \left( \frac{-h}{r} \right) = \frac{2a_{cm} h}{r^2} = \frac{2gMh}{r^2(M + \frac{m}{2})} \]
Kinematics
\[ \mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \]
\[ \mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0t + \frac{1}{2}\mathbf{a}t^2 \]
\[ \mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a} (\mathbf{x} - \mathbf{x}_0) \]
\[ \mathbf{x} - \mathbf{x}_0 = \left( \mathbf{v}_0 + \mathbf{v} \right)t/2 \]
\[ \mathbf{x} - \mathbf{x}_0 = \mathbf{v}t + \frac{1}{2}\mathbf{a}t^2 \]
\[ g = 9.81 \text{ m/s}^2 = GM/R_E^2 \]

2D Motion
\[ h_{\text{max}} = v^2 \sin^2 \theta / 2g \]
\[ \text{Range} = v^2 \sin 2\theta / g \]

Uniform Circular Motion
\[ a = v^2/r = \omega^2 r \]
\[ v = \omega r \]
\[ \omega = 2\pi/T = 2\pi f \]

Galilean Transformations
\[ \mathbf{r}' = \mathbf{r} - \mathbf{v}_0 t \]
\[ \mathbf{v}' = \mathbf{v} - \mathbf{v}_0 \]

Dynamics
\[ \mathbf{F}_{\text{ext}} = m\mathbf{a} = \frac{d\mathbf{p}}{dt} \]
\[ \mathbf{F}_{\text{int}} = -\mathbf{F}_{\text{ext}} \]
\[ F = mg \text{ (gravity near earth's surface)} \]
\[ F_{\text{fric}} = -Gm_1m_2/r^2; \quad G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \]
\[ F_{\text{spring}} = -k(\Delta s) \]

Friction
\[ f = \mu_N N \text{ (kinetic friction)} \]
\[ f \leq \mu_s N \text{ (static friction)} \]
\[ \mathbf{f}_{\text{drag}} = -b\mathbf{v} \text{ (low speed)} \]
\[ |\mathbf{f}_{\text{drag}}| = \frac{1}{2} D \rho A v^2 \text{ (high speed)} \]

Work & Kinetic energy
\[ W = \int \mathbf{F} \cdot d\mathbf{r} \]
\[ W = \mathbf{F} \cdot \mathbf{r} = Fr \cos \theta \text{ (constant force)} \]
\[ W_{\text{grav}} = -mg\Delta y \text{ (near earth surface)} \]
\[ W_{\text{grav}} = GmM(1/r_2 - 1/r_1) \text{ (in general)} \]
\[ W_{\text{spring}} = -k(x^2 - x_0^2)/2 \]
\[ KE = mv^2/2 = p^2/2m \]
\[ W_{\text{ext}} = \Delta KE \]

Potential Energy
\[ U_{\text{grav}} = mg\text{ (near earth surface)} \]
\[ U_{\text{grav}} = -GMm/r \text{ (in general)} \]
\[ U_{\text{spring}} = kx^2/2 \]
\[ \Delta E = \Delta K + \Delta U = W_{\text{ext}} \]
\[ \Delta U = -W \]
\[ F(x) = -dU(x)/dx \]

Power
\[ P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v} \text{ (for constant force)} \]

System of Particles
\[ \mathbf{R}_{CM} = \sum m_i \mathbf{r}_i/\sum m_i \]
\[ V_{CM} = \sum m_i v_i/\sum m_i \]
\[ \mathbf{A}_{CM} = \sum m_i \mathbf{a}_i/\sum m_i \]
\[ \mathbf{P} = \sum m_i \mathbf{v}_i \]
\[ \sum F_{\text{ext}} = M \mathbf{A}_{CM} = d\mathbf{P}/dt \]

Collisions
\[ \text{If } \sum F_{\text{ext}} = 0 \text{ in some direction, then} \]
\[ \mathbf{P}_{\text{before}} = \mathbf{P}_{\text{after}} \text{ in this direction} \]
\[ \sum m_i \mathbf{v}_i \text{ (before)} = \sum m_i \mathbf{v}_i \text{ (after)} \]

In addition, if collision is elastic
* \[ KE_{\text{before}} = KE_{\text{after}} \]

Rotation Kinematics
\[ s = R\theta, \quad v = Rw, \quad a = R\alpha \]
\[ \theta = \theta_0 + \omega t + \frac{1}{2} \alpha t^2 \]
\[ \omega = \omega_0 + \alpha t \]
\[ \omega^2 = \omega_0^2 + 2\alpha \Delta \theta \]

Rotational Dynamics
\[ I = \sum m_i r_i^2, \quad I_{\text{parallel}} = I_{CM} + MD^2 \]
\[ I_{\text{disk}} = \frac{1}{2} MR^2, \quad I_{\text{bipod}} = MR^2 \]
\[ I_{\text{solid sphere}} = \frac{2}{5} MR^2, \quad I_{\text{hollow sphere}} = \frac{2}{3} MR^2 \]
\[ I_{\text{red cm}} = \frac{1}{12} ML^2, \quad I_{\text{red cen}} = \frac{1}{3} ML^2 \]
\[ \tau = I\alpha \text{ (rotation about a fixed axis)} \]
\[ \tau = r \times F, \quad |r| = rF \sin \phi \text{ (definition of torque)} \]
Work & Energy

\[ K_{\text{rotation}} = \frac{1}{2} I \omega^2 \]

\[ K_{\text{translation}} = \frac{1}{2} M v_{CM}^2 \]

\[ K_{\text{total}} = K_{\text{rotation}} + K_{\text{translation}} \]

\[ W = \tau \theta \]

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
\tan \theta &= \frac{y}{x} \\
\end{align*}
\]

\[ x^2 + y^2 = r^2 \]

\[ \sin 2\theta = 2 \sin \theta \cos \theta \]

\[ \sin 30^\circ = 0.50; \quad \cos 30^\circ = 0.866 \]

\[ \sin 60^\circ = 0.866; \quad \cos 60^\circ = 0.50 \]

if \( ax^2 + bx + c = 0 \) then

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]