Homework #8. November 18.


1. Consider the scattering of electrons of momentum \( k \) by a spherically symmetric potential

\[
V(r) = \begin{cases} 
V_0 \frac{a}{r} & r \leq a \\
0 & r > a. 
\end{cases}
\]

a) Using first-order plane wave Born approximation, show that the differential cross section for scattering through an angle \( \theta \) is

\[
\frac{d\sigma}{d\Omega} = \frac{4m^2V_0^2a^2}{\hbar^4k^4(1-\cos\theta)^2}\sin^4\left(\frac{ka}{\sqrt{2}}\cos\theta\right).
\]

b) Evaluate the differential cross section for scattering of a 100eV electron through an angle of 90° by a potential with \( V_0 = -10 \) eV and \( a=2\text{Å} \).

**NOTE: THIS IS A DOUBLE POINT PROBLEM. DO NOT OMIT.**

2. A point particle is scattered by a second particle with a rigid core, i.e. the scattering potential is

\[
V(r) = \begin{cases} 
\infty & r < a \\
0 & r > a. 
\end{cases}
\]

The energy of the scattering particle satisfies \( ka=1 \).

a) Find the expression for the phase shifts \( \delta_l \). Using Maple or Mathematica calculate the phase shifts for \( l=0, 1 \) and, 2.

b) Calculate the differential cross sections for \( \theta=0 \) and \( \theta=\pi \) (write the formula and obtain the values in units of \( a^2 \)) taking into account the waves with \( l=0 \) and \( l=1 \).

3. For the potential and conditions of Problem #2:

a) Calculate the total cross section taking into account the partial waves with \( l=0, l=1, \) and \( l=2 \) using Maple or Mathematica.

b) Determine how many partial waves you need to take into account to obtain the total cross section with 0.0001 accuracy (i.e. how many partial waves do you need to include so the
Hints for Problems 2 and 3:

1) Maple has intrinsic procedures for the Bessel functions of the first and second kind. Use them to define spherical Bessel and Neumann functions.

2) You can define your own function in Maple. Example: to define $f(x)=x^2$ do

   ```maple
   > f := x -> x^2;
   f := x → x^2
   ```

   Then, you can use your function to evaluate it for the different values of your variable:

   ```maple
   > f(2);
   4
   ```

   So you can define the formula for the phase shift or total cross section depending on $l$ and then just ask Maple what is the value for the particular $l$. You may need to use `evalf()` command.

4. Consider elastic scattering of a spinless low energy particle on a spherical well potential

   $$V(r) = \begin{cases} -V_0 & r < a \\ 0 & r > a. \end{cases}$$

   where $V_0$ is a positive constant. Find first the s-wave phase shift and then the differential and total scattering cross sections (assume that the s-wave dominates).

   **NOTE:** THIS IS A DOUBLE POINT PROBLEM. DO NOT OMIT.

5. Show that for the low energies the result of Problem#4 agrees with the low-energy approximation of the Born approximation result.