Postulates of quantum mechanics

Uncertainty relations
Hydrogen-like atoms
Addition of angular momenta
Dipole transitions

What does a physical theory involves?

Basic physical concepts

Set of rules which map the physical concepts to the corresponding mathematical objects

Mathematical formalism

L. Ballentine, Quantum Mechanics, A Modern Development
Chapter 2
How are the problems solved?

Express physical problem in mathematical terms → Solve it using mathematical techniques → Set of rules which relate mathematical formalism to observable reality → Translate mathematical solution back into the physical world

Postulates of quantum mechanics

- L. Ballentine, Quantum Mechanics, A Modern Development, Chapter 2, pages 42-50
- R. Liboff, Introductory quantum mechanics, Chapter 3, pages 68-84 (4th edition)
- M.A. Nielsen and I.L. Chuang, Quantum computation and quantum information, Chapter 2, pages 80-86, 93-96, 101-102
Postulates of quantum mechanics

**Postulate 1.** To each dynamical variable (physical concept) there corresponds a linear operator (mathematical object) and the possible values of the dynamical variable are the eigenvalues of the operator.

**Postulate 2.** The measurement of the observable $A$ that yields the value $\alpha$ leaves the system in the state $\varphi_\alpha$, where $\varphi_\alpha$ is the eigenfunction of $A$ that corresponds to eigenvalue $\alpha$.

**Postulate 3.** The state of a system at any instant of time may be represented by a state or wave function $\psi$ which is continuous and differentiable.

System in state $\psi(r,t)$ \(\rightarrow\) \(\langle A \rangle = \int \psi^* \hat{A} \psi dr\)

Average (expectation value) of observable $A$
Postulates of quantum mechanics

Postulate 4. Development of state in time:

\[ i\hbar \frac{\partial}{\partial t} \psi(r,t) = \hat{H}\psi(r,t) \]

Another version (N&C)

- **Postulate 1:** Associated to any isolated physical system is a complex vector space with inner product (a Hilbert space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system’s state space.
Another version (N&C)

- **Postulate 3**: Quantum measurements are described by a collection \( \{M_m\} \) of measurement operators. These are operators acting on the state space of the system being measured. The index \( m \) refers to the outcome that may occur in the experiment. If the state of the quantum system is \( |\psi\rangle \) immediately before the measurement then the probability that the result \( m \) occurs is given by
  \[ p(m) = \frac{\langle \psi | M_m \dagger M_m | \psi \rangle}{\langle \psi | M_m \dagger M_m | \psi \rangle}. \]
  The state of the system after the measurement is
  \[ \frac{M_m |\psi\rangle}{\sqrt{\langle \psi | M_m \dagger M_m | \psi \rangle}}. \]
  Completeness equation: \( \sum_m M_m \dagger M_m = I \).

- **Postulate 2**: The evolution of a closed quantum system is described by a unitary transformation. That is, the state \( |\psi\rangle \) of the system at time \( t_1 \) is related to the state \( |\psi'\rangle \) of the system at the time \( t_2 \) by a unitary operator \( U \) which depends only on the times \( t_1 \) and \( t_2 \), \( |\psi'\rangle = U |\psi\rangle \).

- **Postulate 2**: The time evolution of the state of a closed quantum system is described by a Schrödinger equation,
  \[ i \hbar \frac{d|\psi\rangle}{dt} = H |\psi\rangle. \]
  \( H \) is a fixed Hermitian operator known as the Hamiltonian of the closed system.
Another version (N&C)

- **Postulate 4**: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. If we have systems numbered 1 through $n$, and system number $i$ is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

Density matrix

- Suppose a quantum system is in one of a number of states $\psi_i$ with probabilities $p_i$, respectively. The $\{p_i, |\psi_i\rangle\}$ is ensemble of pure states.

- The density operator $\rho$ for the system is defined by the equation

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|.$$ 

- A quantum system which state $|\psi\rangle$ is known exactly is said to be in a pure state and the corresponding density operator is $|\psi\rangle \langle \psi|$. The system is in a mixed state otherwise.
Postulates: Summary

Observables <-> Linear operators

State of the system <-> State or wave function

Evolution of the system (development of state in time)

Measurement and its interpretation