AC Power: instantaneous and average power

\[ v(t) = V_p \cos(\omega t) \]

\[ Z = R + jX \]

\[ = \sqrt{R^2 + X^2} \left( \frac{R}{|Z|} + j \frac{X}{|Z|} \right) \]

\[ = |Z| \angle \theta \]

\[ \cos \theta = \frac{R}{|Z|} \]

\[ \tan \theta = \frac{X}{R} \]; use this to determine the sign

\[ I = \frac{V}{Z} = \frac{V_p}{|Z| \angle \theta} = I_p \angle -\theta \]

\[ i(t) = I_p \cos(\omega t - \theta) \]
Instantaneous and average power: cont.

**Instantaneous power (pulsating nature)**

\[
P(t) = i(t)v(t) = I_p V_p \cos(\omega t)\cos(\omega t - \theta)
\]

\[
= \frac{I_p V_p}{2} \cos \theta + \frac{I_p V_p}{2} \cos(2\omega t - \theta).
\]

**Average power (aka: real power)**

\[
\langle P(t) \rangle = \langle i(t)v(t) \rangle = I_p V_p \langle \cos(\omega t)\cos(\omega t - \theta) \rangle
\]

\[
= \frac{I_p V_p}{2} \cos \theta = I_{rms} V_{rms} \cos \theta
\]

\[
= |I| |V| \cos \theta = |I| |V| \frac{R}{|Z|} = |I|^2 R
\]

Example 7.2, P333
Complex Power

- real power $P_{av}$ (unit: watts): power absorbed by the load resistance.
- reactive power $Q$ (unit: volt-amperes-reactive, VAR): exchange of energy between the source and the reactive part of the load. No net power is gained or lost during the process. $Q=Q_L-Q_C$, if $Q<0$, the load is capacitive, $Q>0$, the load is inductive
- Apparent power: $|S|=|V_{rms}||I_{rms}|$ (unit: volt-amperes, VA): computed by measuring the rms load voltage and currents without regard for the phase angle.
The power factor: \( \cos(\theta) \)

- Power factor is defined as a ratio between \( P_{av} \) and \( |S| \).
- \( \cos\theta \leq 1 \), dependent on the complex load.
- Ideal power factor: \( \cos\theta = 1 \), \( Z=R \), pure resistive load.

\[
\langle P(t) \rangle = \left| \bar{I} \right| \left| \bar{V} \right| \cos \theta
\]

\[
\cos \theta = \frac{P_{av}}{\left| \bar{I} \right| \left| \bar{V} \right|}
\]

\[
= \frac{P_{av}}{|S|} = \frac{R}{|Z|}
\]

\[
\tan \theta = \frac{X}{R} = \frac{Q}{P_{av}}
\]

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Relationship among various ac power terms

\[ P(t) = P_{av} \left[ 1 + \cos 2(\omega t + \theta) \right] + Q \sin 2(\omega t + \theta) \]

- \( P(t) \): instantaneous power (W)
- \( P_{av} \): average power, power dissipated on resistive load (W)
- \( Q \): reactive power (volt-ampere-reactive, VAR)
Example

Calculate the instantaneous power flow into the load. Assume that the voltage source has RMS amplitude of 120 volts and $R$ and $X$ are both $100\,\Omega$.

\[ v(t) = \sqrt{2} \times 120 \cos \omega t = 170 \cos \omega t \]

\[ \frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{100} - \frac{j}{100} \]

\[ I = \frac{V}{Z} = 1.7 - j1.7 \]

\[ S = \overline{\overline{VI^*}} = \frac{V I^*}{2} = \frac{1}{2} \times 170 \times (1.7 + j1.7) \]

\[ P_{av} = 144 \, W \]

\[ Q = 144 \, VAR \]

\[ \theta = \tan^{-1} \frac{Q}{P_{av}} = 45^\circ \]

\[ P(t) = 144 \left[ 1 + \cos(\omega t - 45^\circ) \right] + 144 \sin(\omega t - 45^\circ) \]
Concept Check: thinking with phasors

The power factor in this series circuit is less than ideal. Increasing the capacitance is found to improve the power factor. *Is the impedance in the box inductive or capacitive?*

Answer: inductive