PHYS245 Exam I, Spring 2004

This is a closed book exam. Programmable calculators and graphing calculators may be used during this exam.

Since this exam booklet may be separated for grading; it is important to:

Show ALL work on problem sheet and only on that sheet.

Please read questions carefully. Ask the proctor if you are not clear about the questions.

Credit may be lost inadvertently if solutions are not neat and orderly.

Be careful with units, signs, and significant figures.

1. (10 pts) Short questions:

(a) The ______________ of a piece of metal wire is proportional to its length and inversely proportional to its cross sectional area.
   a. Resistance  
   b. Resistivity  
   c. Conductance  
   d. Conductivity  
   Answer: (a)

(b) The capacitance $C = \frac{Q}{V}$, so if I increase the voltage, the capacitance will decrease.
   a. True  
   b. False  
   (b)

(c) The ______________ across a capacitor has to be continuous.
   a. Current  
   b. Voltage  
   (b)

(d) In a steady state, the current going through an inductor is zero.
   a. True  
   b. False  
   (b)

(e) If an inductor is in series with a capacitor, the total impedance can be zero at a particular frequency.
   a. True  
   b. False  
   (a)
2 (30 pts) Assume that \( R_1 = 2 \, \Omega \), \( R_2 = 2 \, \Omega \), \( R_3 = 4 \, \Omega \), \( R_4 = 3 \, \Omega \), \( I = 2 \, A \), and \( V = 3 \, V \). Calculate a) current through \( R_1 \), \( R_2 \), voltage source, and \( R_3 \) (indicate the direction as well).

Solution:
If we use mesh method, we can get
\[
\begin{align*}
I_1 (R_1 + R_2) - I_2 R_2 &= -V \\
I_2 &= -2 \\
I_3 (R_3 + R_4) - I_2 R_3 &= V
\end{align*}
\]

Input values:
\[
\begin{align*}
I_1 \times 4 - I_2 \times 2 &= -3 & \Rightarrow & I_1 = -1.75 \, A \\
I_2 &= -2 \\
I_3 \times 7 - I_2 \times 4 &= 3 & \Rightarrow & I_3 = -0.714 \, A
\end{align*}
\]

So
\[
\begin{align*}
I_{R1} &= I_1 = -1.75 \, A \quad \text{(Right \rightarrow Left)} \\
I_{R2} &= I_2 - I_1 = -0.25 \, A \quad \text{(Right \rightarrow left)} \\
I_{R3} &= I_2 - I_3 = -1.286 \, A \quad \text{(Bottom \rightarrow top)} \\
I_v &= I_3 - I_1 = 1.036 \, A \quad \text{(Left \rightarrow right)}
\end{align*}
\]

3 (30 pts) Now replace \( R_1 \) in problem 2 by a capacitor \( C = 1 \, \mu F \), calculate the time constant of the circuit.

Solution:
Change to Thevenin Circuit, then time constant \( \tau = R_{TH} C \), so we only need to calculate \( R_{TH} \) in the Thevenin Circuit.

After “kill” the sources, we can find \( R_{TH} = R_2 = 2 \, \Omega \), hence,
\[
\tau = R_{TH} C = 2 \times 1 \times 10^{-6} = 2 \times 10^{-6} \, s = 2 \, \mu s
\]
4a (20 pts) Using the phasor techniques, solve for $v$ in the circuit.

Solution 1:

\[ Z_{eq} = j \omega L_3 + \frac{1}{j \omega C} = j(3 \times 3 - \frac{1}{3 \times 1/18}) = 3j \]

\[ Z_{eq} = Z_{L2} \parallel Z_{eq} = \frac{j \omega L_2 \times Z_{eq}}{j \omega L_2 + Z_{eq}} = 2.25j \]

\[ Z_{total} = Z_{R1} + Z_{L1} + Z_{eq} = 9 + 11.25j \]

Voltage divider:

\[ V'_{eq} = \frac{Z_{eq}}{Z_{total}} \times V = \frac{2.25j}{9 + 11.25j} \times 36 \angle -60^\circ = 5.625 \angle -21.34^\circ \]

Voltage divider again:

\[ V_C = \frac{Z_C}{Z_{L3} + Z_C} \times V'_{eq} = \frac{-6j}{3j} \times 5.625 \angle -21.34^\circ = -11.25 \angle -11.25^\circ = 11.25 \angle 158.7^\circ \]

So, \[ V_C = 11.25 \cos (3t + 158.7^\circ) \]

Solution 2:

Use mesh, \[
\begin{align*}
(Z_{R1} + Z_{L1} + Z_{L2})I_1 - Z_{L2}I_2 &= V \\
(Z_{L3} + Z_C + Z_{L2})I_2 - Z_{L2}I_1 &= 0
\end{align*}
\]

\[ \Rightarrow V_C = I_2 \times Z_C = 11.25 \angle 158.7^\circ \]

(\textbf{NOTE:} If use $V = 30 \cos (3t - \pi/3)$, then the result is $V_C = 9.375 \angle 158.7^\circ$.)
4b (10 point) If a voltage source of $v_2(t) = 10\cos(3t)$ V is insert in the circuit as shown in the figure, write down the equations to solve the current through the capacitor. (you do not have to solve them, just list equations).

Solution:

\[
\begin{align*}
(Z_{R_1} + Z_{L_1} + Z_{L_2})I_1 - Z_{L_2}I_2 &= V_1 \\
(Z_{L_3} + Z_C + Z_{L_2})I_2 - Z_{L_2}I_1 &= -V_2
\end{align*}
\]

(where

\[
\begin{align*}
Z_{R_1} &= R_1 = 9 \\
Z_{L_1} &= j\omega L_1 = 9j \\
Z_{L_2} &= j\omega L_2 = 9j \\
Z_{L_3} &= j\omega L_3 = 9j \\
Z_C &= \frac{1}{j\omega C} = -6j \\
V_1 &= 36\angle 60^\circ \\
V_2 &= 10\angle 0^\circ
\end{align*}
\] )