What is Electricity

- Static effects known since ancient times.
- Static charges can be made by rubbing certain materials together.
- Described by Benjamin Franklin as a fluid.
  - Excess of the fluid was described as positive.
  - Deficit of the fluid was described as negative.
- Like charges repel, unlike charges attract.
A Quantitative Approach

- *Charles Augustin de Coulomb* first described relationship of force and charges.
- Coulomb invented the torsion balance to measure the force between charges.
- Basic unit of charge in SI system is called the Coulomb.
- One of the basic irreducible measurements.
  - Like mass, length and time.

Fundamental Charge

- Due to atomic structure.
- Positively charged heavy nuclei and negative charged electrons.
- Charged objects have excess or deficit of electrons.
- Protons and electrons have equal and opposite charge: $e = 1.602 \times 10^{-19} \text{ C}$
- Rubbing some objects causes electrons to move from one object to another.
- *Negatively charged* objects have an excess of electrons.
- *Positively charged* objects have a deficit of electrons.
- Net change in charge in a closed system must be zero.
Fundamental Charge

Problem 18.6 - Water has a mass per molecule of 18 g/mol and each molecule has 10 electrons. How many electrons are in one liter of water?

\[
N = \frac{10 \text{ electrons}}{1 \text{ molecule}} \times \frac{6.022 \times 10^{23} \text{ molecules}}{1 \text{ mol}} \times \frac{1 \text{ mol}}{18 \text{ g}} \times \frac{1 \text{ g}}{1 \text{ ml}} \times \frac{10^3 \text{ ml}}{1 \text{ liter}}
\]

\[
N = 3.3 \times 10^{26} \text{ electrons per liter}
\]

What is the net charge of the electrons?

\[
q = Ne = \left(3.3 \times 10^{26} \text{ electrons}\right)\left(-1.602 \times 10^{-19} \text{ C/electron}\right)
\]

\[
q = -5.4 \times 10^7 \text{ C}
\]

Conservation of Charge

- Net change in charge in any closed system is zero.
- Or, the change of charge in any volume equals the net charge entering and leaving the system:

\[
\left(\text{Charge}\right) = \left(\text{Original Charge}\right) + \left(\text{Charge in}\right) - \left(\text{Charge out}\right)
\]

- We denote the amount of charge by the letter \(Q\) or \(q\)
- If Charge is created, it must be equal and opposite.

\[
q = q_0 + q_{in} - q_{out}
\]
Conservation of Charge

Two identical metal spheres of charge 5.0 C and -1.6 C are brought together and then separated. What is the charge on each sphere?

\[ q_{total} = q_1 + q_2 = 3.4 \text{C} \]

Since the charge is distributed equally, each must have an equal charge:

\[ q_1 = q_2 = \frac{q_{total}}{2} = 1.7 \text{C} \]

Conductors and Insulators

- **Insulator**
  - Charge stays where it is deposited.
  - Wood, mica, Teflon, rubber.

- **Conductor**
  - Charge moves “freely” within the object.
  - Metals exhibit high conductivity

- **Due to atomic structure.**
  - Is the outermost electron(s) tightly bound?
Charging Objects

• By friction
  – Rubbing different materials
  – Example: fur and ebonite

• By contact
  – Bringing a charged object into contact with another (charged or uncharged) object.

• By induction
  – Bringing a charged object near a conductor temporarily causes opposite charge to flow towards the charged object.
  – Grounding that conductor “drains” the excess charge leaving the conductor oppositely charged.

Force Between Charges

• Coulomb found that the force between two charges is proportional to the product of their charges and inversely proportion to the square of the distance between them:

\[ F \propto \frac{q_1 q_2}{r_{12}^2} \]

Where \( q_1 \) and \( q_2 \) represent the charges and \( r_{12} \) the distance between them.
Coulomb’s Law

• By using the torsion balance, Coulomb discovered the constant of proportionality: $k = 8.99 \text{ N} \cdot \text{m}^2/\text{C}^2$

$$F = k \frac{q_1 q_2}{r_{12}^2}$$

• If both $q_1$ and $q_2$ are both positive, the force is positive.
• If both $q_1$ and $q_2$ are both negative, the force is positive.
• If both $q_1$ is positive and $q_2$ is negative, the force is negative.
• Hence, the sign of the force is positive for repulsion and negative for attraction.

Coulomb’s Law

Principle of Superposition

• Electric force obeys the law of linear superposition – the net force on a charge is the vector sum of the forces due to each charge:

$$\vec{F}_i = \sum_{i \neq j} k \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij}$$

• Add the vectors:

The net force on charge 4 is $F_{14} + F_{24} + F_{34} = F_{\text{net}}$

Red are positive charges
Blue are negative charges
Coulomb’s Law
Principle of Superposition

• The forces add for as many charges as you have:

\[
\vec{F}_i = \sum_{i \neq j} k \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij}
\]

Red are positive charges
Blue are negative charges

Coulomb’s Law
Principle of Superposition

• Electric force obeys the law of linear superposition – the net force on a charge is the vector sum of the forces due to each charge:

\[
\vec{F}_i = \sum_{i \neq j} k \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij}
\]

The net force on charge 5 is the vector sum of all the forces on charge 5.

Red are positive charges
Blue are negative charges
Coulomb’s Law

18.14 – Two tiny conducting spheres are identical and carry charges of -20.0 μC and +50.0 μC. They are separated by a distance of 2.50 cm. (a) What is the magnitude of the force that each sphere experiences, and is the force attractive or repulsive?

\[ F = \frac{k |q_1 q_2|}{r^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times (\pm 20.0 \times 10^{-6} \text{ C}) \times (\pm 50.0 \times 10^{-6} \text{ C})}{(2.50 \times 10^{-2} \text{ m})^2} \]

\[ F = -1.44 \times 10^4 \text{ N} \]

The net force is negative so that the force is attractive.

(b) The spheres are brought into contact and then separated to a distance of 2.50 cm. Determine the magnitude of the force that each sphere now experiences, and state whether the force is attractive or repulsive.

Since the spheres are identical, the charge on each after being separated is one-half the net charge, so \( q_1 + q_2 = \pm 15 \mu \text{C} \).

\[ F = \frac{k q_1^2}{r^2} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \times (15.0 \times 10^{-6} \text{ C}) \times (15.0 \times 10^{-6} \text{ C})}{(2.50 \times 10^{-2} \text{ m})^2} = 3.24 \times 10^3 \text{ N} \]

The force is positive so it is repulsive.

Coulomb’s Law

18.21 – An electrically neutral model airplane is flying in a horizontal circle on a 3.0 m guideline, which is nearly parallel to the ground. The line breaks when the kinetic energy of the plane is 50.0 J. Reconsider the same situation, except that now there is a point charge of +q on the plane and a point charge of –q at the other end of the guideline. In this case, the line breaks when the kinetic energy of the plane is 51.8 J. Find the magnitude of the charges.

From the definition of kinetic energy, we see that \( mv^2 = 2(\text{KE}) \), so that Equation 5.3 for the centripetal force becomes

\[ F_c = \frac{mv^2}{r} = \frac{2(\text{KE})}{r} \]

So,

\[ \frac{T_{\text{max}}}{\text{Centripetal force}} = \frac{2(\text{KE})_{\text{charged}}}{r} \]  \hspace{2cm} (1) \]

\[ \frac{T_{\text{max}}}{\text{Centripetal force}} = \frac{2(\text{KE})_{\text{uncharged}}}{r} \]  \hspace{2cm} (2)
Coulomb’s Law

18.21 – (cont.)

\[ T_{\text{max}} + \frac{k|q|^2}{r^2} = \frac{2(KE)_{\text{charged}}}{r} \quad (1) \]
\[ T_{\text{max}} + \frac{k|q|^2}{r^2} = \frac{2(KE)_{\text{uncharged}}}{r} \quad (2) \]

Subtracting Equation (2) from Equation (1) eliminates \( T_{\text{max}} \) and gives

\[ \frac{k|q|^2}{r^2} = \frac{2[(KE)_{\text{charged}} - (KE)_{\text{uncharged}}]}{r} \]

Solving for \(|q|\) gives

\[ |q| = \sqrt{\frac{2r[(KE)_{\text{charged}} - (KE)_{\text{uncharged}}]}{k}} = \sqrt{\frac{2(3.0 \text{ m})(51.8 \text{ J} - 50.0 \text{ J})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2}} = \frac{3.5 \times 10^{-5} \text{ C}}{}} \]

The Electric Field

• Electric charges create and exert and experience the electric force.

\[ \vec{F}_i = \sum_{i \neq j} k \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij} \quad \Rightarrow \quad \vec{F}_i = q_i \sum_{i \neq j} k \frac{q_j}{r_{ij}^2} \hat{r}_{ij} \]

• We define the Electric Field as the force a test charge would feel if placed at that point:

\[ \vec{E} = \sum_{i \neq j} k \frac{q_j}{r^2} \hat{r} \quad \Rightarrow \quad \vec{F} = q \vec{E} \]

• The electric field exists around all charged objects.
Electric Force Lines

• Michael Faraday saw the force caused by one charge to be from lines emanating from the charges.

• Lines begin on Positive charges and end on Negative charges, flowing smoothly through space, never crossing.

Electric Force Lines

• Faraday imagined these lines and the force was greater where the lines are denser, weaker where they are spread out.
Electric Field Lines

• Go out from positive charges and into negative charges.

The Inverse Square Law

• Many laws based on the geometry of three-dimensional space:

\[ F_g = -\frac{Gm_1m_2}{r^2} \]

\[ I_{\text{sound}} = \frac{P}{4\pi r^2} \quad I_{\text{light}} = \frac{L}{4\pi r^2} \]

\[ F_e = k \frac{q_1q_2}{r^2} \]
The Electric Field

18.28 – Four point charges have the same magnitude of $2.4 \times 10^{-12}$ C and are fixed to the corners of a square that is 4.0 cm on a side. Three of the charges are positive and one is negative. Determine the magnitude of the net electric field that exists at the center of the square.

The drawing at the right shows each of the field contributions at the center of the square (see black dot). Each is directed along a diagonal of the square. Note that $E_A$ and $E_B$ point in opposite directions and, therefore, cancel, since they have the same magnitude. In contrast $E_A$ and $E_C$ point in the same direction toward corner A and, therefore, combine to give a net field that is twice the magnitude of $E_A$ or $E_C$. In other words, the net field at the center of the square is given by the following vector equation:

\[ \Sigma E = E_A + E_B + E_C + E_D = E_A + E_B + E_C - E_B = E_A + E_C = 2E_A \]

The magnitude of the net field is \[ \Sigma E = 2E_A = 2 \frac{k|q|}{r^2} \]

In this result $r$ is the distance from a corner to the center of the square, which is one half of the diagonal distance. Using $L$ for the length of a side of the square and taking advantage of the Pythagorean theorem, we have $r = L/\sqrt{2}$. With this substitution for $r$, the magnitude of the net field becomes

\[ \Sigma E = 2 \frac{k|q|}{\left( L/\sqrt{2} \right)^2} = \frac{4k|q|}{L^2} \]

\[ \Sigma E = 4 \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( 2.4 \times 10^{-12} \text{ C} \right) \left( 0.040 \text{ m} \right)^2 \]

\[ \Sigma E = 54 \text{ N/C} \]
Electric Field Lines

18.34 – Review Conceptual Example 12 before attempting to work this problem. The magnitude of each of the charges in Figure 18-21 is $8.6 \times 10^{-12}$ C. The lengths of the sides of the rectangles are 3.00 cm and 5.00 cm. Find the magnitude of the electric field at the center of the rectangle in Figures 18-21a and b.

The figure at the right shows the configuration given in text Figure 18.21a. The electric field at the center of the rectangle is the resultant of the electric fields at the center due to each of the four charges. As discussed in Conceptual Example 11, the magnitudes of the electric field at the center due to each of the four charges are equal. However, the fields produced by the charges in corners 1 and 3 are in opposite directions. Since they have the same magnitudes, they combine to give zero resultant. The fields produced by the charges in corners 2 and 4 point in the same direction (toward corner 2). Thus, $E_c = E_{c2} + E_{c4}$ where $E_c$ is the magnitude of the electric field at the center of the rectangle, and $E_{c2}$ and $E_{c4}$ are the magnitudes of the electric field at the center due to the charges in corners 2 and 4 respectively. Since both $E_{c2}$ and $E_{c4}$ have the same magnitude, we have $E_c = 2E_{c2}$.

Electric Field Lines

18.34 – (cont.)

The distance $r$, from any of the charges to the center of the rectangle, can be found using the Pythagorean theorem:

$$d = \sqrt{(3.00 \text{ cm})^2 + (5.00 \text{ cm})^2} = 5.83 \text{ cm}$$

Therefore, $\frac{d}{2} = 2.92 \text{ cm} = 2.92 \times 10^{-2} \text{ m}$

$$E_c = 2E_{c2} = \frac{2kq_2}{r^2} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-8.60 \times 10^{-12} \text{ C})}{(2.92 \times 10^{-2} \text{ m})^2} = -1.81 \times 10^2 \text{ N/C}$$

The figure at the right shows the configuration given in text Figure 18.21b. All four charges contribute a non-zero component to the electric field at the center of the rectangle. As discussed in Conceptual Example 11, the contribution from the charges in corners 2 and 4 point toward corner 2 and the contribution from the charges in corners 1 and 3 point toward corner 1.
Electric Field Lines

18.34 – (cont.)

Notice also, the magnitudes of $E_{24}$ and $E_{13}$ are equal, and, from the first part of this problem, we know that

$E_{24} = E_{13} = 1.81 \times 10^2 \text{ N/C}$

The electric field at the center of the rectangle is the vector sum of $E_{24}$ and $E_{13}$. The $x$ components of $E_{24}$ and $E_{13}$ are equal in magnitude and opposite in direction; hence

$(E_{13})_x - (E_{24})_x = 0$

Therefore,

$$E_C = (E_{13})_y + (E_{24})_y = 2(E_{13})_y = 2(E_{13})\sin \theta$$

From Figure 2, we have that

$$\sin \theta = \frac{5.00 \text{ cm}}{5.83 \text{ cm}} = 0.858$$

$$E_C = 2(E_{13})\sin \theta = 2(1.81 \times 10^2 \text{ N/C})(0.858) = 3.11 \times 10^2 \text{ N/C}$$

The Electric Field

- The electric field exists around all charged objects.

- This is shown by electric field lines.

- Constructed identically to the force lines.
Drawing Electric Field Lines

- Number of lines at a charge is proportional to the charge.

Electric Field Lines

- The charge with -3q must have three times as many lines as the charge with +q and 1.5 times the lines for +/- 2q.
Parallel Plates of Charge

- Adding fields from each point on surface causes field lines parallel to the plates to \textit{cancel}.

Zoomed image of positively and negatively charged plates with field lines indicated.

- Away from the edges, the field is parallel and perpendicular to the plate(s).

Electric Field & Conductors

- In conductors excess electrons will move when an electric field is present.
- For “steady state” (i.e. no moving charges), the net electric field must be zero.
  \[
  \vec{F} = q\vec{E} \quad \Rightarrow \quad \vec{F} = \vec{E} = 0
  \]
- Therefore the charges move to the surface(s) of the conductor until they cancel the electric field inside the conductor.
- Field lines can start and end on a conductor at right angles to the surface of the conductor.
Electric Field Lines

18.44 – Two particles are in a uniform electric field whose value is +2500 N/C. The mass and charge of particle 1 are \( m_1 = 1.4 \times 10^{-5} \text{ kg} \) and \( q_1 = -7.0 \mu \text{C} \), while the corresponding values for particle 2 are \( m_2 = 2.6 \times 10^{-5} \text{ kg} \) and \( q_2 = +18 \mu \text{C} \). Initially the particles are at rest. The particles are both located on the same electric field line, but are separated from each other by a distance \( d \). When released, they accelerate, but always remain at this same distance from each other. Find \( d \).

The drawing shows the two particles in the electric field \( E_x \). They are separated by a distance \( d \). If the particles are to stay at the same distance from each other after being released, they must have the same acceleration, so \( a_{x,1} = a_{x,2} \). According to Newton’s second law, the acceleration \( a_x \) of each particle is equal to the net force \( \Sigma F_x \) acting on it divided by its mass \( m \), or \( a_x = \frac{\Sigma F_x}{m} \).

![Diagram showing two particles in an electric field](image)

Note that particle 1 must be to the left of particle 2. If particle 1 were to the right of particle 2, the particles would accelerate in opposite directions and the distance between them would not remain constant.

\[
\Sigma F_{x,1} = q_1E_x - k\frac{q_1q_2}{d^2}
\]

\[
a_{x,1} = \frac{\Sigma F_{x,1}}{m_1}
\]

\[
\Sigma F_{x,2} = q_2E_x + k\frac{q_1q_2}{d^2}
\]

\[
a_{x,2} = \frac{\Sigma F_{x,2}}{m_2}
\]

Note the signs indicated the direction of the vector pointing from charge 2 to charge 1 for the force on charge 1.
Electric Field Lines

The acceleration of each charge must be the same if the distance between the charges remains unchanged:

\[ a_{1,x} = a_{2,x} \]

\[ \frac{q_1E_x - k \frac{q_1q_2}{d^2}}{m_1} = \frac{q_2E_x + k \frac{q_1q_2}{d^2}}{m_2} \]

Solving for \( d \),

\[ d = \sqrt{\frac{kq_1q_2(m_1 + m_2)}{E_x(q_1m_2 - q_2m_1)}} = 6.5 \text{ m} \]

Electric Flux

- Flux (from Latin – meaning to flow)
- Electric flux measures amount of electric field passing through a surface.

\[ \Phi_E = EA \]

So, to calculate, find the normal to the surface.

Then find the angle between the field and that normal vector: \( \phi \)

\[ \Phi_E = E \left( \cos \phi \right) A \]
Electric Flux

\[ \Phi_E = E(\cos \phi)A \]

This can be expressed as the vector dot product:

\[ \Phi_E = \vec{E} \cdot \vec{A} \]

\[ \vec{E} \cdot \vec{A} = |E||A| \cos \phi \]

or

\[ \vec{E} \cdot \vec{A} = E_xA_x + E_yA_y + E_zA_z \]

So, to calculate, find the normal to the surface.

Then find the angle between the field and that normal vector: \( \phi \)

Electric Flux

18.48 – A rectangular surface (0.16 m x 0.38m) is oriented in a uniform electric field of 580 N/C. What is the maximum possible electric flux through the surface?

The maximum possible flux occurs when the electric field is parallel to the normal of the rectangular surface (that is, when the angle between the direction of the field and the direction of the normal is zero). Then,

\[ \Phi_E = (E \cos \phi)A = (580 \text{ N/C})(\cos 0^\circ)(0.16 \text{ m})(0.38 \text{ m}) \]

\[ \Phi_E = 35 \text{ N} \cdot \text{m}^2 / \text{C} \]
Gauss’ Law

• Electric Flux through a closed surface is proportional to the charge enclosed by that surface:

\[ \Phi_e = \frac{q_{enc}}{\varepsilon_0} \]

• The fundamental constant \( \varepsilon_0 \) is called the permittivity of free space. It is related to Coulomb’s law by \( k = \frac{1}{4\pi\varepsilon_0} \), \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2) \).

• Gauss Law is most useful when the electric field on the entire surface is constant.

• Take a point charge. Construct a sphere centered on the point charge (all points on the surface equidistant from the charge) – the electric field is constant.

• Since the field is radial, it is perpendicular to the concentric spherical surface everywhere so \( \phi = 0, \cos \phi = 1 \)

\[ \Phi_e = \frac{q_{enc}}{\varepsilon_0} \implies EA \cos \phi = \frac{q_{enc}}{\varepsilon_0} \]

• The direction of the field is radially outward from the charge (or inward towards the charge if \( q \) is negative).

\[ E \left( 4\pi r^2 \right) = \frac{q_{enc}}{\varepsilon_0} \implies E = \frac{q}{4\pi\varepsilon_0 r^2} \]

Gauss’ Law for a plate of charge

• In this case we take our closed surface to be a box oriented such that two faces are parallel to the plate of charge and the others are perpendicular to the plate.

• Since the electric field from a plate of charge is perpendicular to the surface of the plate, the sides that are perpendicular to the plate have no flux.

• The charge enclosed by the box is the surface charge density multiplied by the area the box passes through the plate: \( \sigma A \).

\[ \Phi_e = \frac{q}{\varepsilon_0} \implies EA = \frac{\sigma A}{\varepsilon_0} \implies E = \frac{\sigma}{\varepsilon_0} \]
Gauss’ Law for a line of charge

18.54 – A long, thin, straight wire of length $L$ has a positive charge $Q$ distributed uniformly along it. Use Gauss’ law to find the electric field created by this wire at a radial distance $r$.

As the hint suggests, we will use a cylinder as our Gaussian surface. The electric field must point radially outward from the wire. Because of this symmetry the field must be the same strength pointing radially outward on the surface of the cylinder regardless of the point on the cylinder, thus we are justified in using Gauss’ Law.

Let us define the flux through the surface. Note that the field is parallel to the endcaps, thus the angle between the normal from the surface (which is parallel to the wire) is perpendicular to the field, thus $\phi = \pi/2$, so, $\cos(\phi)=0$ for the ends. For the side if the cylinder, the flux is

$$\Phi_E = EA_{\text{side}} = E(2\pi rL)$$

Thus, from Gauss’ Law,

$$\Phi_E = \frac{q_{\text{enc}}}{\varepsilon_0} \quad \Rightarrow \quad E(2\pi rL) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi \varepsilon_0 rL}$$

It is customary to refer to charge density $\lambda = Q/L$, thus

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$
Work

• Let us take an object with charge $q$ in a constant electric field $E$.
• If we release this charge, the electric field causes a force to move the charge.

$F = qE \quad \Rightarrow \quad a = \frac{q}{m}E$

• Let us define the field as $E = E\hat{x}$, so there is only one dimension to consider. Let us also set the initial velocity and position to be both zero.

• Let the object move through a distance $d$. The equations of motion are

\[
x = \frac{1}{2}at^2 \quad v = at
\]

• So, after a time $t$, the object moves $x = d$, $d = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2d}{a}}$

\[
\frac{v}{a} = \frac{2d}{a} \quad \Rightarrow \quad v = \sqrt{2ad} \quad v = \sqrt{\frac{2qEd}{m}} \quad \Rightarrow \quad \frac{1}{2}mv^2 = qEd
\]

Conservation of Energy

\[
\frac{1}{2}mv^2 = qEd
\]

• The left hand side is the kinetic energy the particle has after moving a distance $d$.

• That means, the right hand side must represent some form of potential energy the object had prior to being released.

• Types of Energy already covered:
  - Translational kinetic energy
  - Rotational kinetic energy
  - Gravitational potential energy
  - Elastic (spring) potential energy

\[
E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + \frac{1}{2}kx^2 + (EPE)
\]

• Electric Potential Energy
  - Cutnell & Johnson use (EPE), however, it is often more convenient to use $U$. 
Electric Potential Energy

• Like gravitational potential energy, $EPE$ is defined with respect to the energy at some location.

• Importance lies in the difference between electric potential energy between two points.

Example Problem

19.4 – A particle has a charge of $+1.5 \mu C$ and moves from point A to point B, a distance of 0.20 m. The particle experiences a constant electric force, and its motion is along the line of action of the force. The difference between the particle’s electric potential energy at A and B is $U_A - U_B = +9.0 \times 10^{-4} J$. Find the magnitude and direction of the electric force that acts on the particle and the magnitude and direction of the electric field that the particle experiences.

The work done must be equal to the change in potential energy:

$$\text{Work} = U_A - U_B \quad \Rightarrow \quad F\Delta x = U_A - U_B$$

$$F = \frac{U_A - U_B}{\Delta x} = \frac{9.0 \times 10^{-4} J}{(0.20 \text{ m})} = 4.5 \times 10^{-5} \text{ N} \quad \text{(from A to B)}$$

But, we know that for a particle in an electric field: $F = qE$

$$qE = \frac{U_A - U_B}{\Delta x} \quad \Rightarrow \quad E = \frac{U_A - U_B}{q\Delta x}$$

$$E = \frac{9.0 \times 10^{-4} J}{(1.5 \times 10^{-8} \text{ C})(0.20 \text{ m})} = 3.0 \times 10^3 \text{ N/C}$$
Electric Potential

• The electric potential energy of an object depends on the charge of an object.

• Once again, we can find a measurement independent of the objects charge: \( V = \frac{E_{PE}}{q} \)

• The SI unit for \( V \) is measured in Volts ( = Joules/Coulomb)

• A common unit of energy is the electron Volt.

• Specifically, How much potential energy does an electron get if moved through a potential of 1 Volt.

\[
1 \text{eV} = 1.602 \times 10^{-19} \text{ J}
\]

Electric Potential

• Like the Electric field, the electric potential is independent of any test charge placed to measure a force.

• Unlike the electric field, the potential is a scaler, not a vector.

• By using calculus methods it can be shown that the electric potential due to a point charge is

\[
V = \frac{kq}{r}
\]

• Note that the value of the electric potential depends on the sign of the charge.
Electric Potential for a point charge

• Example: What was the potential difference due to a charge $q_A = 10.0 \text{ nC}$, at the distances $r_0 = 10.0 \text{ cm}$ and $r_1 = 20.0 \text{ cm}$?

$$\Delta V = V_0 - V_1 = \frac{kq_A}{r_0} - \frac{kq_A}{r_1}$$

$$\Delta V = kq_A \left( \frac{1}{r_0} - \frac{1}{r_1} \right)$$

$$\Delta V = \left( 8.99 \times 10^9 \text{ N m}^2 / \text{C}^2 \right) \left( 10^{-5} \text{ C} \right) \left( \frac{1}{0.1 \text{ m}} - \frac{1}{0.2 \text{ m}} \right)$$

$$\Delta V = 450 \text{ V}$$

Electric Potential for a point charge

• Example: A charge $q_A$ is held fixed. A second charge $q_B$ is placed a distance $r_0$ away and is released. What is the velocity of $q_B$ when it is a distance $r_1$ away?

• We use the conservation of energy:

$$Energy = \frac{1}{2} mv^2 + U \quad \Rightarrow \quad \frac{1}{2} mv_0^2 + U_0 = \frac{1}{2} mv_1^2 + U_1$$

$$\frac{1}{2} mv_1^2 = U_0 - U_1 = q_B (V_0 - V_1)$$

$$v_1 = \sqrt{\frac{2q_B}{m} (V_0 - V_1)}$$
Electric Potential for a point charge

\[ v_1 = \sqrt{\frac{2q}{m}} (V_0 - V_1) \]

- Find the electric potential at \( r_0 \) and \( r_1 \):
  \[ V_0 = \frac{kq_A}{r_0} \quad \text{and} \quad V_1 = \frac{kq_A}{r_1} \]
  \[ v_1 = \sqrt{\frac{2q}{m}} \left( \frac{kq_A}{r_0} - \frac{kq_A}{r_1} \right) = \sqrt{\frac{2kq_Aq_B}{m} \left( \frac{1}{r_0} - \frac{1}{r_1} \right)} \]
- Or, \( v_1 = \sqrt{2q\Delta V / m} \)

Electric Potential for a point charge

- For a single charge, we set the potential at infinity to zero.
  \[ V = \frac{kq}{r} \]
- So what is the potential energy contained in two charges?
- Bring in the charges from infinity.
- The first charge is free as it does not experience any forces.
- The second moves through the field created by the first.
- So, the energy in the system must be

\[ \Delta U = q_2\Delta V \quad \Delta V = V_r - V_\infty \quad U = \frac{kq_1q_2}{r_{12}} \]
Potential Energy of a collection of charges

• Let’s add a third charge to this group.

\[ \Delta U = q_2 \Delta V_1 + q_3 \Delta V_1 + q_3 \Delta V_2 \]

\[ \Delta V_1 = \frac{kq_1}{r_{13}} \quad \Delta V_2 = \frac{kq_2}{r_{23}} \]

\[ \Delta U = \frac{kq_1 q_2}{r_{12}} + \frac{kq_1 q_3}{r_{13}} + \frac{kq_2 q_3}{r_{23}} \]

• Each successive charge experiences the field (i.e. potential) created by the previous charges. Be careful not to double count!

Example Problem

19.13 – An electron and a proton are initially very far apart (effectively an infinite distance apart). They are then brought together to form a hydrogen atom, in which the electron orbits the proton at an average distance of \(5.29 \times 10^{-11}\) m. What is the change in the electric potential energy?

The first charge comes in for free, the second charge experiences the field of the first, so,

\[ \Delta U = q_2 \Delta V \quad \Delta V = V_r - V_\infty \]

\[ \Delta V = \frac{kq_1}{r_{12}} - \frac{kq_1}{\infty} = \frac{kq_1}{r_{12}} \]

\[ \Delta U = \left( -1.602 \times 10^{-19} \text{ C} \right) \left( \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{1.602 \times 10^{-19} \text{ C}} \right) \left( \frac{5.29 \times 10^{-11} \text{ m}}{1.602 \times 10^{-19} \text{ C}} \right) \]

\[ \Delta U = -4.35 \times 10^{-18} \text{ J} \]
Electric Field and Potential

- Let a charge be moved by a constant electric field from points A to B (let \( \Delta d \) be that distance).
  \[
  W = F \Delta d \quad \Rightarrow \quad W = qE \Delta d
  \]

- Now, we know that the work done must be equal to the negative change in electric potential energy:
  \[-\Delta U_{AB} = qE \Delta d\]

- But potential energy is simply:
  \[-q \Delta V_{AB} = qE \Delta d\]

- Thus:
  \[
  E = -\frac{\Delta V}{\Delta d}
  \]

Electric Field and Potential

- This implies that the field lines are perpendicular to lines (surfaces) of equal potential.

- This equation only gives the component of the electric field along the displacement \( \Delta d \).

- To truly find the electric vector field, you must find the components in each direction:
  \[
  E_x = -\frac{\Delta V}{\Delta x} \quad E_y = -\frac{\Delta V}{\Delta y} \quad E_z = -\frac{\Delta V}{\Delta z}
  \]
Electric Field and Potential

- If the potentials at the following points \((x, y)\) are \(V(1,1) = 30 \text{ V}, \ V(0.9,1) = 15 \text{ V}, \) and \(V(1,0.9) = 40 \text{ V}.\) What is the (approximate) electric field at \((1,1)\)?

\[
E_x = -\frac{\Delta V}{\Delta x} \quad E_y = -\frac{\Delta V}{\Delta y} \quad E_z = -\frac{\Delta V}{\Delta z}
\]

\[
E_x = -\frac{30 \text{ V} - 15 \text{ V}}{1.0 \text{ m} - 0.9 \text{ m}} = -150 \text{ V} / \text{m} \quad E_y = -\frac{30 \text{ V} - 40 \text{ V}}{1.0 \text{ m} - 0.9 \text{ m}} = 100 \text{ V} / \text{m}
\]

\[
E = \sqrt{(-150 \text{ V})^2 + (100 \text{ V})^2} = 180 \text{ V}
\]

\[
\theta_x = \tan^{-1}\left(\frac{100 \text{ V}}{-150 \text{ V}}\right) = -34^\circ
\]

Example

Problem 19.34 – Determine the electric field between each of the points.

\[
E = -\frac{\Delta V}{\Delta x}
\]

\[
E_{AB} = -\frac{5.0 \text{ V} - 5.0 \text{ V}}{0.20 \text{ m} - 0.0 \text{ m}} = 0 \text{ V} / \text{m}
\]

\[
E_{BC} = -\frac{3.0 \text{ V} - 5.0 \text{ V}}{0.40 \text{ m} - 0.20 \text{ m}} = 10.0 \text{ V} / \text{m}
\]

\[
E_{CD} = -\frac{1.0 \text{ V} - 3.0 \text{ V}}{0.80 \text{ m} - 0.40 \text{ m}} = 5.0 \text{ V} / \text{m}
\]
Equipotential Surfaces

- Electric field lines are perpendicular to equipotential surfaces.
- Electric field lines are perpendicular to surfaces of conductors.
- Thus, surfaces of conductors are equipotential surfaces.
- The interior of conductors have no electric field and are therefore equipotential.
- The Electric potential of a conductor is constant throughout the conductor.

Example 19.68 – An electron is released at the negative plate of a parallel plate capacitor and accelerates to the positive plate (see the drawing). (a) As the electron gains kinetic energy, does its electric potential energy increase or decrease? Why?

The electric potential energy decreases. The electric force $F$ is a conservative force, so the total energy (kinetic energy plus electric potential energy) remains constant as the electron moves across the capacitor. Thus, as the electron accelerates and its kinetic energy increases, its electric potential energy decreases.

(b) The difference in the electron’s electric potential energy between the positive and negative plates is $U_{\text{pos}} - U_{\text{neg}}$. How is this difference related to the charge on the electron ($-e$) and to the difference in the electric potential between the plates?

The change in the electron’s electric potential energy is equal to the charge on the electron ($-e$) times the potential difference between the plates.

(c) How is the potential difference related to the electric field within the capacitor and the displacement of the positive plate relative to the negative plate?
Example

19.68 – (cont.) (c) How is the potential difference related to the electric field within the capacitor and the displacement of the positive plate relative to the negative plate?

The electric field $E$ is related to the potential difference between the plates and the displacement $\Delta s$ by

$$ E = -\frac{(V_{\text{pos}} - V_{\text{neg}})}{\Delta s} $$

Note that $(V_{\text{pos}} - V_{\text{neg}})$ and $\Delta s$ are positive numbers, so the electric field is a negative number, denoting that it points to the left in the drawing.

**Problem** The plates of a parallel plate capacitor are separated by a distance of 1.2 cm, and the electric field within the capacitor has a magnitude of $2.1 \times 10^6$ V/m. An electron starts from rest at the negative plate and accelerates to the positive plate. What is the kinetic energy of the electron just as the electron reaches the positive plate?

Use conservation of energy:

$$ KE_{\text{pos}} + eV_{\text{pos}} = KE_{\text{neg}} + eV_{\text{neg}} $$

$$ \Rightarrow KE_{\text{pos}} + eV_{\text{pos}} = KE_{\text{neg}} + eV_{\text{neg}} $$

Example

19.68 – (cont.)

$$ KE_{\text{pos}} + eV_{\text{pos}} = KE_{\text{neg}} + eV_{\text{neg}} $$

But the electron starts from rest on the positive plate, so, the kinetic energy at the positive plate is zero.

$$ eV_{\text{pos}} = KE_{\text{neg}} + eV_{\text{neg}} $$

$$ KE_{\text{neg}} = eV_{\text{pos}} - eV_{\text{neg}} $$

$$ KE_{\text{neg}} = e(V_{\text{pos}} - V_{\text{neg}}) $$

But, $E = -\frac{(V_{\text{pos}} - V_{\text{neg}})}{\Delta s}$, so,

$$ KE_{\text{neg}} = e(-E\Delta s) = (-1.602 \times 10^{-19} \text{ C})(-2.16 \times 10^6 \text{ V/m})(0.012 \text{ m}) $$

$$ KE_{\text{neg}} = 4 \times 10^{-15} \text{ J} $$
Lines of Equal Potential

- Lines of equal elevation on a map show equal potential energy (mgh).

Example

19.29 – Two equipotential surfaces surround a $+1.5 \times 10^8 \, \text{C}$ point charge. How far is the $190$-V surface from the $75.0$-V surface?

The electric potential $V$ at a distance $r$ from a point charge $q$ is $V = kq/r$. The potential is the same at all points on a spherical surface whose distance from the charge is $r = kq/V$. The radial distance $r_{75}$ from the charge to the $75.0$-V equipotential surface is $r_{75} = kq/V_{75}$, and the distance to the $190$-V equipotential surface is $r_{190} = kq/V_{190}$. The distance between these two surfaces is

$$r_{75} - r_{190} = \frac{kq}{V_{75}} - \frac{kq}{V_{190}} = kq \left( \frac{1}{V_{75}} - \frac{1}{V_{190}} \right)$$

$$r_{75} - r_{190} = \left( 8.99 \times 10^9 \, \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left( +1.50 \times 10^8 \, \text{C} \right) \left( \frac{1}{75.0 \, \text{V}} - \frac{1}{190 \, \text{V}} \right) = 1.1 \, \text{m}$$
Capacitors

• A capacitor consists of two conductors at different potentials being used to store charge.

• Commonly, these take the shape of parallel plates, concentric spheres or concentric cylinders, but any conductors will do.

• \( q = CV \), where \( C \) is the capacitance, measured in SI units of Farads (Coulomb / Volt or \( C^2 / J \)), \( V \) is the voltage difference between the conductors and \( q \) is the charge on each conductor (one is positive, the other negative).

• Capacitance is depends only upon the geometry of the conductors.

Parallel Plate Capacitor

• Field in a parallel plate capacitor is \( E = \frac{q}{\epsilon_0 A} \)

• But, the a constant electric field can be defined in terms of electric potential:
  \[
  E = -\frac{V}{d} \quad \Rightarrow \quad V = Ed
  \]

• Substituting into the equation for capacitance,
  \[
  q = CV = CEd
  \]
  \[
  q = C\left(\frac{q}{\epsilon_0 A}\right)d \quad \Rightarrow \quad C = \frac{\epsilon_0 A}{d}
  \]
Other Capacitors

- Two concentric spherical shells
  \[ C = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} \]

- Cylindrical capacitor has a capacitance per unit length:
  \[ \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)} \]

Energy stored in a capacitor

- A battery connected to two plates of a capacitor does work charging the capacitor.

- Each time a small amount of charge is added to the capacitor, the charge increases, so the voltage between the plates increases.

\[ V = \frac{q}{c} \]
Energy stored in a capacitor

- Potential Energy is $U = qV$

- We see that adding up each small increase in charge $\Delta q \times V$ is just the area of the triangle.

$$U = \frac{1}{2} qV \quad q = CV$$

Such that,

$$U = \frac{q^2}{2V} \quad U = \frac{1}{2} CV^2$$

So the energy density (i.e. energy per unit volume) must be

$$U = \frac{1}{2} \varepsilon_0 \varepsilon \left( \frac{V}{d} \right)^2$$

$$U = \frac{1}{2} \varepsilon_0 \varepsilon (Ad)$$

$Ad$ is the area of the capacitor. $u = \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 \varepsilon E^2$

Example

- How much energy is stored in a parallel plate capacitor made of 10.0 cm diameter circles kept 2.0 mm apart when a potential difference of 120 V is applied?

- Find the capacitance:

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi r^2}{d}$$

- The energy stored is

$$U = \frac{1}{2} CV^2 = \frac{\varepsilon_0 \pi r^2 V^2}{2d} = \frac{(8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2) \pi (0.05 \text{ m})^2 (120 \text{ V})^2}{2 (0.002 \text{ m})}$$

$$U = 2.09 \times 10^{-9} \text{ J}$$
Electric Polarization in Matter

- Electric charge distribution in some molecules is not uniform.
- Example: Water – both H atoms are on one side.
- In the presence of an Electric Field, water will align opposite to the field.

Electric Polarization in Matter

- In the absence of a field. Molecules will be oriented randomly.
- In an electric field, they line up opposite to the field.
Electric Polarization in Matter

- The resulting electric field is the sum of the external field and the fields generated by the molecules.
- The electric field strength through the matter is reduced.
- The reduction of the field depends on the energy in the molecules, the strength of the field each one makes and the density of the molecules.
- This can be measured and the material is said to have a dielectric constant - $\kappa$.

Dielectrics

- This dielectric constant changes the value of the permittivity in space ($\varepsilon$)
- Coulomb's law in a dielectric is $F = \frac{q_1 q_2}{4\pi\varepsilon_0 \kappa r^2}$
- Where $\varepsilon$ is related to $\varepsilon_0$ by a constant $\kappa$: $\varepsilon = \kappa \varepsilon_0$
- The dielectric constant for a vacuum is $\kappa = 1$.
- All other materials have a dielectric constant value greater than 1.
- Water at about 20°C has $\kappa = 80.4$.
- Air at about 20°C has $\kappa = 1.00054$. 
Dielectrics

\[ \varepsilon = \kappa \varepsilon_0 \]

- How does this affect the capacitance of parallel plates?

\[ C_d = \frac{\varepsilon A}{d} = \frac{\kappa \varepsilon_0 A}{d} \]
\[ C_d = \kappa C_0 \]

Dielectric Breakdown

- If the field in the dielectric becomes too strong (i.e. voltage is too great), the material begins to conduct current – this is called dielectric breakdown.

- When this happens the charge flows through the material to the other plate and neutralizes the capacitor.
Example

19.57 – Two hollow metal spheres are concentric with each other. The inner sphere has a radius of 0.1500 m and a potential of 85.0 V. The radius of the outer sphere is 0.1520 m and its potential is 82.0 V. If the region between the spheres is filled with Teflon, find the electric energy contained in this space.

The electric energy stored in the region between the metal spheres is

\[ u = \frac{1}{2} \kappa \varepsilon_0 E^2 \quad \Rightarrow \quad U = \frac{1}{2} \kappa \varepsilon_0 E^2 (Volume) \]

The volume between the spheres is the difference in the volumes of the spheres:

\[ (Volume) = \frac{4}{3} \pi r_2^3 - \frac{4}{3} \pi r_1^3 = \frac{4}{3} \pi (r_2^3 - r_1^3) \]

Since \( E = \Delta V/\Delta s \) and \( \Delta s = r_2 - r_1 \), so, \( E = \Delta V/r_2 - r_1 \)

\[ U = \frac{1}{2} \kappa \varepsilon_0 \left( \frac{\Delta V}{r_2 - r_1} \right)^2 \left[ \frac{4}{3} \pi (r_2^3 - r_1^3) \right] = 1.2 \times 10^{-8} \text{ J} \]