Problem 1. If $|\alpha\rangle$ is a state vector of a quantum system, then what is the value of the following expression $\text{Tr} \left( |\alpha\rangle\langle\alpha| \right)$ (i.e., trace over states in the Hilbert space of the quantum system).

Problem 2. Suppose a sudden electric field pulse is applied to a material, $\vec{E} = \vec{E}_0 \delta(t)$. Sketch qualitatively, as a function of time, the current $j(t)$ that would develop in

(a) metal
(b) insulator
(c) superconductor
Label the time scales in your sketch, both in qualitative and quantitative terms.

Problem 3. A spin state of an electron is described by the following density matrix:

$$\hat{\rho}_s = \frac{1}{3} \begin{pmatrix} 2 & 1 + i \\ 1 - i & 1 \end{pmatrix},$$

which is represented here in the basis $|\uparrow\rangle$, $|\downarrow\rangle$ of the eigenstates of $\hat{\sigma}_x$ (that is, $\hat{\sigma}_x |\uparrow\rangle = +|\uparrow\rangle$, $\hat{\sigma}_x |\downarrow\rangle = -|\downarrow\rangle$).

In order to characterize this state, examine the following properties:

(a) Find explicit representation of the Pauli operators in this basis, in which $\hat{\sigma}_x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is diagonal.

(b) What are the expectation values of the spin operator $\langle \vec{S} \rangle = \text{Tr} (\hat{\rho}_s \vec{S})$ where spin-$\frac{1}{2}$ operator is defined in terms of the Pauli operators $(\hat{S}_x, \hat{S}_y, \hat{S}_z) = \frac{\hbar}{2} (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$? Note that $\langle \vec{S} \rangle \equiv \frac{\hbar}{2} \vec{P}$ where $\vec{P}$ is the Bloch vector, so this calculation should yield the direction and the intensity of $\vec{P}$ inside the Bloch sphere.

(c) Is this spin in a pure or in a mixed quantum state?