Mini test

Methods of Mathematical Physics

Problem 1. Evaluate the following expression involving the Dirac $\delta$-function:

$$\int_0^{1000} dx \sin(x)\delta(x) =$$

Problem 2. If $|a_i\rangle$ are vectors comprising a full orthonormal base of states in the Hilbert space $\mathcal{H}$, then what is the result of the following operations:

- $(|a_j\rangle\langle a_k|)^\dagger =$
- $\text{Tr} \ (|a_j\rangle\langle a_k|) =$

Problem 3. What are the eigenvalues of the following $2 \times 2$ (Pauli) matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \text{ eigenvalues : }$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ eigenvalues : }$$