Proximity Effect in Strongly Correlated Nanostructures

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Resurrecting an Old Problem?

- "Ancient" **proximity effect**: $F(z)$ decays from $SN$ boundary into $N$ on the scale set by $\xi_N = \sqrt{\hbar D}/2\pi kT$ (diffusive; $D$ is diffusion constant) or $\xi_N = \hbar v_F/2\pi kT$ (ballistic), while $\Delta(z) = |U(z)|F(z)$ is suppressed on the $S$ side ("inverse proximity effect").
Mesoscopic SC: Exchange of Quantum Phase Information

- Mesoscopic $SN$ nanostructures unveil **real-space picture** of how $S$ imparts superconducting-type correlations in the adjacent $N$: **Andreev reflection + mesoscopic phase coherence** over $L_\omega = \sqrt{\hbar D/2\omega} \ (= \xi_N$ for thermal electrons with energy $\omega \simeq \pi k_B T) \Leftrightarrow$ **long-range PE** for $\omega < E_{\text{Th}}$.

Andreev Retro-Reflection in Pictures

- $e$ with excitation energy $\omega$ is scattered into $h$ with energy $-\omega$, which acquires a scattering phase shift $\pi/2 - \phi$ ($\Delta = |\Delta|e^{i\phi}$).

- AR (left) and typical pair of Feynman paths (right) that generate $F = \langle \Psi^\dagger \Psi^\dagger \rangle \neq 0$.

- **Andreev bound states** $\Rightarrow$ DOS in clean SN (Saint-James and McMillan in the 60’s):
Minigap in the 90’s: Quantum Disorder

- Hard gap opens in finite-size proximity coupled $N$, having chaotic classical dynamics, in closed geometries:
  
  → $I$/diffusive $N/S$ structure: $E_g = 0.78E_{Th}^{diff}$.
  
  → $S$/diffusive $N/S$ junction [$\ell \ll L$, $E_{Th}^{diff} = \hbar D/L^2 \ll$ bulk superconducting gap $\Delta$]: $E_g = 3.18E_{Th}^{diff}$ (depends on $\phi_L - \phi_R$).
Minigap in the 90’s: Quantum Chaos


- **Ballistic chaotic cavities:** $E_g = \hbar/\tau_c$ ($\tau_c$ is the typical time to establish contact with $S$, $\tau_c = L/v_F$ with no tunnel barriers at the contact with $S$, or Ehrenfest time $\tau_E$ instead when $\tau_E \gg \tau_c$).

- **Quantum Chaos** ⇒ understand statistics of energy levels and eigenfunctions and relate them to chaotic classical dynamics.
Minigap in the III Millennium

- **2001**: Quasiclassical minigap is smeared by mesoscopic fluctuations due to prelocalized states found even in metals $G \gg 2e^2/h$.

Example of a prelocalized state captured in an ensemble of 30000 nanoconductors $(12 \times 12 \times 12)$ and $2 \cdot 10^6$ eigenstates.
Minigap in Experiments

- Local spectroscopy of SN structure—STM at 60mK:

SCmS Josephson Junction

Attractive $U < 0$ Hubbard model
Falicov-Kimball interaction

\[
H = -\sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i U_i \left( c_{i\uparrow}^\dagger c_{i\uparrow} - \frac{1}{2} \right) \left( c_{i\downarrow}^\dagger c_{i\downarrow} - \frac{1}{2} \right) + \sum_{i\sigma} U_i^{FK} c_{i\sigma}^\dagger c_{i\sigma} \left( w_i - \frac{1}{2} \right)
\]
Lattice Hamiltonian Approach

- $2U_{\text{FK}}$ describes real-space pairing of electrons due to a local instantaneous attractive interaction—in Hartree-Fock approximation this gives the standard BCS superconductor with cutoff provided by the electronic bandwidth rather than the phonon frequency.

- $U < 0$ describes real-space pairing of electrons due to a local instantaneous attractive interaction—in Hartree-Fock approximation this gives the standard BCS superconductor with cutoff provided by the electronic bandwidth rather than the phonon frequency.
S and Cm Properties

- **Superconductor** for $U = -2t$ at half-filling $\mu = 0$: $\Delta \approx 0.2t$; $T_c = 0.11t$; $\xi_S \approx 4a$; $\Delta/E_F \approx 0.03$ invalidates Andreev quasiclassics.

- **Correlated metal** $\Leftrightarrow$ **Falicov-Kimball model** [mobile electrons interact with static ions (or $f$ electrons) through on-site Coulomb interaction $U_{FK}$]:

![Graphs](a) and (b)
Solving Lattice BdG Equations Self-Consistently

- Nambu-Gorkov Green function and the local self-energy
\[ \hat{\Sigma}(r_i, r_j, i\omega_n) = \hat{\Sigma}(r_i, i\omega_n) \delta_{ij} : \]

\[
\begin{pmatrix}
G(r_i, r_j, i\omega_n) & F(r_i, r_j, i\omega_n) \\
\overline{F}(r_i, r_j, i\omega_n) & -G^*(r_i, r_j, i\omega_n)
\end{pmatrix},
\begin{pmatrix}
\Sigma(r_i, i\omega_n) & \phi(r_i, i\omega_n) \\
\phi^*(r_i, i\omega_n) & -\Sigma^*(r_i, i\omega_n)
\end{pmatrix}
\]

- DMFT for macroscopically inhomogeneous systems:

\[ \Sigma^\alpha = G^{-1}_\alpha - G^{-1}_{\alpha} \]

Dyson's equation for the atomic problem
What is Dynamical Mean-Field Theory?

- **Non-perturbative** technique for correlated fermions: neglects spatial correlations, but fully retains local quantum dynamics ⇔ exact in $\infty$-dimensions.

- **Dynamical** $\equiv$ energy scale dependent MF theory.

- Single out one lattice site $\Rightarrow$ self-consistent impurity model with $S_{\text{eff}}(G_0)$ and effective “Weiss function” $G_0$.

- **Key ingredient**: local $\Sigma(i\omega_n) = G_0^{-1} - G^{-1}$ (replacement for dispersion relation of quasiparticles).
LDOS within Proximity Coupled Cm

- The center of $SCmS$ Josephson junction is at $z = 0$. 

![Graph showing LDOS within Proximity Coupled Cm for different values of U and L.](image)
‘‘Inverse’’ Proximity Effect in LDOS

- LDOS on the $S$ side of $SCm$ boundary in $SCmS$ junction.

![Graph showing LDOS on the S side of SCm boundary in SCmS junction.](image-url)
Heuristic Analysis: Part I

- **Strongly correlated systems**: either introduce “better” quasiparticles, or **forget** about quasiparticles.

- Phenomenological way out (employed for JJ with underdoped cuprates as interlayer): extract $D$ from the Einstein formula $\sigma = 2e^2N(0)D$.

  — Semiclassical diffusivity: $D = v_F\ell/3$

  — Quantum diffusivity: $D^x_\alpha = \pi\hbar \sum_{\alpha'} |\langle \alpha|\hat{v}_x|\alpha'\rangle|^2 \delta(E_\alpha - E_{\alpha'})$ via Kubo.

- **Does it help in the case of $SCmS$ junction** [as $U_{FK} \to 0$, $Cm \to$ Fermi liquid]?

  $\Rightarrow U_{FK} = 2.0 \Rightarrow \rho_{FK} \simeq 240 \mu\Omega\text{cm}$ (for $a = 3 \text{\AA}$) $\Rightarrow D \approx 2ta^2/\hbar$ and $\xi_N \simeq 5.6a$ (which is surprisingly close to the true $\xi_N \approx 6.7a$ extracted from the decay of Josephson critical current $I_c$ in the $SCmS$ junction as a function of the $Cm$ interlayer thickness).

  $\Rightarrow$ Quasiclassics for a mesoscopic diffusive junction of the same resistivity and $L = 10a$: $E_{Th} \approx 0.1\Delta$ and $E_g \approx 0.32\Delta$.

- **At $U_{FK} = 2.0$ ⇒ no gap in the $Cm$ spectrum!**
Heuristic Analysis: Part II

- $E_g^a/E_g^b$ for $L_a = 5a$ and $L_b = 10a$:
  
  1° $U_{FK} = 0.1 \Rightarrow E_g^a/E_g^b = 2.9$
  
  2° $U_{FK} = 0.25 \Rightarrow E_g^a/E_g^b = 3.1$
  
  3° $U_{FK} = 0.5 \Rightarrow E_g^a/E_g^b = 4.4$.

- **Finite $\Delta/E_F$** causes normal scattering which opens a minigap $\sim \Delta^2/E_F$ (coexisting with Andreev bound states) in the Fermi-liquid phase.

- To be contrasted with **quasiclassical minigap**:

![Graph showing the relationship between $E_{Gap}/E_A$ and $d/l_N$.]

References, Figures, etc.

