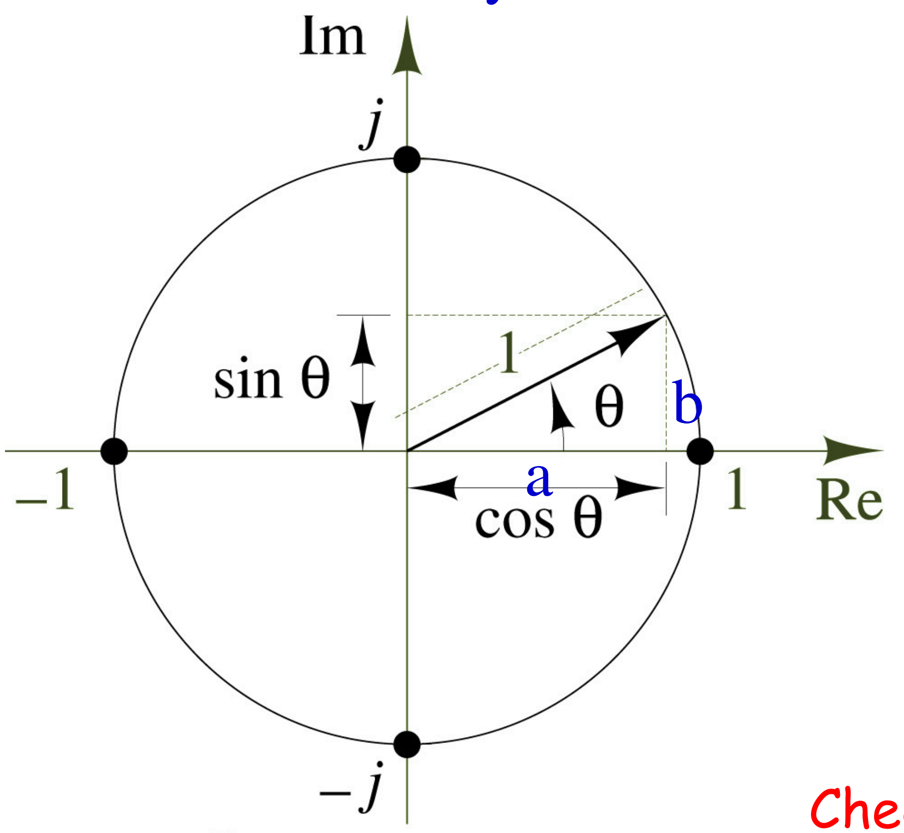
Complex number review

AC signal x(t)=Acos(ωt+φ)

- *freq. does not change in linear circuit*
- *need compute amplitude A and phase φ → need math representation that deal with both quantities → complex number*

Euler's identity



$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$a + jb = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + j\frac{b}{\sqrt{a^2+b^2}}\right)$$

$$= A(\cos\theta + j\sin\theta)$$

$$= Ae^{j\theta}$$

$$= A\angle\theta$$

A: amplitude, θ : phase

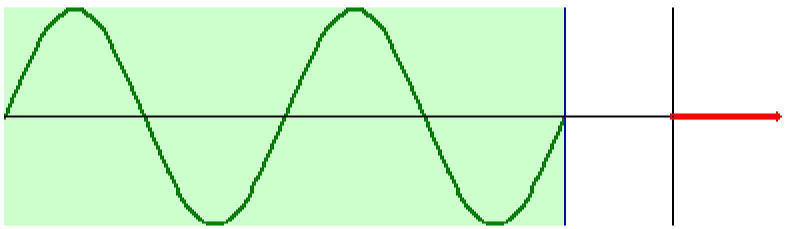
$$c_1 = A_1 e^{j\theta_1} = A_1\angle\theta_1, c_2 = A_2 e^{j\theta_2} = A_2\angle\theta_2$$

$$c_1 \times c_2 = A_1A_2e^{j(\theta_1+\theta_2)} = A_1A_2\angle(\theta_1+\theta_1)$$

$$\frac{c_1}{c_2} = \frac{A_1}{A_2}e^{j(\theta_1-\theta_2)} = \frac{A_1}{A_2}\angle(\theta_1-\theta_1)$$

Check Appendix A for complex number help

Phasor Animation



sinPlotting1.swf

Phasor

How can an ac quantity be represented by a complex number?

$$A\cos(\omega t+\theta)=Re[A\cos(\omega t+\theta)+ jA\sin(\omega t+\theta)]= Re(Ae^{j(\omega t+\theta)})=Re(Ae^{j\omega t}e^{j\theta})$$

Wait, if you only take real part, why do I need imaginary part. This is only way I can compute the phase, θ, conveniently.

Since Re and $e^{j\omega t}$ always exist, for simplicity

$$A\cos(\omega t+\theta) \rightarrow Ae^{j\theta}=A\angle\theta \rightarrow$$ Phasor representation *(amplitude & Phase)*

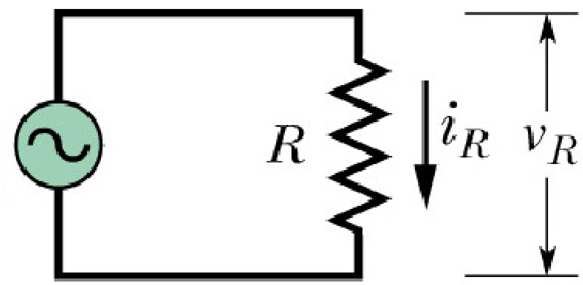
phasor is a complex number in polar form used to assist the calculation in ac circuits

In text book, bold uppercase quantity indicate phasor voltage or currents

Note the specific frequency ω of the sinusoidal signal, since this is not explicit apparent in the phasor expression

AC i-V relationship for R, L, and C

Resistive Load

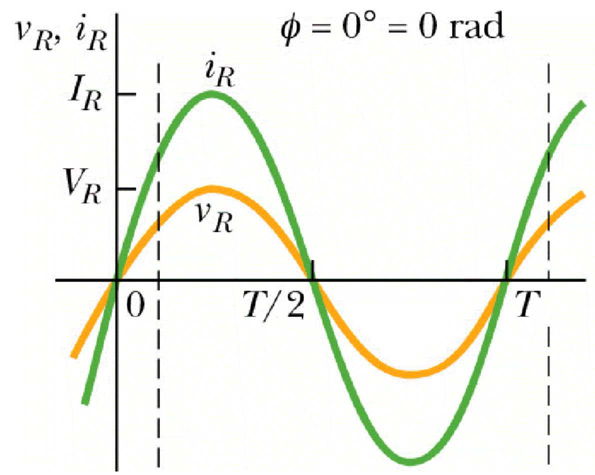


Source $v_S(t)$ =Asinωt

$$v_R = v_S(t) = A\sin\omega t$$

$$i_R = \frac{v_R}{R} = \frac{A}{R}\sin\omega t$$

v_R and i_R are in phase



Phasor representation: $v_S(t)$ =Asinωt = Acos(ωt-90°)= A ∠-90°=$\mathbf{V}_S$(jω)

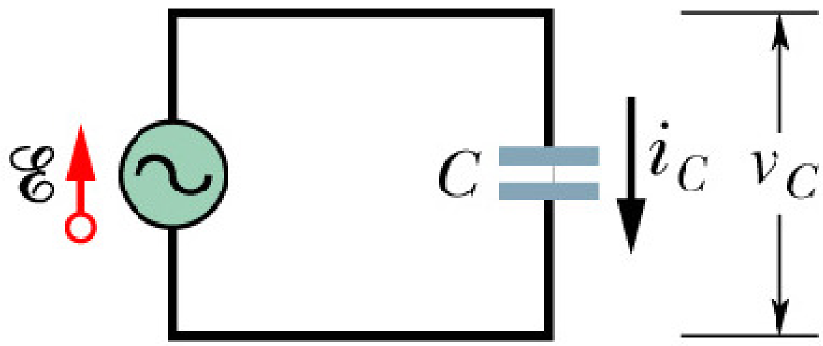
$\mathbf{I}_S$(jw) =(A / R)∠-90°

Impendence: complex number of resistance $Z=\mathbf{V}_S(j\omega)/\mathbf{I}_S(j\omega)$=R

Generalized Ohm's law $\mathbf{V}_S(j\omega) = Z\,\mathbf{I}_S(j\omega)$

Everything we learnt before applies for phasors with generalized ohm's law

Capacitor Load



$$v_C = A\sin\omega t$$

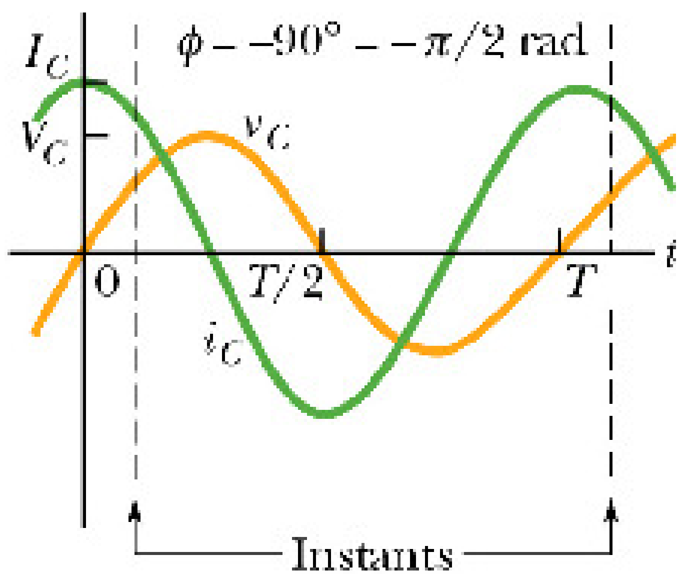
$$q_C = C v_C$$

$$i_C = \frac{dq_C}{dt} = \omega C A\cos\omega t$$

$$i_C = \frac{A}{1/\omega C}\sin(\omega t + 90)$$

ICE

$V_C(j\omega) = A\ \angle{-90°}$



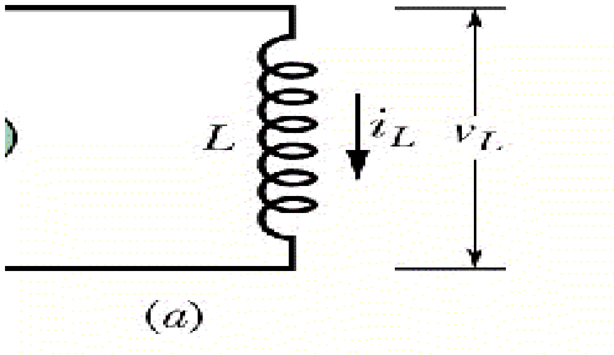
$$I_C(j\omega) = \frac{A}{X_c}\angle 0^o$$

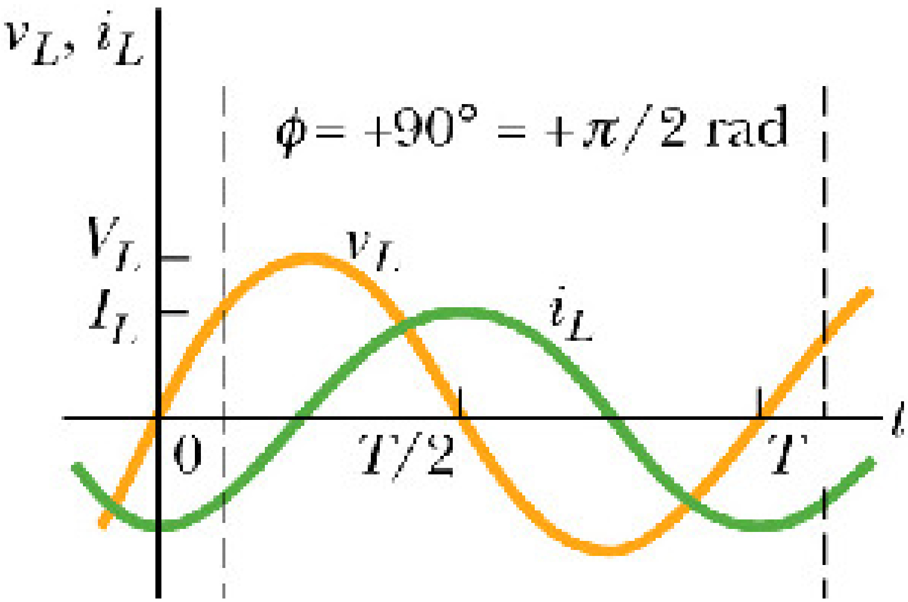
$$Z_C = \frac{V_C(j\omega)}{I_C(j\omega)} = X_C\angle -90^o$$

$$= \frac{-j}{\omega C} = \frac{-j \bullet j}{j\omega C} = \frac{1}{j\omega C}$$

Notice the impedance of a capacitance decreases with increasing frequency

Inductive Load





$$v_L = A\sin\omega t$$

$$v_L = L\frac{di_L}{dt}$$

$$i_L = \frac{A}{L}\int \sin\omega dt = -\left(\frac{A}{\omega L}\right)\cos\omega t$$

$$i_L = \frac{A}{\omega L}\sin(\omega t - 90)$$

ELI

Phasor: $\mathbf{V}_L(j\omega)=A\angle{-90°}$

$\mathbf{I}_L(j\omega)=(A/\omega L)\angle{-180°}$

$Z_L=j\omega L$

Opposite to Z_C, Z_L increases with frequency

Impedance

$$Z_R = R \quad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C} \quad Z_L = j\omega L$$

- Why Z_c and Z_L are complex number
  - Since they change the phase of signal. The phase is represented by the imaginary part of the complex number.
- Related questions: What does impedance physically change in a circuit?
  - Real part: amplitude, Imaginary part: phase.