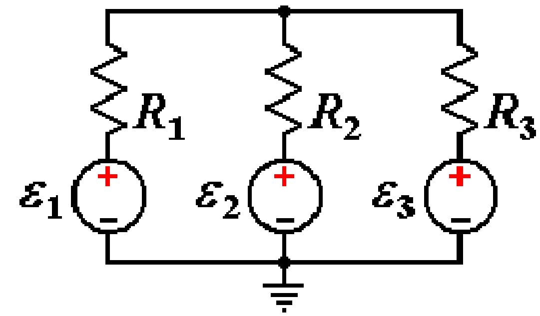
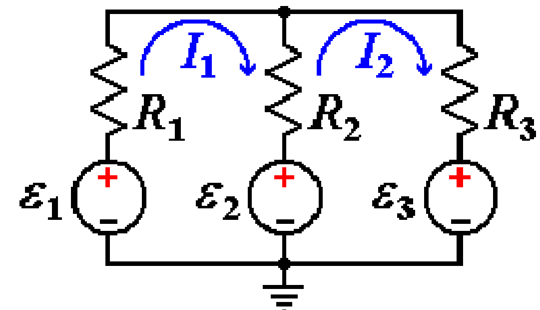
Method 2b: Mesh analysis





Example: 2 meshes

Step 1: Assignment of mesh currents (clockwise) (mesh is a loop that does not contain other loop).

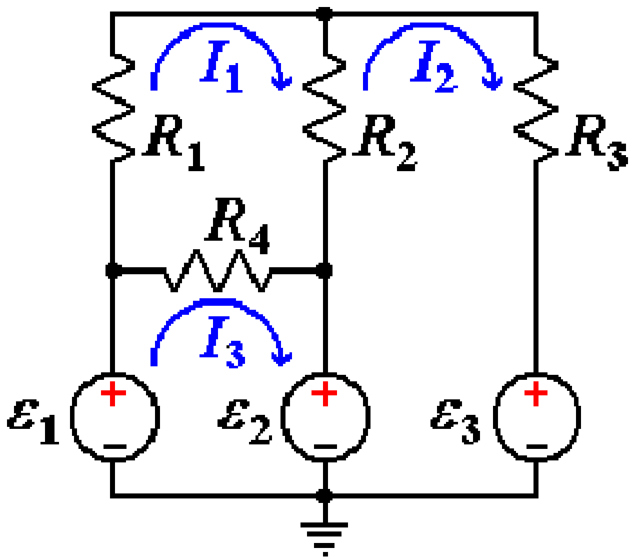
Step 2: Apply KVL to each mesh

- The so-called self-resistance is the effective resistance of the resistors in series within a mesh. The mutual resistance is the resistance that the mesh has in common with the neighboring mesh.
- To write the mesh equation in standard form, evaluate the self-resistance, then multiply by the mesh current. This will have units of voltage.
- From that, subtract the product of the mutual resistance and the current from the neighboring mesh for each such neighbor.
- Equate the result above to the driving voltage, taken to be positive if its polarity tends to push current in the same direction as the assigned mesh current.

Mesh 1	$(R_1+R_2)I_1$	$- R_2I_2$	$=\varepsilon_1 - \varepsilon_2$
Mesh 2	$- R_2I_1$	$(R_2+R_3)I_2$	$=\varepsilon_2 - \varepsilon_3$

Step 3: Solve currents

Sample circuit: 3 meshes



Mesh 1:

$$(R_1 + R_2 + R_4) I_1 \quad - R_2 I_2 - R_4 I_3 = 0$$

Mesh 2:

$$-R_2 I_1 + (R_2 + R_3) I_2 = \varepsilon_2 - \varepsilon_3$$

Mesh 3:

$$-R_4 I_1 \quad + R_4 I_3 = \varepsilon_1 - \varepsilon_2$$

Detailed Mesh Analysis Example

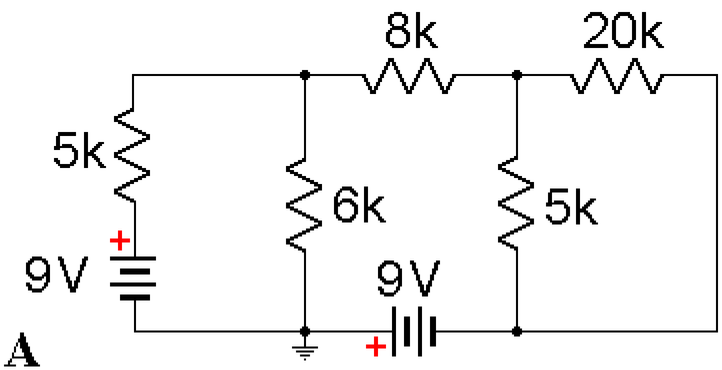
Find currents in each branches

Step 1: Replace any combination of resistors in series or parallel with their equivalent resistance.
Step 2: Choose clockwise mesh currents for each mesh and label accordingly.
Step 3: Write the mesh equation for each mesh.

Left mesh: $11\ I_1 - 6\ I_2 = 9$
Right mesh: $-6\ I_1 + 18\ I_2 = 9$

Here I suppress the "k" for each resistor; the final currents will be in milliamps.
Step 4: Solve the equations

Solution: I_1 : 4/3 mA 1.33 mA
*I*2 : 17/18 mA 0.94 mA



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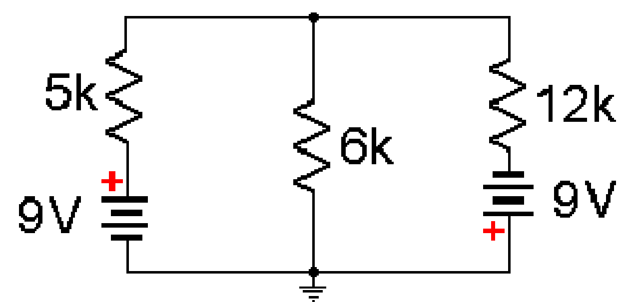
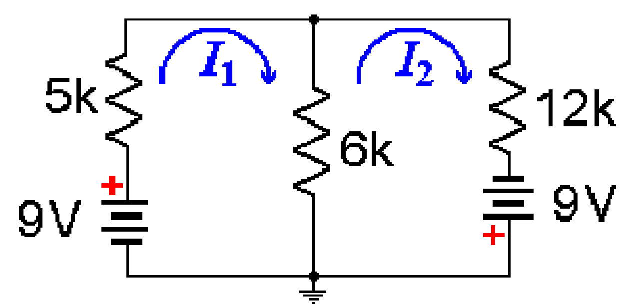
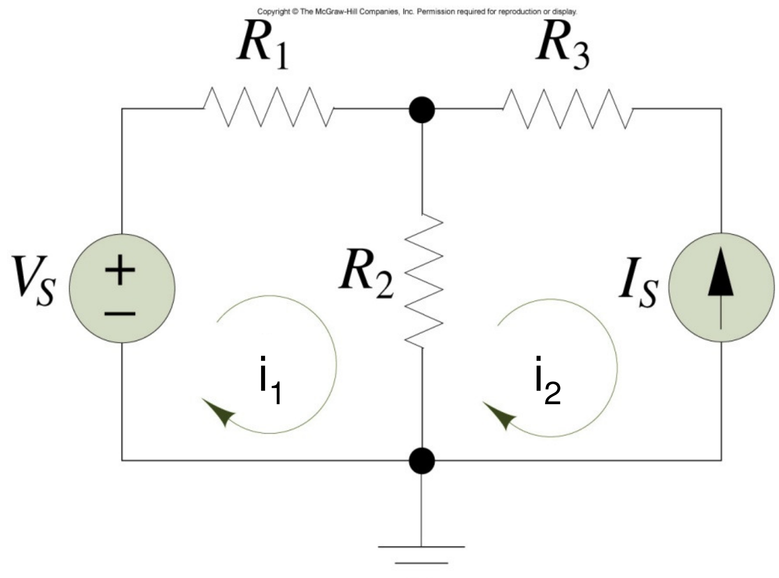




Mesh Analysis with Current Source



$$i_2 = -I_S$$

$$(R_1 + R_2)i_1 - R_2 i_2 = V_S$$

$$i_1 = \frac{V_S + R_2 i_2}{R_1 + R_2} = \frac{V_S - R_2 I_s}{R_1 + R_2}$$

Example with mixed sources: mesh analysis

- Identify mesh currents and label accordingly.
- Write the mesh equations

Mesh 1: $I_1 = -2$

mesh 2: $-4I_1 + 8I_2 - 4I_4 = 12$

Mesh 3: $8I_3 = -12$

Mesh 4: $-2I_1 - 4I_2 + 6I_4 = 10$

Check the calculation, they are wring

I_1 = - 2.0 A

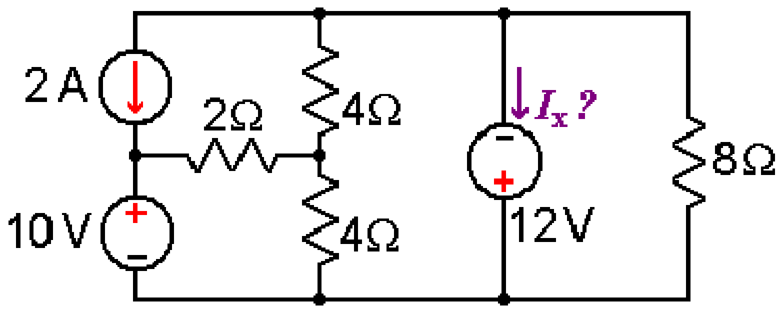
I_2 = 1.5 A

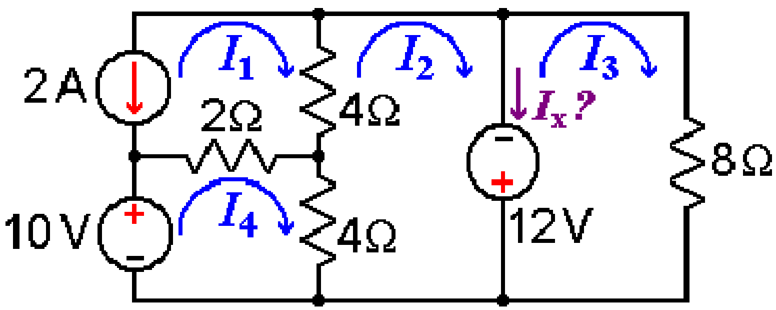
I_3 = - 1.5 A

I_4 = 2.0 A

$I_x = I_2 - I_3$

I_x = 3.0 A

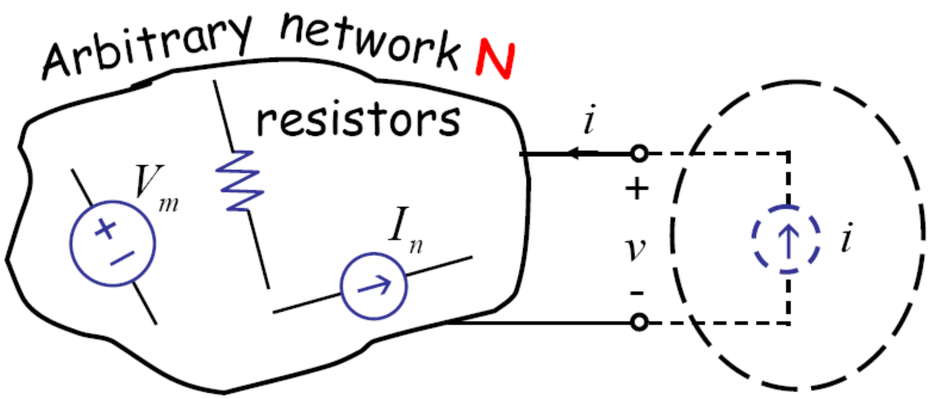




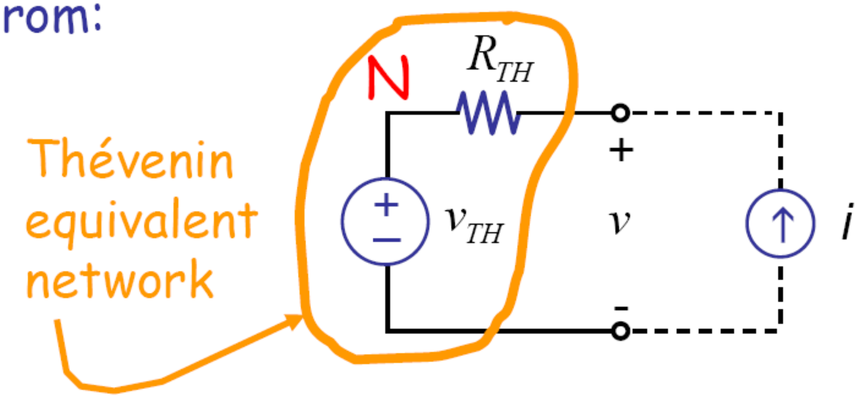
A Comparison between KVL/KCL method and mesh analysis

	KVL/KCL	Mesh analysis
Independent variables	Branch currents	Mesh currents
Number of equations	Number of branches	Number of meshes
Underlying principles	KCL and KVL laws	KVL explicit KCL implicit
Sign conventions	Voltage direction, loop direction	Self resistance, mutual resistance, source values
When to use what	A supplement to mesh analysis	A simpler method, always try it first
How to deal with current source	Current in branch is known. Assign voltage to branch as a variable. Then write KVL equations	

Yet another method: Thévenin and Norton Equivalent Circuits



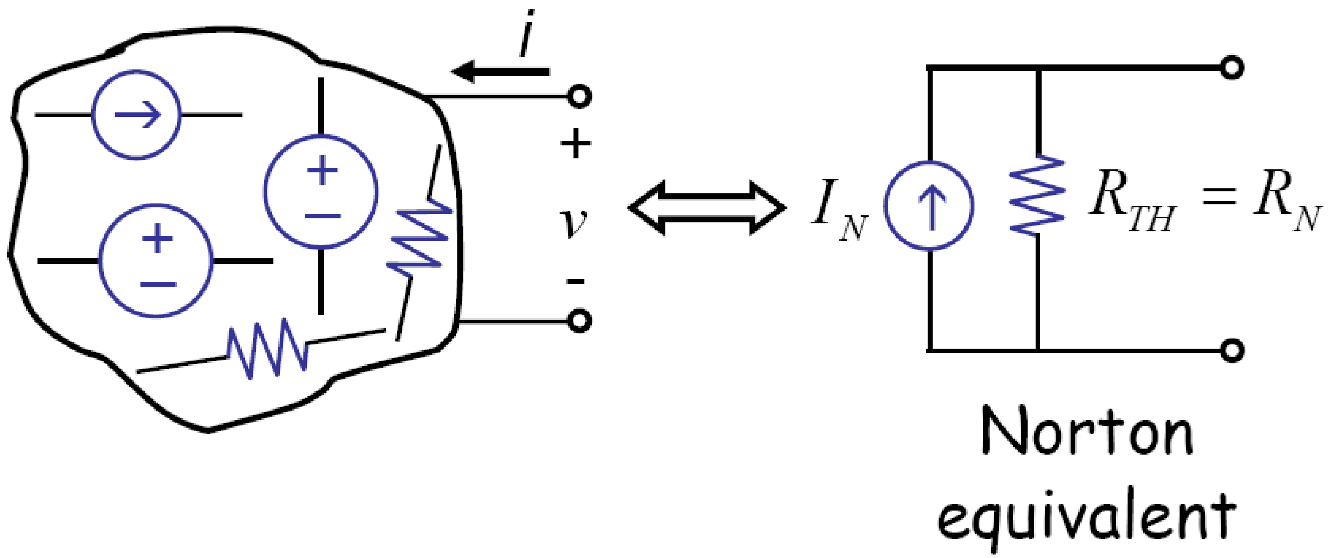
"Arbitrary network N" is indistinguishable from:



v_{TH} ⟶ open circuit voltage at terminal pair (a.k.a. port)

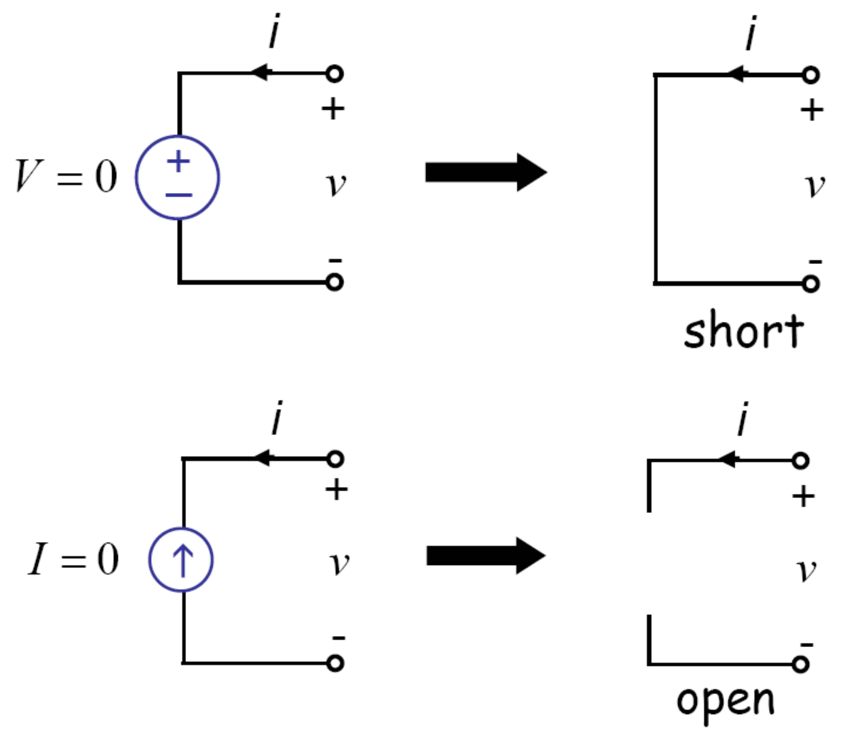
R_{TH} ⟶ resistance of network seen from port (V_m's, I_n's set to 0)

Norton Equivalent Circuit



Calculation of R_T and R_N

- $R_T = R_N$, same calculation
- Setting all sources to be zero ("killing" the sources)
  - Voltage source: short
  - Current source: open



Calculation of R_T and R_N: Cont.

- Calculate equivalent resistance seen by the load.

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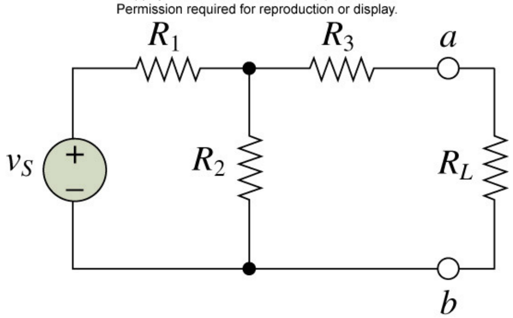
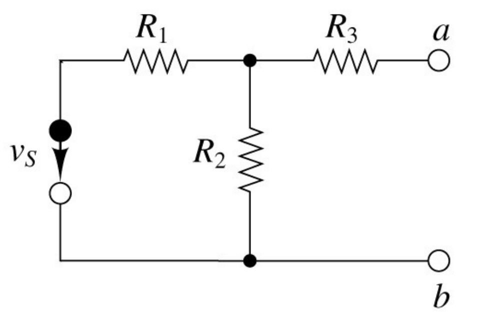


Complete circuit



Circuit with load removed for computation of R_T. The voltage source is replaced by a short circuit.

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