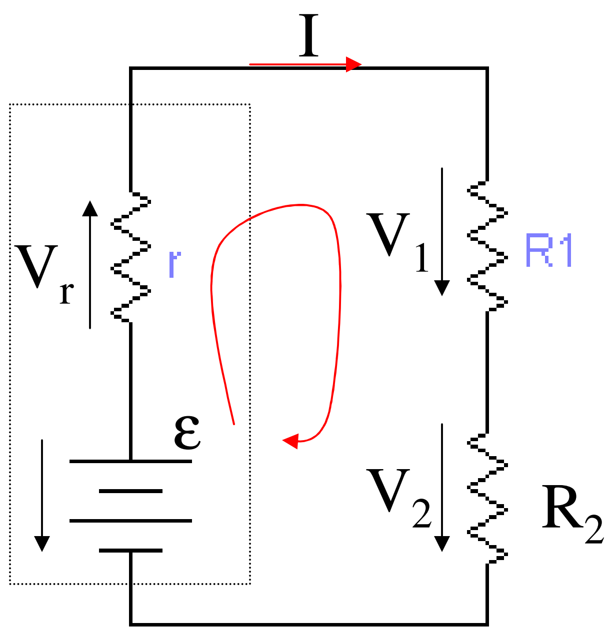
Method 2a: KVL & KCL

Kirchhoff's Voltage Law (KVL)

The algebraic sum of voltages around a closed loop must be zero

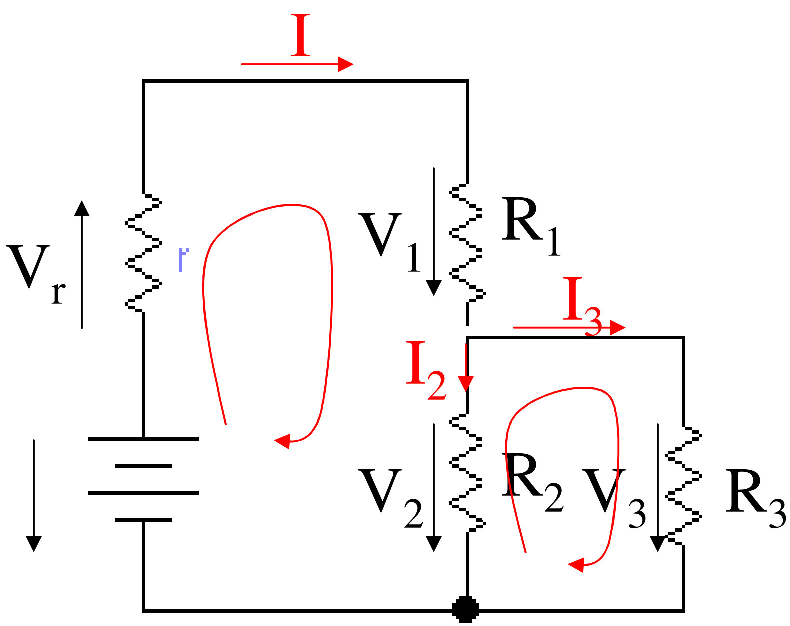


$-\varepsilon+V_r+V_1+V_2=0$

$\varepsilon=V_r+V_1+V_2$
$=I(r+R_1+R_2)$
$I=\varepsilon/(r+R_1+R_2)$

1. For each branch, draw current direction (arbitrary) and label the voltage direction (determined by the current direction). Voltage on a voltage source is always from positive to negative end.
2. Define either clockwise or counterclockwise direction as positive loop direction. Once the direction is defined, you have to use the same convention in every loop. The sign for the voltage cross each component is positive if voltage direction is the same as the defined loop direction, negative other wise. (can use current direction to determine the sign)
3. Apply KVL: +V if V is in the same direction defined above.

Kirchhoff's Voltage Law: multiloop

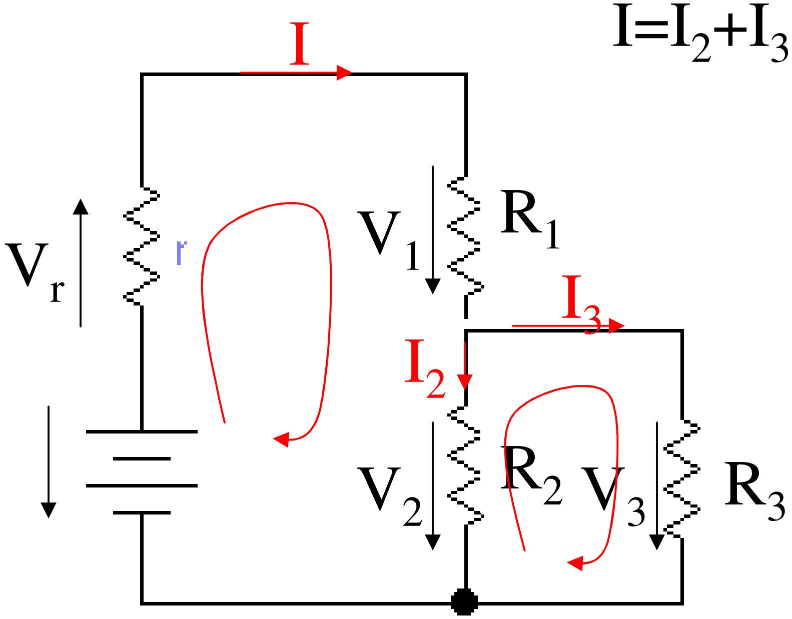


$-\varepsilon+V_r+V_1+V_2=0$
$-V_2+V_3=0$

$\varepsilon=Ir+IR_1+I_2R_2$
$-I_2R_2+I_3R_3=0$

Kirchhoff's Current Law (KCL)

The algebraic sum of current at a node must be zero: $I_{in}=I_{out}$



$I=I_2+I_3$ (1)

$\varepsilon=Ir+IR_1+I_2R_2$ (2)

$-I_2R_2+I_3R_3=0$ (3)

$\varepsilon=3$ V, $r=1\ \Omega$, $R_1=3\ \Omega$, $R_2=5\ \Omega$, $R_3=10\ \Omega$

$$\begin{pmatrix} 1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & -5 & 10 \end{pmatrix}\begin{pmatrix} I \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$$

$I - I_2 - I_3 = 0$ (4)

$4I + 5I_2 - 0I_3 = 3$ (5)

$0I - 5I_2 + 10I_3 = 0$ (6)

Cramer's Rule: Append. A

~~$0(5\times10-(-5)\times0)$~~ $-(-1)(3\times10-0\times0)$ $+(-1)(3\times(-5)-0\times5)$

$$I=\frac{\begin{vmatrix} 0 & -1 & -1 \\ 3 & 5 & 0 \\ 0 & -5 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & -5 & 10 \end{vmatrix}}=\frac{15+30}{50+20+40}=0.41 \qquad I_2=\frac{\begin{vmatrix} 1 & 0 & -1 \\ 4 & 3 & 0 \\ 0 & 0 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & -1 \\ 4 & 5 & 0 \\ 0 & -5 & 10 \end{vmatrix}}=\frac{30-0}{50+20+40}=0.27$$

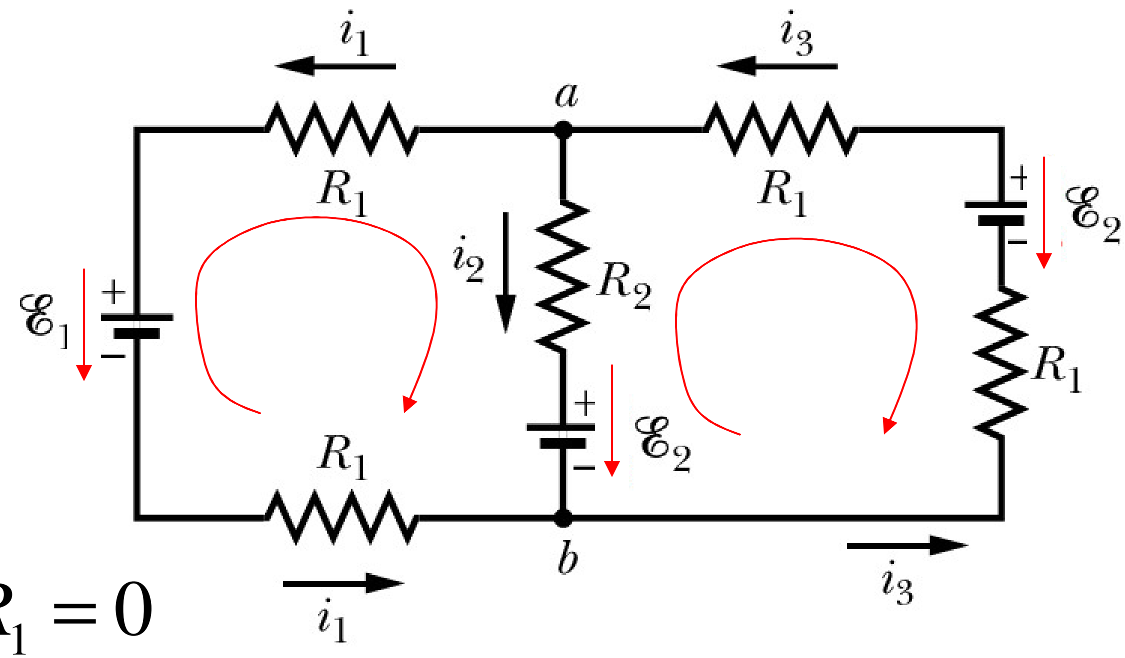
Last note on KVL & KCL

- If solutions to currents or voltages are negative, they mean the real directions are opposite to what you have defined!

- Deal with current source: current is known, but assign a voltage across the source that has to be solved

Sample Problem

Find magnitude and direction of currents



$\varepsilon_1 = 3.0V$, $\varepsilon_2 = 6.0V$

$R_1 = 2.0\Omega$, $R_2 = 4.0\Omega$

$$-\varepsilon_1 - i_1R_1 + i_2R_2 + \varepsilon_2 - i_1R_1 = 0$$

$$-\varepsilon_2 - i_2R_2 - i_3R_1 + \varepsilon_2 - i_3R_1 = 0$$

$$i_1 + i_2 = i_3$$

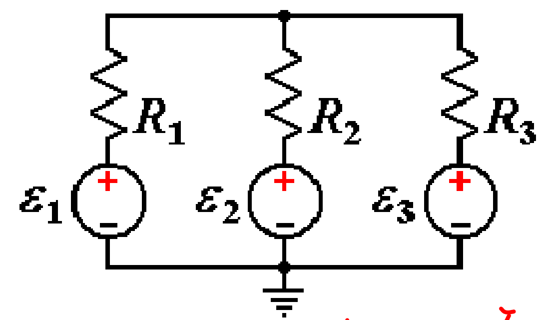
$$4i_1 - 4i_2 + 0i_3 = 3$$

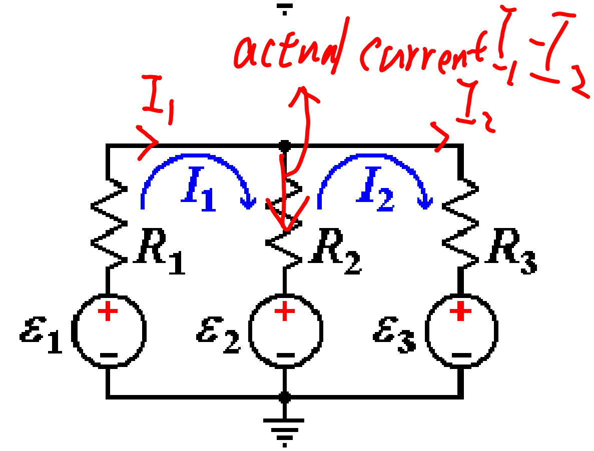
$$0i_1 + 4i_2 + 4i_3 = 0$$

$$i_1 + i_2 - i_3 = 0$$

$$i_1 = \frac{\begin{vmatrix} 3 & -4 & 0 \\ 0 & 4 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 4 & -4 & 0 \\ 0 & 4 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{-12-12}{-16-16-16} = 0.5(A)$$

Method 2b: Mesh analysis





Example: 2 meshes

Step 1: Assignment of mesh currents (clockwise) (mesh is a loop that does not contain other loop).

Step 2: Apply KVL to each mesh

- The so-called self-resistance is the effective resistance of the resistors in series within a mesh. The mutual resistance is the resistance that the mesh has in common with the neighboring mesh.
- To write the mesh equation in standard form, evaluate the self-resistance, then multiply by the mesh current. This will have units of voltage.
- From that, subtract the product of the mutual resistance and the current from the neighboring mesh for each such neighbor.
- Equate the result above to the driving voltage, taken to be positive if its polarity tends to push current in the same direction as the assigned mesh current.

KCL is implicit

$R_2(I_1 - I_2)$

Mesh 1	$(R_1+R_2)I_1$	$-R_2I_2$	$=\mathcal{E}_1-\mathcal{E}_2$
Mesh 2	$-R_2I_1$	$(R_2+R_3)I_2$	$=\mathcal{E}_2-\mathcal{E}_3$

$I_1 = I_2 + (I_1 - I_2)$

$I_1 = I_1$

Step 3: Solve currents