1. Consider a simple system with energy $E$ and volume $V$, containing $N$ particles.

(a) Formulate (briefly) the main postulates of thermodynamics.

(b) Formulate (briefly) the main postulates of statistical mechanics.

(c) By considering two simple systems separated by a partition which transmits energy, relate the entropy $S$ to the number of microstates $\Omega$.

(d) Using the relation from point (c), derive the postulates of thermodynamics from statistical mechanics (for $\partial S/\partial E$ give heuristic argument).

2. Consider a system of $N$ independent distinguishable particles, where $N$ is large. Each particle can be in two possible states with energies $\pm \epsilon$.

(a) Assuming a microcanonical ensemble, find the number of possible states of the system for a given total energy $E$.

(b) Find the entropy $S$ and the temperature $T$ of the system.

(c) Assume that $E > 0$, i.e., the number of particles in the state $\epsilon$ is larger than at $-\epsilon$ (this can be achieved by laser pumping). What temperature range does this state correspond to? What is the condition that such temperatures are meaningful?

(d) Draw $S/kN$ as a function of $E/\epsilon N$ and relate $T$ to various parts of this graph.

3. The material constants are often connected by rigorous relations. Find a relation connecting the isothermal compressibility $\kappa_T$, adiabatic compressibility $\kappa_S$, heat capacity at constant pressure $C_p$, and heat capacity at constant volume $C_V$ for a system with a fixed number of particles. The coefficients are defined as follows:

$$\kappa_T = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T, \quad \kappa_S = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_S, \quad C_p = T \left( \frac{\partial S}{\partial T} \right)_p, \quad C_V = T \left( \frac{\partial S}{\partial T} \right)_V.$$

*Hint:* Consider $S$ and $V$ as functions of $T$ and $p$. 