PHYS 813: Statistical Mechanics and Thermodynamics
Exam I

March 20, 2008, 75 minutes, closed book.

Please start each solution on a fresh sheet of paper, use only one side of the paper. Try to show how well you understand the problems. Always justify your reasoning. Assemble your solutions in the increasing numerical order. Derive all thermodynamic formulas beyond definitions of thermodynamic functions.

1. Consider a simple system with energy E and volume V, containing N particles.
   (a) Formulate (briefly) the main postulates of thermodynamics.
   (b) Formulate (briefly) the main postulates of statistical mechanics.
   (c) By considering two simple systems separated by a partition which transmits energy, relate the entropy S to the number of microstates Ω.
   (d) Using the relation from point (c), derive the postulates of thermodynamics from statistical mechanics (except for $S(E)$ being a monotonously increasing function).

2. The equation of state can constrain the forms of internal energy $U = E$. Assume that the number of particles N is constant.
   (a) Show that the equation of state of an ideal gas: $pV = NkT$, where p, T, and k are the pressure, temperature, and Boltzman's constant, implies that U depends only on T (i.e., is independent of V). *Hint:* Find $(\partial U/\partial V)_T$.
   (b) What is the most general equation of state that implies $U = U(T)$?

3. The elasticity of a rubber band can be described by a "one-dimensional" model of a polymer involving $N + 1$ molecules linked together end-to-end. There are N such links, each of length d. The angle between successive links is equally likely to be 0 or π. Assume there are no interactions between the molecules.
   (a) Find the number of arrangements $g(N, m)$ that give an overall length $L = 2md$.
   (b) Show that for $m \ll N$, $g(N, m)$ can be written as
   $$g(N, m) = g(N, 0)\mathrm{e}^{-4m^2/N}.$$
   (c) Find the entropy of the system as a function of L for $N \gg 1$ and $L \ll Nd$.
   (d) Assume that the system is in equilibrium with a heat bath of temperature T. Using the expression derived in (c) and appropriate thermodynamic relations, find the external force required to maintain the length L.