1. A surface with \( N_0 \) adsorption centers has \( N < N_0 \) gas molecules adsorbed on it, at most one molecule per center. Assume that the canonical partition function for a single adsorbed molecule is known and denote it by \( a(T) \). Neglect interactions between adsorbed molecules. Find the chemical potential of the system:

(a) using the canonical partition function;
(b) using the grand canonical partition function. In this case \( N \) can vary from 0 to \( N_0 \).

2. As a continuation of the previous problem, consider the (ideal) gas which is in equilibrium with the surface. Find the pressure of the gas as a function of the fraction of average number of occupied sites and of temperature.

3. For a single electron in a magnetic field \( B \), the Hamiltonian is

\[
\hat{H} = -\mu_B \mathbf{\sigma} \cdot \mathbf{B}
\]

where \( \mu_B \) is a constant and \( \mathbf{\sigma} \) is the spin operator. Choose the \( z \) axis to be directed along \( B \) and find the expression for the canonical density matrix elements \( \rho_{mn} \) in the representation in which

(a) \( \hat{\sigma}_z \) is diagonal;
(b) \( \hat{\sigma}_x \) is diagonal.

Next, calculate the average value of \( \hat{\sigma}_z \) in both representations.

4. A system with two energy levels is populated by \( N \) distinguishable noninteracting particles at temperature \( T \) with occupations determined by the canonical distribution.

(a) Find the average energy per particle.
(b) Find the behaviour of this energy as \( T \to 0 \) and \( T \to \infty \).
(c) Find the specific heat to the system.
(d) Find the behaviour of the specific heat as \( T \to 0 \) and \( T \to \infty \). Interpret the obtained results.

5. Consider a dilute, noninteracting gas of \( N \) distinguishable diatomic molecules of mass \( m \). Each molecule is a rigid rotor with \( (2J + 1) \)-degenerate energy levels \( \epsilon_J = \hbar^2 J(J+1)/2I \), where \( I \) is the moment of inertia. Start from the general expression for the canonical partition function \( Q_N(V, T) = \sum_n e^{-E_n/kT} \) where \( E_n \) is the total energy of \( N \)-molecule system. Calculate then the average energy, the specific heat at constant volume, and the entropy of the system in the limit \( kT \gg \hbar^2/2I \). You may use without proof the partition function for the ideal monoatomic gas \( Q_N^{\text{ideal}}(V, T) = \left[ V(2\pi mkT)^{3/2}/\hbar^3 \right]^N \). Hint: In the assumed limit, \( \sum_n f(n) = \int_0^\infty f(x)dx \).
6. Use the Debye model to calculate the internal energy and heat capacity of a one-dimensional atomic solid with length $L$ for both high and low temperature. Assume periodic boundary conditions. The Debye model treats the solid as a set of coupled harmonic oscillators and approximates the unknown normal modes of the system as plane waves propagating with the velocity of sound.

7. Consider a free particle in a box with periodic boundary conditions, in the momentum representation. The particle is in equilibrium with a heat bath and therefore is described by the canonical distribution. Denote the eigenfunctions of the Hamiltonian as

$$
\phi_k(r) = \frac{1}{L^{3/2}} e^{i k \cdot r}
$$

(a) Evaluate matrix elements of $e^{-\beta \hat{H}}$ in the basis $\phi_k(r)$.

(b) Find the canonical partition function in terms of $L$ and $\lambda = \hbar / \sqrt{2\pi mkT}$.

(c) Find the density operator $\hat{\rho}$ in the basis $\phi_k(r)$.

(d) Calculate the average value of $\hat{H}$ as $\text{Tr}(\hat{\rho} \hat{H})$. Express your answer in terms of $L$, $\lambda$, and $T$. 