1. Taylor: Problem 5.1.

2. Taylor: Problem 5.18.

3. Taylor: Problem 5.25.
   (d) In another oscillator, the amplitude drops by a factor of $e$ after four periods. Find the ratios $\beta/\omega_0$ and $\omega_1/\omega_0$. Discuss how the oscillatory character of the motion depends on these ratios.

4. Taylor: Problem 5.27.

5. Consider an underdamped harmonic oscillator.
   (a) Find an expression for the energy of an underdamped harmonic oscillator as a function of time and then, by calculating its derivative, an expression for the rate of energy loss due to dissipation.
   (b) Make an approximate plot of the latter expression assuming the damping parameter $\beta$ equal about 0.1 of the natural frequency $\omega_0$. Interpret the minima and maxima appearing on the plot.
   (c) For a small $\beta$, calculate the average rate at which the oscillator loses energy, i.e., the time average of the energy loss in one cycle. In this limit, one can assume that the exponential factor in the solution is approximately constant over one cycle.

6. A grandfather’s clock has a pendulum of length $l$ with a bob of mass $m$ and is in a constant gravitational field of acceleration $g$. To counteract damping, a mechanism gives an almost instantaneous “kick” to the pendulum at the end of each cycle, restoring the amplitude to a constant value $\theta_m$. The mechanism uses the gravitational energy of a weight of mass $M$ which falls a distance $h$ in time $d$. Since this amplitude is small, one can assume that the pendulum behaves like the standard damped harmonic oscillator between the kicks. Assume that the only energy losses in the system are due to the damping of the pendulum. In other words, approximate the system as a damped harmonic oscillator whose amplitude is instantaneously restored to the initial value at the end of each cycle.
   (a) Write down the Newton equation for the pendulum in terms of the angular deviation $\theta$ from the equilibrium and then the solution to this equation (no need to derive these equations here).
   (b) Find the energy loss of the weight per one period of the pendulum.
   (c) Define, in general terms, the mechanical energy dissipated by the pendulum in one period. Note that to calculate this quantity, one needs to use only the potential energy.
(d) Now find the detailed expression for the potential energy of the pendulum $U(t)$ in terms of $m$, $g$, $l$, and $\theta(t)$.

(e) Use the expression for the positions of maxima of a damped harmonic oscillator derived earlier to calculate the dissipated energy. *Hint:* Show that although the cosine function appearing in the solution is not equal to 1 at the maxima of $\theta(t)$, its value is the same at all the maxima.

(f) Use the given quantity $\theta_m$ to eliminate the arbitrary constants in the expression for the dissipated energy.

(h) Compare the gravitational and the dissipated energy to find the damping constant of the pendulum. Then find an approximate expression for small damping.

7. Taylor: Problem 5.43. Interpret “a couple of centimeters” as 2 cm.

8. Taylor: Problem 5.44. The constants listed in the problem are defined by Eqs. (5.57), (5.64), and (5.77) in the text.

9. A particle of mass $m$ slides without friction on horizontal table. The particle is attached to one end of a massless spring of equilibrium length $a$ and spring constant $k$. The other end of the spring is attached to a point on the table, such that the spring can rotate around this point without friction.

(a) What is the net force acting on the particle? Note that the gravity force is balanced by the reaction of the table and there is no friction.

(b) Find the expression for the kinetic energy $T$, the potential energy $U$, and the magnitude of the angular momentum $|l|$ of the particle in cylindrical coordinates. Then express $T$ in terms of $|l|$.

(c) Define the effective potential energy $U_{\text{eff}}$ as the sum of $U$ and the part of $T$ depending on $|l|$. Sketch the $U$ and $U_{\text{eff}}$ potential energy functions.

(d) Find the angular velocity of the rotational motion of the particle $\omega_0$ such that the particle moves exactly on a circular orbit of radius $r_0$. Note that $\omega_0$ is not equal to $\sqrt{k/m}$.

(e) Relate $\omega_0$ to $U_{\text{eff}}$. Interpret this result physically.

(f) Argue qualitatively that for angular velocities $\omega$ larger than $\omega_0$, the particle will perform some kind of oscillatory motion in the radial coordinate.

(g) Show that if $\omega$ is close to $\omega_0$, the particle will oscillate around the orbit of radius $r_0$ and the motion in the radial coordinate will be that of a simple harmonic oscillator. Find the frequency of such small oscillations. *Hint:* Show that $U_{\text{eff}}$ can be approximated by a harmonic oscillator potential. Then find the solution from the total energy expression derived in points (b) and (c).

Express all you answers in term of $m$, $k$, $a$, and $r_0$. 

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