1. Two blocks of unequal masses $m_1$ and $m_2$ are connected by a weightless string over a frictionless, massless pulley, as shown in the figure. The coefficient of kinetic friction between the block and the incline is $\mu_k$. The incline angle is $\theta$. The system is in a uniform gravitational field of strength $g$ directed downward.

(a) If block A slides down the incline at a constant speed, what is the mass of block B in terms of the other parameters given?

(b) The same question for block A moving at a constant speed up the incline.

(c) For the ratio of masses obtained in (b), is it possible for block A to move at a constant speed down the incline? Justify your answer.

2. A jet plane flies at a constant speed of Mach 3. The pilot starts by going vertically down to some altitude where she changes to a circular trajectory (vertical loop). When flying on the circle, the pilot will feel sensation due to the acceleration. (a) Where will this acceleration be the largest? (b) Assume that the highest perceived acceleration that the pilot can withstand is $9g$ (we assume that a person at rest on the Earth is feeling $1g$). What is the minimum radius of the loop that the pilot can take?

3. In a classic movie scene, a actor weighting 100 kg jumps from a bridge onto the platform of an open railroad car moving with a constant speed of 1.20 m/s and after a period of slipping on the floor comes to rest with respect to the car. The coefficient of kinetic friction between the car and the soles of actor’s boots is 0.400.

(a) What is the final kinetic energy of the actor with respect to the Earth?
(b) What is the work done by the engine of the train on accelerating the actor?

(c) The answers obtained in (a) and (b) should be different. Explain what happens to the balance between the two values.

4. A homogenous spherical bead of mass $m$ and radius $r$ starts to roll without slipping from the top of a sphere of radius $R$ (see the figure). The initial velocity of the bead is $v_0 = 0$. Find the angular velocity $\omega$ of the bead immediately after it leaves the sphere.

(a) Draw a diagram showing the forces acting on the bead at the beginning of the motion and some time later when the bead is still in contact with the sphere.

(b) Explain the relations between the forces at the moment when the bead leaves the sphere and write the proper equation resulting from the balance of the forces (this equation should relate the linear velocity $v$ of the bead to the angle $\theta$).

(c) What is the relation between the linear and angular velocities of the bead when it is still in contact with the sphere (note that the motion is without slipping).

(d) Write the equation relating the potential energy loss to the kinetic energy of the bead.

(e) Find the angular velocity of the bead immediately after it leaves the sphere.

5. The recoil of a cannon has to be taken into account in order to hit the intended target. Consider a 1400 kg cannon which fires 70.0 kg shells with the speed of 556 m/s relative to the muzzle. The cannon is mounted on frictionless rails, so that it recoils freely. The cannon is set at an elevation angle of 39.0° above the horizontal at rest with respect to the Earth and then fired.

(a) What is the speed of the shell with respect to the Earth?
(b) What is the angle with respect to the ground that the shell is projected?

6. An electron of mass \( m \) traveling with speed \( v_i \) collides head-on with an atom of mass \( M \), initially in ground state, and excites the atom to an energy level lying \( \Delta E \) above the ground state. After the collision, the electron bounces back with (unknown) speed \( v_f \) and the atom, which was initially at rest, travels with (unknown) speed \( V_f \). Find the minimum value of \( v_i \) making the excitation possible (note that it is not just the speed corresponding to the kinetic energy of electron equal to \( \Delta E \)). The masses and the excitation energy are known.

7. A small solid marble of mass \( m \) and radius \( r \) rolls without slipping along the track (similar to the “loop-the-loop” circus constructions) shown in the figure. The marble was released from rest somewhere on the straight section of the track. What is the minimum height \( h \) at which the marble can be released so that it does not leave the track at the top of the loop? The radius of the loop \( R \) is much larger than \( r \).

![Diagram of a marble rolling without slipping along a track](image)

8. A damped harmonic oscillator consists of a block of mass \( m = 2.00 \text{ kg} \) attached to a spring with the harmonic constant \( k = 10 \text{ N/m} \) (another end of the spring is fixed). The oscillator is in a medium providing a damping force \( F = -bv \) at velocity \( v \), where \( b \) is a constant. At zero time, the system oscillates with an amplitude of 25.0 cm. After the completion of four periods \( T \), the amplitude falls to 20.0 cm.

(a) Explain the notion of the amplitude for a damped oscillator.

(b) Find the value of \( b \).

(c) Should the mechanical energy of the oscillator (i.e., the oscillator energy due to conservative forces) be conserved? Justify your answer.

(d) Compare the mechanical energies at times \( t = 0 \) and \( t = 4T \).
9. A 2200-lb car carrying four 180-lb people travels over a rough “washboard” dirt road with corrugations 13 ft apart. The car bounces with maximum amplitude when its speed is 10 mi/h. The car now stops and the four people get out. How much does the car body rise on its suspension owing to this decreased weight? Assume the simplest approximate relation between the maximum amplitude and the frequencies of oscillations (satisfied in the limit of negligible damping).

10. A planet consists of a spherical core and two outer spherical layers with increasing radiiuses $R_1$, $R_2$, and $R_3$. The corresponding densities are $\rho_1$, $\rho_2$, and $\rho_3$. Assume that the planet is at rest.

   (a) Calculate $g$ at the surface of the planet.

   (b) A tunnel is drilled through the center of the planet. Find $g$ at $R_2$.

   (c) Suppose the planet is instead a uniform sphere of the same mass and radius $R_3$. What is $g$ at the same depth as in (b).

   (d) Using the values $10^6$, $3 \times 10^6$, and $6 \times 10^6$ m for $R_1$, $R_2$, and $R_3$, respectively, and the values $14 \times 10^3$, $11 \times 10^3$, and $4 \times 10^3$ kg/m$^3$ for $\rho_1$, $\rho_2$, and $\rho_3$, respectively, calculate the numerical values for steps (b) and (c). Rationalize your findings and explain how measurements of gravity can give information about the composition of this planet.

11. A particle of mass $m$ moves on a circle of radius $r$ with a constant speed $v$. By differentiating the position vector of the particle, find its acceleration. Then use Newton’s laws to find the force that acts on the particle. If this force is not zero, explain why this is not in contradiction to the fact that the speed of the particle is constant.

12. A thin rod of length $L$ and mass $m$ is suspended at its midpoint from a long wire, see the figure. The rod performs simple angular harmonic motion with the period $T_a$. The moment of inertia of a thin rod about its perpendicular axis can be shown to be $I = mL^2/12$. Then, an irregular object is hung from the same wire and its period of oscillations is measured to be $T_b$. What is the moment of inertia of this object about its suspension axis?
13. Calculate the moment of inertia (rotational inertia) of a solid thin rod of length $L$ and uniform density $\rho$ with respect to an axis of rotation through the center of the rod and perpendicular to its length. The rod has a uniform cross-sectional area $A$ (perpendicular to the length).

14. Two point-like objects, each with mass $m$, are connected by massless, stretchless rope of length $l$. The objects are suspended vertically near the surface of the Earth, so that one object is hanging below the other, and then released. Find the tension in the rope after the objects are released. Use proper approximations to express the tension as a function of $m$, $l$, mass of the Earth $M$, radius of the Earth $R$, and gravitational constant $G$. 