1. Two equal masses, \( m_1 = m_2 = m \), are joined by a massless string of length \( L \) that passes through a hole in a frictionless horizontal table in Earth gravitational field with the gravity acceleration \( g \). The first mass slides on the table, while the second hangs below the table and moves up and down in a vertical line.

(a) Assuming the string remains taut, write down the Lagrangian in terms of the polar coordinates \((r, \phi)\) of the mass on the table.

(b) Find the two Lagrange’s equations and interpret the \( \phi \) equation in terms of the angular momentum \( l \) of the mass remaining on the table.

(c) Express \( \dot{\phi} \) in terms of \( l \) and eliminate \( \dot{\phi} \) from the \( r \) equation. Now use the \( r \) equation to find the value \( r = r_0 \) at which the mass on the table can move in a circular path.

2. Consider a mass \( m \) which is constrained to move on the frictionless surface of a fixed vertical cone in a uniform gravitational field with the gravity acceleration \( g \) (directed downwards). The angle between the symmetry axis of the cone and any straight line lying on the cone is \( \alpha \). Choose a coordinate system with the \( \hat{z} \) axis along the symmetry axis of the cone, tip of the cone at the center of this system, and the cone on the positive part of \( \hat{z} \).

(a) Write down the Hamiltonian \( H \) for the mass using the cylindrical coordinates \( z \) and \( \phi \) as generalized coordinates \( q_i \) and the corresponding generalized momenta \( p_i = \partial L / \partial \dot{q}_i \) where \( L \) is the Lagrangian.

(b) Show that for any given solution there are minimum and maximum heights \( z_{\text{min}} \) and \( z_{\text{max}} \) between which the motion is confined. \textit{Hint:} \( H = E \), the total energy of the system.

(c) Write down the four Hamilton’s equations. Briefly discuss these equations, but do not solve them.

(d) Use the results obtained above to qualitatively describe the motion of the mass on the cone.

3. Consider an inertial coordinate system \( S_0 \) fixed in space, with axes along unit vectors \( \hat{x}_i, \ i = 1, 2, 3 \), and a rotating coordinate system \( S \), with its origin at the origin of \( S_0 \) and axes along unit vectors \( \hat{e}_i, \ i = 1, 2, 3 \). \( S \) rotates around an axis \( \omega = \sum_{i=1}^{3} \omega_i \hat{x}_i \) with the angular velocity \( \omega = |\omega| \). Find the relation between the time derivatives of a time-dependent vector \( \mathbf{A} \) computed in systems \( S_0 \) and \( S \). You may use without proof the relation \( \dot{\mathbf{r}} = \omega \times \mathbf{r} \).