1. Taylor: Problem 3.27.


3. A system consists of $N$ masses $m_\alpha$ at positions $r_\alpha$ relative to a fixed origin $O$. Let $r'_\alpha$ denote the positions of the masses $m_\alpha$ relative to the center of mass (COM); that is, $r'_\alpha = r_\alpha - R$, where $R$ is the COM position.

   (a) Make a sketch to illustrate this last equation.

   (b) Prove the useful relation that $\sum m_\alpha r'_\alpha = 0$. Can you explain why this relation is nearly obvious?

   (c) Use this result to show that the rate of change of the angular momentum about COM is equal to the total external torque about the COM:

   $$\dot{L}(\text{COM}) = \Gamma^{\text{ext}}(\text{COM}).$$

   Note that this result is nontrivial since the COM may be accelerating, so it is not necessarily a fixed point in any inertial frame.

4. A juggler is juggling a uniform rod of mass $m$, one end of which is coated in tar (of negligible mass) and burning. The juggler is holding the rod by the opposite end and throws it up. At the moment of release, the rod is horizontal, its center of mass is traveling vertically up at speed $v_0$, and it is rotating in a vertical plane with angular velocity $\omega_0$. To catch the rod, the juggler wants to arrange that when it returns to his hand, it will have made an integer number of complete rotations. For a given $\omega_0$, what should $v_0$ be for the rod to have made exactly $n$ rotations when it returns to his hand? Neglect air resistance. Quote all the laws used to solve this problem.

5. Taylor: Problem 4.3.


8. Taylor: Problem 4.23.