1. Using the method of undetermined Lagrange multipliers, find the dimensions of the rectangular box of maximum volume circumscribed by

   (a) A sphere of radius $R$.
   (b) An ellipsoid with semiaxes $a$, $b$, and $c$.

2. Taylor: Problem 7.52.

3. Consider the simple pendulum with the length $l$ and the mass of the bob $m$. Do not assume small angles. Write down the Lagrangian for the system and the equation of the constraint in terms of the Cartesian coordinates $x$ and $y$. Find the set of Lagrange equations with the undetermined multiplier. Use these equations and the relation between the angle of oscillation and the Cartesian coordinates to obtain the familiar equation for a simple pendulum in terms of the angle.

4. Consider a double Atwood machine, in Earth’s gravitational field, constructed as follows. A mass $m_1$ is suspended from a string that passes over a massless pulley on frictionless bearings. The other end of this string supports a second similar pulley, over which passes a second string supporting a mass of $m_2$ at one end and $m_3$ at the other end. Using as the coordinates the vertical positions of the three masses, write down the Lagrange equations of motion with an undetermined Lagrange multiplier. Use these equations to find the tensions in both moving strings.

5. Taylor: Problem 13.5.
