
2. A particle of mass \( m \) can slide freely along a straight wire placed in the \( x - y \) plane whose perpendicular distance to the origin \( O \) is \( h \). Denote the projection of \( O \) on the wire by \( C \). The line \( OC \) rotates around the origin (in \( x - y \) plane) at a constant angular velocity \( \omega \). The particle is subject to a gravitational force acting down the \( y \) axis. Find the equations of motion under the initial conditions \( \theta(0) = 0, q(0) = 0, \text{ and } \dot{q}(0) = 0 \), where \( \theta \) is the polar angle of \( OC \) and \( q \) is the distance of the particle from \( C \). Sketch the solution.


5. A uniform density cube of mass \( m \), side \( 2b \), and center at \( C \) is placed on a fixed horizontal cylinder of radius \( r \) and center \( O \) in Earth’s gravitational field. The cube is originally put so that \( C \) is centered above \( O \), but it can roll from side to side without slipping. Use the Lagrangian approach to find the angular frequency of small oscillations about the top position. Make the small angle approximation at the level of the Lagrangian and retain only the leading term separately in the kinetic and potential energy. **Hints:** The kinetic energy of the cube consists of the kinetic energy of the center of mass and of the kinetic energy of rotational motion of the cube around its center of mass with the moment of intertia of the cube \( I = 2mb^2/3 \). Find \( x \) and \( y \) coordinates of COM from the triangles marked on the second figure. For the small angle approximation: \( \sin \theta \approx \theta \) and \( \cos \theta \approx 1 - \theta^2/2 \).

6. A small object of mass \( m \) is initially at rest, located at a distance \( d \) from one edge of a frictionless, horizontal surface in Earth gravitational field with a constant acceleration \( g \). At time \( t = 0 \), the surface begins to pivot about this edge with a constant angular speed \( \omega \).

   (a) Find an expression for the position of the object (distance \( r \) from the pivot axis) as a function of time.

   (b) Find an expression for the normal force acting on the object as a function of time.