ASYMMETRIC MAGNETIC RECONNECTION:
A PARTICLE-IN-CELL STUDY

by

Kittipat Malakit

A dissertation submitted to the Faculty of the University of Delaware in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Physics

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DEDICATION

To Sumitda and Khanchai Malakit, whose love toward me
is the foundation of my life.

And

To Michael Shay, an adviser who I cannot thank enough.
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I am now at the end of a journey in my life. However, it is important to note that this long journey cannot be completed alone by myself. May I pay tribute to those who has helped bring me to this point in this special part of the dissertation.

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ABSTRACT

Magnetic reconnection is an important plasma process for which the conversion from the magnetic energy into the flow energy occurs. Conventionally, studies of magnetic reconnection have mainly focused on symmetric reconnection where the inflow magnetic field, density, and temperature are identical on the two upstream sides. In nature, however, these ideal conditions are not normally realized. Nevertheless, recently, the asymmetry of the inflow has been taken into account. By performing a Sweet-Parker-type analysis, Cassak and Shay [12] arrived at scaling relations for both gross properties (reconnection rate, outflow speed, and outflow density) and the diffusion region structure (the locations of the X-line and stagnation point) of asymmetric reconnection. The scaling relations have been tested by a number of following studies. However, a systematic test of the theory has been limited to only the fluid approach.

By performing fully kinetic particle-in-cell simulations with inflow variables varying systematically, we are able to show that the fluid physics used in deriving the scaling relations for the gross properties is still valid in the fully kinetic picture. However, there is definitely room for improvement indicating that there is other physics playing a role. Furthermore, we demonstrate that the Hall signatures and the structure of the diffusion region are fundamentally linked. We find that to maintain 1D equilibrium across the diffusion region along the inflow direction, the strong magnetic gradient is required at the stagnation point where there is transition between the two populations of the inflow plasmas. The magnetic gradient, in turn, creates the out-of-plane current. The electron flow required for carrying the current then creates the Hall electric and magnetic fields. We note that the Hall signatures in asymmetric reconnection are not as clean as the symmetric case due to other physics involved in asymmetric reconnection. In addition to signatures derived from the Hall physics, some
signatures, namely the kinetic electric field and the electron anisotropy, are found to be resulting from kinetic physics. The kinetic electric field is spawned from the effect of finite Larmor radius. The electron anisotropy results from two-dimensional nature of the X-line.
Chapter 1

INTRODUCTION

To our eyes, the Sun has a hot and bright surface that seems to release a steady stream of energy. In reality, the Sun is not like this. It is rather an erupting mass of plasma. Solar flares, or bursts on the Sun’s surface, are common especially during solar maximums (a period where sunspots are most abundant). These flares are often associated with Coronal Mass Ejections (CME), which are clouds of particles released from solar corona in the vicinity of the flares. When the solar energetic particles from flares and CMEs hit the Earth, they can disturb the magnetospheric environment and lead to many possible consequences, such as satellite failures, general communication failures, widespread power blackouts, and radiation risks to astronauts and plane passengers. Furthermore, the energetic particles can introduce and spawn the beautiful auroras [19]. All the solar flares and CMEs responsible for these effects have something in common. They originate from a plasma mechanism which converts the solar magnetic energy into the kinetic energy of the particles. This mechanism is called “Magnetic Reconnection” and will be our subject of study in this dissertation.

1.1 Reconnection at a Glance

Magnetic reconnection is associated with the magnetic field and is a common process in plasmas. It occurs at the boundary of two regions with opposite (or partially opposite) magnetic field directions. To get an idea of what it is, we consider two parallel wires which contain identical currents flowing out of the page. These two wires, shown in Figure 1.1, produce a magnetic field that has a neutral point denoted with an × in the figure. Let us label a magnetic field line between wire number 1 and the neutral point with red and a field line between wire number 2 and the neutral point with blue
(Figure 1.1(a)). When we move the wires toward the neutral point, the red and blue field lines will do the same. Eventually, the two field lines will meet and form an X-shape field line, with the center of the X being the neutral point (Figure 1.1(b)). At this point, the field line of the same color can break and reconnect with the field line of the other color. Detecting the words “break” and “reconnect” one might wonder whether this process violates $\nabla \cdot \mathbf{B} = 0$, where $\mathbf{B}$ is the magnetic field? The answer is “No.” The neutral point perfectly complies with $\nabla \cdot \mathbf{B} = 0$. The number of field lines (i.e., the magnetic flux) pointing toward are equal to the number of field lines pointing away from the neutral point. The breaking and reconnecting of magnetic field lines can therefore be done at this point without violating any fundamental physical law. Finally, moving the wires toward the neutral point even further results in the newly reconnected red-blue field lines advancing to the sides (Figure 1.1(c)). Although a trivial one, this process where there is breaking and reforming of the magnetic field is called magnetic reconnection.

Things get more interesting when the space between the wires is filled with a plasma. The interactions between the plasma and the magnetic field now need to be taken into account. Since plasma is highly conducting, the electric field in the frame of the plasma is zero. Any presence of the internal electric field will be quickly countered by the motions of the charged particles. According to Faraday’s law, when the electric field is zero, there must be no change in the magnetic field. This means the magnetic field is stuck in the plasma. When a plasma moves, the magnetic field has to move along. The plasma that co-moves with the magnetic field is said to be under the “frozen-in” condition. We call the plasma that is perfectly under frozen-in condition “ideal” plasma.

We emphasize that under the frozen-in condition, the electric field is zero only in the plasma frame. If an element of plasma moves with the velocity $\mathbf{u}$ as observed in the lab frame, the electric field of the plasma element in the lab frame $\mathbf{E}$ can be thought to come into existence purely by means of the frame transformation, which
Figure 1.1: Magnetic field created by two parallel wires with identical current flowing out of the page. (a) At the initial time, we color a magnetic field line between wire number 1 and the neutral point with red. Its counterpart between wire number 2 and the neutral point is colored with blue. (b) At a later time after we move the wires closer to each other, the red and blue line meet at the neutral point $X$. (c) After making the wires to be even closer, the blue and red lines join and move out to the sides. Throughout the whole process, the field lines at the neutral point are perpendicular to one another.
follows
\[ E = -\frac{u}{c} \times B, \quad (1.1) \]
where \( B \) is the magnetic field of the plasma element measured in the lab frame and \( c \) is the speed of light. Note that the expressions in this dissertation are in \( cgs \) units unless stated otherwise.

The frozen-in condition can be viewed from another perspective. By crossing Equation 1.1 with \( B \), using a vector identity, and rearranging the terms, we can also show that,
\[ u_\perp = \frac{c(E \times B)}{B^2}, \quad (1.2) \]
where \( u_\perp \) is the plasma velocity in the direction perpendicular to the magnetic field. So, when a plasma is under the frozen-in condition, its perpendicular velocity is solely governed by \( E \times B \) drift. Since, the \( E \times B \) drift does not differentiate between the positive and negative charges. This implies that when the plasma is frozen in, the guiding centers of ions and electrons co-move with the same velocity.

The conservation of magnetic flux through a plasma element also has another important consequence - two magnetic flux tubes in a plasma do not merge, where a flux tube is defined as a tube-like object that has no magnetic field perpendicular to its side surface and equal magnetic fluxes coming in and going out through its end surfaces. If they can merge into one, then the amount of the magnetic flux from the two tubes can combine. This breaks the conservation of the flux within each tube. Figure 1.2(b) illustrates how merging of two flux tubes does not comply with the flux conservation. From the frozen-in view point, when two flux tubes collide, they do not cross each other (Figure 1.2(c)). This implies no magnetic reconnection (see Figure 1.3(a)).

Nevertheless, the situation in Figure 1.3(a) does not actually occur in the real plasma because the frozen-in condition is not always valid. One of the reasons is that plasma is not a perfect conductor. At the region near the neutral point where the magnetic field is weak and the current is strong, the value of \( -(u/c) \times B \) becomes small. So, rather than following Equation 1.1, the electric field follows Ohm’s law,
Figure 1.2: Interactions of two flux tubes illustrated in both 3D pictures and top-view 2D cross section. (a) Two flux tubes away from each other. Tube 1 has 7 flux units out-of-the-page (from 2D view). Tube 2 has 7 units into-the-page. (b) Two flux tubes are merging. The net number of flux contained in each tube is not conserved. Tube 1 and 2 now have net flux of 6 in the out-of-the-page and into-the-page directions respectively. (c) From the viewpoint of frozen-in theorem, two flux tubes can be pressed toward each other but cannot merge. Without merging, each tube conserves the amount of flux no matter how it is pressed.
Figure 1.3:  (a) When the space between the two wires in Figure 1.1 is filled with plasma, the red and blue field lines can only be pressed toward each other, and the into-the-page current will be created at the boundary between plasmas from the two sides. As long as the frozen-in condition holds, they will not meet, merge, break, and reconnect. The energy from moving the wires toward each other is stored in the system as magnetic energy. (b) The frozen-in condition breaks in the region called diffusion region (the yellow box), where the electric field does not follow $\mathbf{E} = -\left(\frac{\mathbf{u}}{c}\right) \times \mathbf{B}$. This allows magnetic reconnection to occur. The reconnected field lines are shot out to the sides along with the plasma. The stored magnetic energy can be converted into the heat and kinetic energy of the flow.
\( E = \eta J \) (or \( J = \sigma E \)), where \( J \) is the current density, \( \eta \) is the resistivity, and \( \sigma = 1/\eta \) is the conductivity. In this region, which we refer to as the “diffusion region,” the frozen-in condition breaks, and magnetic reconnection is allowed (Figure 1.3)(b)). In contrast to the reconnection in vacuum, where there is no interaction between the magnetic field and particles, the reconnection in plasma can convert the energy stored in the magnetic field to the flow and thermal energy of the plasma. The magnetic field lines do not just break and reconnect, but they also drag the plasma out along with them creating the plasma outflow (Figure 1.3)(b)). When outgoing plasma has been jetted out, the new plasma has to flow in due to continuity. This new plasma brings the new magnetic field lines along with it. The new field lines then break, reconnect, snap out, and create outflow again. The reconnection will continue, even if we no longer move the wires closer to each other, until the system cannot release the magnetic energy any further. At that point, the X-configuration will be unpressed and feature the perpendicular shape (the shape shown in Figure 1.1).

1.2 Symmetric Reconnection

1.2.1 Steady-State Reconnection: General Analysis

Now that we have a rough idea of what magnetic reconnection is, we will next do an analysis on the reconnection. Our focus will be on the reconnection in steady-state, at which there are equal incoming and outgoing mass, momentum, and energy fluxes through the diffusion region. The reconnection rate in this state is normally used as a characteristic of a reconnection.

What we want to achieve is to find the expressions for the outflow speed and the reconnection rate. However, before we begin our analysis, let us define what they are. It is clear that the outflow speed is the speed of the plasma that comes out of the diffusion region on the outflow edges. Defining the reconnection rate is a bit more complicated, but can be done as follows. To measure the reconnection rate, we measure the amount of the magnetic flux flowing into or away from the neutral point (referred
to as the “X-line.”) If we make an integration loop shown in Figure 1.4, the Faraday’s law reads

\[ \frac{d\phi_B}{dt} = -c \oint E \cdot dl, \]  

(1.3)

where \( \phi_B \) is the magnetic flux through the loop, \( t \) is the time, \( c \) is the speed of light, \( E \) is the electric field, and \( dl \) is the line element of the integral. We can see that the \( d\phi_B/dt \) of the loop is exactly what we want to measure - the rate of the magnetic flux flowing away from the X-line. If we make side III of the loop to be at infinity (or somewhere not related to the situation happening around side I), where the electric field equal to zero, the contribution to the integral from side III of the loop will be zero. Since we assume no variation in the \( z \)-direction, the contributions from side II and IV will cancel completely. This leaves side I to be the only contributing term. The rate of change of the magnetic flux going through this loop per unit length in the \( z \)-direction can then be written as

\[ \frac{1}{h} \frac{d\phi_B}{dt} = \frac{\partial}{\partial t} \int B_y dx = -cE_z, \]  

(1.4)

where \( h \) is the length of the loop on side I. This implies that the out-of-plane electric field \( E_z \) can be used as an indicator of how fast the rate of reconnection is. Note that if the region around side I is locally in steady state, the out-of-plane electric field will be space independent in that region. This can be shown by moving side III from the infinity to somewhere in the vicinity of side I. If the system is locally in steady state, the magnetic flux will not change in time in the integration loop. Therefore, \( E_z \) at side I and III will be equal, implying uniform \( E_z \) in the region. Another point of view of looking at \( E_z \) as the reconnection rate is to consider the amount of the magnetic flux convected by the frozen-in inflow plasma. If the upstream plasma has the magnetic field \( B_x \), and the magnetic field is being convected toward the diffusion region with the velocity \( u_y \), the amount of flux being fed into the diffusion region per unit length in the \( z \)-direction per unit time will be \( u_y B_x \), which can then be written as \(-cE_z\) using Equation 1.1. This out-of-plane electric field \( E_z \) is sometimes referred to as the reconnection electric field.
Following the Sweet-Parker analysis \cite{65, 46}, the expression of the outflow speed and the reconnection rate can be obtained by considering the conservation of mass and energy of the diffusion region. We model the diffusion region as a rectangular box with the thickness of $2\delta$ and the length of $2L$ as shown in Figure 1.5. At this point, we do not specify the mechanism that breaks the frozen-in condition inside the diffusion region. The results from the following analysis are general. Due to the symmetry of the problem, we can apply the conservation laws to any one of the four quarter of the diffusion region. The conservation of mass reads,

$$\rho(u_{in} L) \sim \rho(u_{out} \delta),$$

which can also be rearranged as

$$u_{in} \sim \frac{\delta}{L} u_{out},$$

where $u_{in}$ is the inflow speed, $u_{out}$ is the outflow speed, $\rho$ is the density of the plasma, and the symbol “$\sim$” means “scales as.” Assuming that the energy upstream of the
diffusion region is mainly magnetic energy, and the flow energy dominates downstream, the conservation of energy yields,

\[ \frac{B_{up}^2}{8\pi} (u_{in} L) \sim \frac{1}{2} \rho u_{out}^2 (u_{out} \cdot \delta), \tag{1.7} \]

Using Equations 1.5 and 1.7 together, we arrive at an expression for the outflow velocity

\[ u_{out} \sim \frac{B_{up}}{\sqrt{4\pi \rho}} = c_{A,up}, \tag{1.8} \]

where \( c_{A,up} \) is the Alfvén speed based on the upstream magnetic field. Substituting Equation 1.8 into Equation 1.6, then using the substituted Equation 1.6 to Equation 1.1, we finally obtain an expression for the reconnection rate

\[ E_z \sim \left( \frac{c_{A,up} B_{up}}{c} \right) \frac{\delta}{L} \]. \tag{1.9} \]

Again, Equation 1.8 and 1.9 are independent of the mechanism breaking frozen-in in the diffusion region. We have not yet made any specification on the mechanism.

1.2.2 Collisional Reconnection

As mentioned earlier, the plasma is not a perfect conductor. So, the electric field in the plasma is not only created by the frame transformation but also from the
effect of the collisions between ions and electrons. The effect of collision comes into the picture through the resistivity. Besides the ideal term $-\mathbf{u}/c \times \mathbf{B}$, the resistive term now has to be included, and Equation 1.1 becomes

$$E = -\frac{\mathbf{u}}{c} \times \mathbf{B} + \eta \mathbf{J}. \quad (1.10)$$

In the diffusion region, the current is strong. On the other hand, the magnetic field is weak. So, the resistive term dominates. Substituting $E_z \sim \eta J_z$ into the expression for the reconnection rate (Equation 1.9), we get

$$\left(\frac{c_{A,up} B_{up}}{c}\right) \frac{\delta}{L} \sim \eta J_z. \quad (1.11)$$

Using Ampère’s law $\mathbf{J} = (c/4\pi)(\nabla \times \mathbf{B})$ leads us to the scaling relation

$$J_z \sim \frac{c}{4\pi} \left(\frac{B_{up}}{\delta}\right). \quad (1.12)$$

Getting rid of $J_z$ by substituting Equation 1.12 in the Equation 1.11 yields

$$\delta \sim \sqrt{(\eta L)(\frac{c^2}{4\pi c_{A,up}})}. \quad (1.13)$$

So, the thickness of the diffusion region of collisional reconnection is determined by the resistivity, the length of the diffusion region, and the Alfvén speed based on the upstream magnetic field and density. The ratio $\delta/L$ then can be written as

$$\frac{\delta}{L} \sim \sqrt{\frac{\eta c^2}{4\pi c_{A,up} L}}. \quad (1.14)$$

Putting $\delta/L$ in Equation 1.14 back into the general expression for the reconnection rate (Equation 1.9), we arrive at the expression for the reconnection rate for collisional reconnection.

$$E_z \sim \sqrt{\frac{\eta c^2}{4\pi c_{A,up} L}} \left(\frac{c_{A,up} B_{up}}{c}\right) \quad (1.15)$$

The collisional reconnection is often referred to as “Sweet-Parker” reconnection as its mechanism and treatment was introduced by Sweet [65] and Parker [46].

After Giovanelli’s observations [?] suggesting the association between solar flares and magnetic null points and Dungey’s theory [18] linking magnetic reconnection with
strong current sheets, Sweet-Parker reconnection was a big step forward in the quest of explaining the energy release rate of the solar flare. However, it still was not enough to explain the energy release by the solar flare as Parker himself pointed out in [47]. One of the reasons is that the resistivity of the plasma in the vicinity of the solar flare is low. The other key reason is that the length of the current sheet (diffusion region) $L$ of the Sweet-Parker reconnection can be as long as the system size allows, causing the ratio $\delta/L$ to be very small. Since the outflow speed is fixed to the Alfvén speed, the smaller the $\delta/L$, the smaller the inflow speed. Another big step forward came when Petschek [48] introduced the shock configuration into the reconnection (see Figure 1.6) allowing the plasma to turn the corner and gain energy at the shocks without going through the diffusion region. The diffusion region in Petschek’s configuration is localized. The flow, therefore, does not suffer from the bottle neck of the extremely small $\delta/L$ experienced by Sweet-Parker reconnection. Petschek reconnection can achieve a much faster rate than the Sweet-Parker reconnection and succeeded in explaining the energy released rate of the solar flare. Unfortunately, Biskamp [6] found in his numerical simulation that the open-outflow Petschek structure cannot occur unless there is a localized resistivity. As of now, there are no model of reconnection that includes localized resistivity in a self-consistent manner. Furthermore, the Petschek structure has never been seen in experiments.

1.2.3 Collisionless Reconnection

Collisions are a mechanism that breaks frozen-in condition. However, it is not the only one. In the case where there is no collision, the physics of finite Larmor radius comes into play. When the frozen-in condition applies, the particles move under ExB drift. However, near the X-line where the magnetic field significantly changes within the scale comparable to the particle gyro-orbit, the ExB drift will no longer works and particles will start to meander instead (see Figure 1.7). At this point, the magnetic field is no longer attached to the plasma, and the frozen-in condition breaks. We can say that the plasma is “demagnetized.”
Due to the fact that ion mass is greater than the electron mass, the gyroradius of the ions is larger than the gyroradius of the electron. As a result, they start to have the meandering motion at different distances from the X-line, leading to the two-scale diffusion region. The smaller scale is the electron diffusion scale (shown in green area in Figure 1.8), total breaking of the frozen-in condition happens, and the magnetic field can break and reconnect there. The larger scale is the scale of the ion diffusion region (yellow area in Figure 1.8). The region between the bound of the ion diffusion region and the electron diffusion region is called the Hall region. In this region, the electrons are still frozen in and behave like a fluid, but the ions are meandering. The bulk flow speed of the ions is therefore small and can be considered stationary compared to the electrons. The current is, therefore, mostly carried by the electrons: $J \approx -neu_e$.

The physics breaking frozen-in and the Hall physics can be captured by the generalized Ohm’s law for a collisionless plasma [71],

$$E = -\frac{1}{c}(u_i \times B) + \frac{1}{nec}(J \times B) - \frac{1}{ne}(\nabla \cdot P_e) - \frac{me}{e^2} \left(\frac{du_e}{dt}\right),$$

(1.16)

where $u_i$ is the ion bulk flow velocity, $n$ is the number density, $-e$ is the charge of an electron, $P_e$ is the electron pressure tensor, $m_e$ is the mass of the electron. Note that
Figure 1.7: 2D cross-section in the $yz$-plane of a reconnection illustrating the breaking of frozen-in condition due to the effect of finite Larmor radius near the X-line. The magnetic field (blue) is out of (and into) the page ($x$-direction). The reconnection electric field (red) is pointing upward ($z$-direction). In the white region, where the frozen-in applies, the particle moves with the ExB-drift velocity to the right ($y$-direction). In the yellow region, the particle meanders. The frozen-in condition breaks in this region.
Figure 1.8: Two-scale diffusion region. The ion diffusion region is shown by yellow. The electron diffusion region is shown by green.
only the last two terms on the right hand side, namely the electron pressure gradient term and the electron inertia term, can break the frozen-in condition of the electrons. The second term, which can be called the Hall term, does not break frozen-in of the electrons, only the ions. When we combine the first term, or the ion frozen-in term, with the Hall term, we get the electron frozen-in term. The Hall term just describes the effects due to the relative bulk flow of the ions and the electrons.

It has been shown by simulations ([5] and references therein) that no matter what physics breaking the frozen-in condition is, if the Hall physics is included in the system, the reconnection will be fast with the diffusion-region width-to-length ratio $\delta/L$ being of order 0.1. (However, there is no theory yet of why $\delta/L$ is in the order of 0.1) The diffusion region will be localized, and the reconnection will be fast. So, it was concluded that the Hall physics plays an important role in facilitating fast reconnection. Still, the conclusion was not without disagreement. Karimabadi [25] raised a question about the importance of the Hall physics by suggesting that the ions kinetic alone allow reconnection to be fast. The Appendix, however, explicitly demonstrates that the ion kinetic effects alone are not enough to make reconnection fast and reaffirms that the Hall physics plays an important role in allowing fast reconnection.

The fact that the electron is still frozen-in in the Hall region has an important consequence: the quadrupolar out-of-plane magnetic field. Once a newly reconnected field line comes out from the electron diffusion region and then moves into the Hall region, the field and the electrons become attached. Since the electrons have to flow out-of-the plane to create current, the magnetic field is then dragged out of the page by the electron flow, creating the quadrupolar out-of-plane magnetic field [64] (Figure 1.9). When dragged out of the page, the magnetic field plane changes its angle resulting in the redirection of the current. This dynamics is called whistler dynamics, which occurs in the Hall region where the electrons are the major current carrier while still being frozen-in [34]. Another signature of the Hall physics is the normal electric field (normal to the upstream magnetic field). This electric field is derived from the Hall term, $\mathbf{J}/nec \times \mathbf{B}$. Figure 1.9 illustrates how the Hall electric field is created.
Figure 1.9: (a) The newly reconnected field lines (blue lines) just come out from the electron diffusion region (green area) to the Hall region (yellow area). The magnetic field are still in $xy$-plane. The current is pointing in the negative $z$-direction leading to the electron flow (black arrows) in the positive $z$-direction. The Hall electric field (red arrows) is created by $-u_e/c \times B$. (b) With effect of the electron flow, the field is dragged out of the page. The current is redirected due to the new configuration of the magnetic field. So, does the electron flow. The dragged magnetic field creates the quadrupolar structure in the out-of-plane magnetic field, with the top-left and bottom-right quarters having the field pointing out of the page while the top-right and bottom-left quarters having the field pointing into the page.
Although we do not know yet why the ratio $\delta/L$ is of order 0.1, we do know how to determine the width $\delta$. When an ion of mass $m_i$ enters the diffusion region, it will decouple from the magnetic field and get accelerated due to the reconnection electric field. The time it gets accelerated scales as

$$\Delta t \sim \frac{L}{u_{out}}.$$ (1.17)

Therefore, the speed obtained after being accelerated for time $\Delta t$ scales as

$$u_z \sim \frac{eE_z\Delta t}{m_i} \sim \frac{eE_zL}{m_iu_{out}}.$$ (1.18)

This upward accelerated ion is then redirected by the newly reconnected field line causing

$$u_z \sim u_{out}.$$ (1.19)

The electric field providing the acceleration is the reconnection electric field, which scales as

$$E_z \sim \frac{u_{in}B_{up}}{c}.$$ (1.20)

Substituting Equations 1.19 and 1.20 into Equation 1.18, we obtain

$$u_{out} \sim \left(\frac{eB_{up}}{cm_i}\right) \left(\frac{u_{in}}{u_{out}}\right) L.$$ (1.21)

Using $u_{in} \sim (\delta/L)u_{out}$ to get rid of $u_{in}$, the Equation 1.21 becomes

$$u_{out} \sim \left(\frac{eB_{up}}{cm_i}\right) \delta,$$ (1.22)

which can be rearranged into

$$\delta \sim \left(\frac{cm_i}{eB_{up}}\right) u_{out}.$$ (1.23)

With $u_{out} \sim c_{A,up}$, the Equation 1.23 can be rewritten as

$$\delta \sim \left(\frac{cm_i}{eB_{up}}\right) c_{A,up} \sim d_i,$$ (1.24)

where $d_i$ is the ion inertial length, which is the gyroradius of the ion gyrating with Alfvén speed.
1.3 Asymmetric Reconnection

Studies of reconnection are conventionally done in two dimensions with the identical inflow conditions, as seen in the previous sections. This symmetric set up is a good place to begin understanding the nature of reconnections. In reality, however, the symmetric set up is not always present. Under a turbulent condition, for example, the reconnection loses all of its symmetry [56]. In this study, we will focus on a type of reconnection with asymmetric inflow. For convenient, we will simply refer to this type of reconnection as asymmetric reconnection. A good example of this type of reconnection is the reconnection at the day-side magnetopause (see Figure 1.10).

In this section, we will provide an approach to understand the quasi-steady state asymmetric reconnection by constructing a model of the diffusion region and carry out a Sweet-Parker-type scaling analysis as originally done by [12] and [14].

1.3.1 The Model

The following are the full list of features which make up our diffusion-region model of asymmetric reconnection. See Figure 1.11 for the model’s diagram.

- The system is in a steady state.

- The diffusion region is taken to be a rectangular box with a width of $2\delta$ and a length of $2L$, where the length $L$ is assumed to be large compared to $\delta$. The dissipation mechanism that break the frozen-in condition inside the diffusion region need not be specified.

- Outside the diffusion region, the plasma is treated as a single fluid (meaning the ions and electrons are co-moving), and the frozen-in theorem applies $E = -\mathbf{v}/c \times \mathbf{B}$. The governing laws for this fluid are (1) the conservation of mass, (2) the conservation of momentum, (3) conservation of energy, and (4) Maxwell’s equations (Gauss’s law and $\nabla \cdot \mathbf{B} = 0$ do not play an important role in the analysis, however).

- The magnetic fields of the two upstream sides are horizontally anti-parallel with inconsequentially small vertical component and no out-of-plane component. The downstream magnetic field is negligible.

- Each of the two upstream sides has its own inflow speed $v_i$, magnetic field strength $B_i$ and density $\rho_i$ that are not necessarily equal to the ones on the other side. The subscript $i$ denotes the side, which can be either 1 or 2. These properties are assumed to be uniform along the inflow edges of the diffusion region.
The system of magnetosphere. The reconnection at magnetotail is normally symmetric. However, the reconnection at the magnetopause (day-side reconnection) is asymmetric by nature, occurring at the boundary of the magnetosheath and magnetospheric plasma. The upstream plasma on the magnetosheath side has a magnetic field strength of 10-20 nT and a density of 20-30 cm$^{-3}$ while the magnetosphere has a magnetic field strength of 50-60 nT and a density of 0.2-0.3 cm$^{-3}$ (see [50] for actual satellite data).
Figure 1.11: A diffusion-region model for asymmetric reconnection. The rectangular box, with a width of $2\delta$ and a length $2L$, represents the diffusion region. The solid blue line is the magnetic field. The X-line and stagnation point are denoted by points $X$ and $S$ respectively. The upstream magnetic field strength, density, and inflow speed are represented by $B_i$, $\rho_i$, and $v_i$ respectively with the subscript $i$ being either “1” or “2” specifying the side of the upstream. $\rho_{out}$ and $v_{out}$ are the outflow density and the outflow speed. $\delta_{Xi}$ is the distance from the X-line to the edge of the diffusion region of side $i$. Similarly, $\delta_{Si}$ is the distance from the stagnation point to the edge of the diffusion region of side $i$.

- Plasma mass fluxes from the two sides flow into the diffusion region and meet at the stagnation point ($S$). This stagnation point is not necessarily co-located with the X-line ($X$), where the magnetic fluxes from the two side meet.

- The plasmas from the two upstream side are assumed to be completely mixed before going out of the diffusion region, leading to the uniform outflow speed $v_{out}$ and density $\rho_{out}$.

- The magnetic energy is much greater than the kinetic energy on the upstream. The opposite is true for the downstream. All the magnetic inflow energy is assumed to be converted to the kinetic energy of the outflow. The pressure is assumed to play no major role in energy conversion.

Now that we have the model in place, we are ready to carry out our analysis.
1.3.2 The Analysis I: The Gross Properties

First, let us consider Faraday’s law in a steady state $\oint \mathbf{E} \cdot d\mathbf{l} = 0$. If we make an integration loop as shown in Figure 1.12, the top and bottom parts of the loop will cancel since our model has no variation in the out-of-plane direction. This leads to $E_1 = E_2$. At the edges of the diffusion region, the frozen-in theorem applies ($E_1 \sim v_1 B_1$ and $E_2 \sim v_2 B_2$), suggesting

$$v_1 B_1 \sim v_2 B_2.$$  \hfill (1.25)

More intuitively, we can arrive at this relation by simply arguing that the magnetic fluxes coming toward the diffusion region ($vB$) from the two sides have to be equal, otherwise, there will be a magnetic pile-up, and the system will not be in a steady state. In any case, our first conclusion is that the ratio of the inflow speeds is controlled by the ratio of the upstream magnetic field strengths.

Next, we will analyze the outflow density. As the old plasma is jetted downstream, the new plasmas must enter the diffusion region to replace the old plasma.
However, the volumes of the replacing plasmas do not have to be equal on the two upstream sides. Consider a transit time, represented by $\Delta t_T$, which is the amount of time the new plasmas take to totally replace the old plasma. Figure 1.13(a) shows the beginning of this interval. The yellow and red boxes represent the new plasmas from side 1 and 2 that are about to flow into the diffusion region filling up the void. The volumes of the boxes are controlled by their inflow speeds. In the figure, the inflow speed $v_1$ is greater than the inflow speed $v_2$, so the expected volume of the incoming plasma from side 1 (yellow box) will be greater than its counterpart from side 2 (red box). Figure 1.13(b) shows the situation after a time $\Delta t_T$ has passed. The plasmas from the two upstream sides have come in and filled the void. They are entering not only with different volumes but different densities as well. Given the plasma densities of side 1 and 2 are $\rho_1$ and $\rho_2$, the amount of plasma coming in from side 1 and 2 will be $\rho_1 v_1 \Delta t_T$ and $\rho_2 v_2 \Delta t_T$. These two plasma populations are assumed to completely mix into a single plasma before leaving the diffusion region as an outflow, as shown in Figure 1.13(c). To calculate the outflow density $\rho_{\text{out}}$ of this plasma we need to divide the total amount of the plasma by the whole volume,

$$\rho_{\text{out}} \sim \frac{\rho_1 v_1 \Delta t_T + \rho_2 v_2 \Delta t_T}{v_1 \Delta t_T + v_2 \Delta t_T}$$  \hspace{1cm} (1.26)

$$\rho_{\text{out}} \sim \frac{\rho_1 v_1 + \rho_2 v_2}{v_1 + v_2}.$$  \hspace{1cm} (1.27)

So, our conclusion is that the outflow density is the average of the two inflow densities weighted by their inflow speeds. If we proceed further by using the conclusion that the ratio of the inflow speeds is controlled by the ratio of the upstream magnetic field strengths, the outflow density can be expressed in terms of the upstream densities and magnetic fields,

$$\rho_{\text{out}} \sim \frac{\rho_1 B_2 + \rho_2 B_1}{B_1 + B_2},$$  \hspace{1cm} (1.28)

which is a key result that will be tested in Chapter 3.

Finally, we consider the conservation laws in a steady state. The conservation of momentum in a steady state just means that the system requires total pressure balance along the directions of both inflow and outflow. The conservation of mass dictates that
Figure 1.13: Cycle of incoming and outgoing plasma. Here, only the right half of the diffusion region is shown. (a) The new plasmas are to replace the old plasma. (b) After a time interval $\Delta t_T$, the plasmas from the two side have come in to occupy the diffusion region. (c) The two populations of the plasmas are assumed to completely mix before leaving the diffusion region as an outflow. Note: This replacing process is actually done continuously. There is no actual void when the old plasma leaves the diffusion region downstream.
the two incoming mass fluxes combined have to be equal to the outgoing mass flux (see Figure 1.14(a)),

$$\rho_1(v_1L) + \rho_2(v_2L) \sim \rho_{out}v_{out}. \quad (1.29)$$

From Equation 1.29, expressing $v_2$ in terms of $v_1$ using Equation 1.25 and substituting the outflow density using Equation 1.28, we can write the inflow speed of side 1 in terms of the outflow speed as

$$v_1 \sim \left(2B_2 \over B_1 + B_2\right) \left(\delta \over L\right) v_{out}. \quad (1.30)$$

Similarly, we can also obtain

$$v_2 \sim \left(2B_1 \over B_1 + B_2\right) \left(\delta \over L\right) v_{out}. \quad (1.31)$$

The conservation of energy says the sum of energy fluxes from the inflow equals the energy flux of the outflow (see Figure 1.14(b)),

$$\frac{B_1^2}{8\pi}(v_1L) + \frac{B_2^2}{8\pi}(v_2L) \sim \frac{1}{2}\rho_{out}v_{out}^2(v_{out}\delta). \quad (1.32)$$

Here, we use our assumption that the pressure does not play a significant role in energy conversion. Only the magnetic energy will be converted to the kinetic energy of the outflow.

If we divide Equation 1.32 by Equation 1.29 and then use Equation 1.25 to replace $v_1$ and $v_2$ with $B_1$ and $B_2$, we will arrive at the scaling relation for the outflow speed,

$$v_{out} \sim \sqrt{\frac{B_1B_2}{4\pi} \left(\frac{B_1 + B_2}{\rho_1B_2 + \rho_2B_1}\right)} \sim \sqrt{\frac{B_1B_2}{4\pi\rho_{out}}}. \quad (1.33)$$

This is another key result. When $B_{up} = B_1 = B_2$ and $\rho_{up} = \rho_1 = \rho_2$, the relation reduces to $v_{out} \sim B_{up}/\sqrt{4\pi\rho_{up}} = c_{A,up}$, where $c_{A,up}$ is the Alfvén speed based on the upstream magnetic field and density, as we expect from a symmetric case.

Going back to the Equation 1.29 again, we eliminate $v_1$ and $v_2$ in the equation by applying the frozen-in theorem on the upstream edges $E \sim v_1B_1/c \sim v_2B_2/c$, where $E$ is the out-of-plane electric field, which indicates reconnection rate. Substituting the
Figure 1.14: Diagrams of the incoming and outgoing (a) mass fluxes and (b) energy fluxes for the right half of the diffusion region.
outflow density $\rho_{out}$ on the right hand side of the equation by Equation 1.28. We then obtain the last key result for the gross-property section, the relation for reconnection rate

$$E \sim B_{red} \left( \frac{v_{out}}{c} \right) \left( \frac{\delta}{L} \right),$$

where $B_{red} = 2B_1B_2/(B_1 + B_2)$ is the reduced magnetic field. If $B_{up} = B_1 = B_2$ and $\rho_{up} = \rho_1 = \rho_2$, then $B_{red} = B_{up}$ and $v_{out} = c_{A,up}$ leading to the relation of the reconnection rate for the symmetric case $E \sim B_{up}(c_{A,up}/c)(\delta/L)$.

1.3.3 The Analysis II: The Diffusion Region Structure

As seen in the previous section, the asymmetry found in the upstream affects the gross properties of reconnection - the outflow density, outflow speed, and reconnection rate. We, therefore, sensibly expect that the inner structure of the diffusion region will be affected as well. By doing an analysis similarly to what has been done in the previous section, we can derive the locations of the X-line and stagnation point in terms of magnetic field strengths and the densities. The thickness of the diffusion region for asymmetric Sweet-Parker reconnection and Hall reconnection will be determined later in the analysis.

To figure out where the X-line is, we analyze the energy fluxes. Since the magnetic field on the two inflow sides are oppositely directed, there must be a point within the diffusion region where the magnetic field is zero. This point is the X-line. At this point, there cannot be a magnetic energy flux passing through. Therefore, the X-line acts as a border of the incoming energy flux (see Figure 1.15(b)). If $\delta_{X1}$ and $\delta_{X2}$ are the distance from the X-line to the upstream edges of side 1 and side 2 respectively, using the conservation of energy, we get

$$\left( \frac{B_1^2}{8\pi} \right) v_1 L \sim \left( \frac{1}{2} \rho_{out} v_{out}^2 \right) v_{out} \delta_{X1}$$

$$\left( \frac{B_2^2}{8\pi} \right) v_2 L \sim \left( \frac{1}{2} \rho_{out} v_{out}^2 \right) v_{out} \delta_{X2}.$$
Figure 1.15: Diagrams of the incoming and outgoing (a) mass fluxes with the stagnation point being the point where no incoming mass flux passes through (b) energy fluxes with the X-line being the point where no incoming magnetic energy flux passes through. This Figure demonstrates only the right half of the diffusion region.

Since the outflow is assumed to be uniform, the ratio of $\delta_{X1}$ to $\delta_{X2}$ is controlled by the ratio of the incoming magnetic energy flux from side 1 to its match from side 2. Mathematically, this can be shown by dividing Equation 1.35 by Equation 1.36. Replacing the inflow speeds by the magnetic field strengths using Equation 1.25, we then arrive at

$$\frac{\delta_{X1}}{\delta_{X2}} \sim \frac{B_1}{B_2}. \quad (1.37)$$

Similarly, by analyzing the mass fluxes, we can predict the location of the stagnation point. Taking that the stagnation point as the border of the two incoming mass fluxes (see Figure 1.15(a)), using the conservation of mass, we obtain
Again, since the outflow is assumed to be uniform, the ratio of $\delta_{S1}$ to $\delta_{S2}$ is simply determined by the ratio of the mass flux from side 1 to its counterpart from side 2. This can be mathematically shown by taking the ratio of Equation 1.38 to Equation 1.39. Replacing the inflow speeds by the magnetic field strengths using Equation 1.25, we finally arrive at

\[
\frac{\delta_{S1}}{\delta_{S2}} \sim \frac{\rho_1 B_2}{\rho_2 B_1}.
\] (1.40)

Note that the Equation 1.37 and Equation 1.40 only determine the relative locations of the X-line and stagnation point. To get the absolute locations, we have to know thickness of the diffusion region $2\delta$, where

\[
2\delta = \delta_{X1} + \delta_{X2} = \delta_{S1} + \delta_{S2}.
\] (1.41)

Unlike the previous analysis we have done so far, which is applicable to any mechanism that breaks the frozen-in condition, the analysis of the diffusion region thickness requires the knowledge of the mechanism breaking frozen-in.

For asymmetric Sweet-Parker reconnection, the frozen-in condition is broken by the effect of the resistivity. So, inside the diffusion region, the dominant term controlling the electric field is the resistive term $E \sim \eta J$, where $\eta$ is a constant and uniform resistivity and $J = (c/4\pi)\nabla \times B$ is the current density. Considering the out-of-plane component, the scaling relation becomes

\[
E \sim \eta J.
\] (1.42)

Substituting the out-of-plane electric field using the reconnection-rate relation (Equation 1.34) and the similarly directed current density using $J \sim (c/4\pi)(B_1/\delta_{X1})$ yields

\[
\left( \frac{2B_1 B_2}{B_1 + B_2} \right) \left( \frac{v_{out}}{c} \right) \left( \frac{\delta_{X1} + \delta_{X2}}{2L} \right) \sim (c/4\pi)(B_1/\delta_{X1}).
\] (1.43)
where $\delta$ on the left-hand side has been replaced with $(\delta_{X1} + \delta_{X2})/2$. Expressing $\delta_{X2}$ in terms of $\delta_{X1}$ using Equation 1.37 and then solving for $\delta_{X1}$ will lead us to

$$\delta_{X1} \sim \sqrt{\frac{\eta c^2 L}{4\pi v_{out}} \frac{B_1}{B_2}}.$$  \hspace{1cm} (1.44)

By substituting $J$ in Equation 1.42 with $J \sim (c/4\pi)(B_2/\delta_{X2})$ and doing similar analysis, we can also arrive at

$$\delta_{X2} \sim \sqrt{\frac{\eta c^2 L}{4\pi v_{out}} \frac{B_2}{B_1}}.$$ \hspace{1cm} (1.45)

The half thickness $\delta$ of the diffusion region for Sweet-Parker reconnections is, therefore,

$$\delta \sim \frac{1}{2} \left( \sqrt{\frac{B_1}{B_2}} + \sqrt{\frac{B_2}{B_1}} \right) \sqrt{\frac{\eta c^2 L}{4\pi v_{out}}},$$ \hspace{1cm} (1.46)

which can be reduced to

$$\delta \sim \sqrt{\frac{\eta c^2 L}{4\pi c_{A,up}}},$$ \hspace{1cm} (1.47)

as is expected from symmetric upstream conditions.

For asymmetric Hall reconnection, the frozen-in condition is broken by the effect of particles’ finite Larmor radius (as explained in Section 1.2). When an ion of mass $m$ and charge $e$ enters the diffusion region it will no longer be tied to the magnetic field and can be accelerated by the out-of-plane electric field. Nonetheless, this ion cannot accelerate forever. It has to leave as new particles come in. Therefore, the time the ion spends accelerating scales like the transit time $\Delta t_T$. The accelerated ion will eventually be redirected by the newly reconnected magnetic field toward the outflow direction. So, the ion’s speed after getting accelerated for $\Delta t_T$ scales like the outflow speed. This gives us

$$v_{out} \sim \left( \frac{eE}{m} \right) \Delta t_T.$$ \hspace{1cm} (1.48)

Since the transit time scales like the time the ion takes to go out of the diffusion region, $\Delta t_T \sim L/v_{out}$, we arrive at

$$\delta \sim \left( \frac{mc}{eB_{red}} \right) v_{out},$$ \hspace{1cm} (1.49)
where $E$ is substituted using Equation 1.34. With $v_{out} \sim \sqrt{B_1 B_2/(4 \pi \rho_{out})}$, the relation becomes

$$
\delta \sim \frac{1}{2} \left( \sqrt{\frac{B_1}{B_2}} + \sqrt{\frac{B_2}{B_1}} \right) d_{i, out}, \quad (1.50)
$$

where $d_{i, out} = (m_i^2 c^2 / 4 \pi e^2 \rho_{out})^{1/2}$ is the ion inertial length based on the outflow density.

### 1.3.4 The Hall Signatures

An important step forward of the last decade in the field of magnetic reconnection is that we were able to confirm the Hall model of reconnection by observing its signatures. Specifically, the quadrupolar Hall magnetic and bipolar Hall electric field expected in symmetric reconnection have been observed on the day-side magnetosphere, the magnetotail, and in laboratory plasmas (e.g., [40, 41, 35, 53, 73], and references therein). One of the cleanest signatures occurred in a crossing of the day-side magnetopause with the POLAR satellite [40]. However, when Mozer sought to find statistical evidence for the Hall quadrupolar magnetic fields, he found that these “clean” crossings were quite uncommon, and almost unique. The question was, if Hall physics was playing such an important role in reconnection, why was reconnection at the magnetopause not showing the expected Hall signatures? To answer this question, we need to take the nature of reconnection at the magnetopause into account - the inflows are normally asymmetric. When the plasma conditions are very different on the two upstream sides, the Hall electric and magnetic fields can become strongly skewed. Figure 1.16 shows how the quadrupolar Hall magnetic field get skewed when the inflow conditions become similar to ones of a typical dayside reconnection. Figure 1.17 shows the same comparison for the Hall electric field. At this point, we might want to ask what controls the skewness of the fields? Is it the density asymmetry, the magnetic field asymmetry, or both that control how the Hall signatures behave? This topic will be discussed in more detail in Chapter 4.
Figure 1.16: The out-of-plane magnetic field of (a) symmetric reconnection with the inflow conditions $[B_1, B_2, \rho_1, \rho_2] = [1,1,1,1]$ (b) asymmetric reconnection similar to a typical dayside reconnection with the inflow conditions $[B_1, B_2, \rho_1, \rho_2] = [1,2,1,0.1]$. The colors red and blue represent the out-of-the-page and into-the-page directions respectively. The darker the color, the stronger the field. (More details on these simulations are available in Chapter 2 and 4)
Figure 1.17: The Hall electric field of (a) symmetric reconnection with the inflow conditions $[B_1, B_2, \rho_1, \rho_2] = [1,1,1,1]$ (b) asymmetric reconnection similar to a typical dayside reconnection with the inflow conditions $[B_1, B_2, \rho_1, \rho_2] = [1,2,1,0.1]$. The colors red and blue represent the up and down directions respectively. The darker the color, the stronger the field. (More details on these simulations are available in Chapter 2 and 4)
1.4 Dissertation Outline

This dissertation is an attempt to understand asymmetric reconnection in collisionless plasma, employing systematic particle-in-cell simulations. We lay out the necessary theoretical background in the previous sections of this chapter. Chapter 2 provides the description of the particle-in-cell code P3D utilized in this dissertation. Verifications of the scaling relations [12] of the gross properties, i.e., reconnection rate, outflow speed, and outflow density, are done in Chapter 3 (adapted from [33]). The diffusion region structure and signatures are investigated in Chapter 4. Summary is then given in Chapter 5. The Appendix shows the importance of the Hall physics vs. ion kinetic physics in allowing fast reconnection using hybrid simulations (adapted from [32]).
Chapter 2
SIMULATION

Magnetic reconnection is a plasma process. To exactly simulate reconnection, we need to be able to follow the trajectories of each and every particle in the plasma system. This is not quite a practical task. Approximations are therefore needed. The three most popular approximation models used in reconnection simulations are fluid, hybrid, and particle-in-cell models.

The fluid model, which is also called the magnetohydrodynamics (MHD) model, is probably the most widely used model out of the three. Instead of calculating the trajectories of each individual particle, this model treats the plasma as a single fluid, where ions and electrons are co-moving. This fluid is governed by the conservation of mass, the conservation of momentum, the conservation of energy, Maxwell’s equations with the limit that the displacement current is negligible and the electric field follows the frozen-in condition, and quasi-neutrality assumption. Collectively, these governing rules are called the MHD equations. Some modifications to the MHD equation can be made to include more physics. The two most popular MHD variations used in reconnection simulations are known as “resistive MHD” and “Hall MHD.” The resistive MHD model includes the physics of collisions via the effect of resistivity while the Hall MHD model, which is sometimes referred to as a “two-fluid model”, includes the relative motion of ions and electrons. Due to the fact that the MHD model and its variations can capture the large scale dynamics of the system well and are computationally cheap compared to other models, they have been successfully used to study the dynamics of large scale systems, e.g., the magnetosphere. However, when small scale physics is important, we have to look for an alternative model.
One possibility to introduce more physics into the system is that we represent ions as non-interacting particles whose motions are affected by the electric and magnetic fields at the location of the particles while electrons are still represented as a fluid. This kind of simulation is called a hybrid simulation. This model not only contains physics from the fluid models like the resistive and Hall physics, but it also contains all of the ion kinetic physics neglected in the fluid descriptions. The Appendix, which investigates the role of ion kinetic physics vs. the role of the Hall physics in facilitating fast reconnection, is an example of a reconnection study using a hybrid model. The study of kinetic dissipation at sub-ion scales in collisionless turbulent plasmas [45] is another example that makes use of hybrid simulations. Such studies can not be done using fluid simulations. However, the hybrid model is not without its limitations. Some studies, e.g. study of diffusion region details, require the kinetic physics of both ions and electrons. For these studies, the physics contained in the hybrid model is inadequate - a fully kinetic model such as the particle-in-cell (PIC) model, which treats both ions and electrons as particles, is required.

In this chapter, we will discuss the detail of a particle-in-cell code, called P3D [60, 75], which is used for all of the studies in this dissertation.

2.1 The Governing Equations

P3D represents both ions and electrons as non-interacting particles. The position and velocity of each particle are governed by the relativistic form of Newton’s second law of motion. In our situations of interest, gravity plays no role. This leaves the Lorentz force as the only force of interest. The equations written in the cgs units are shown as follows:

\[
\frac{dx}{dt} = \mathbf{v},
\]

\[
\frac{d(\gamma \mathbf{v})}{dt} = \frac{q}{m} \left[ \mathbf{E} + \left( \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \right],
\]

where \(\mathbf{x}\) is the position of the particle, \(\mathbf{v}\) is the velocity of the particle, \(t\) is the time, \(\mathbf{E}\) is the electric field, \(\mathbf{B}\) is the magnetic field, \(c\) is the speed of light, \(\gamma = [1 - (v/c)^2]^{-1/2}\)
is the Lorentz factor, \( q \) is the charge of the particle, and \( m \) is the mass of the particle. The electric and magnetic fields are governed by Maxwell’s equations:

\[
\begin{align*}
\frac{\partial \mathbf{B}}{\partial t} &= -c(\nabla \times \mathbf{E}) \\
\frac{\partial \mathbf{E}}{\partial t} &= c(\nabla \times \mathbf{B}) - 4\pi \mathbf{J} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \cdot \mathbf{E} &= 4\pi \rho_c,
\end{align*}
\]

where \( \rho_c \) is the charge density, \( \mathbf{J} = q(n_i \mathbf{u}_i - n_e \mathbf{u}_e) \) is the current density, \( n_i \) and \( n_e \) are the number densities of ions and electrons, and \( \mathbf{u}_i \) and \( \mathbf{u}_e \) are the bulk flow velocities of ions and electrons. Note that the only equations that we need for stepping forward the electric field and magnetic field in time are Ampere’s and Faraday’s laws, owing to the fact that they are the only terms with time derivatives. Although not directly used in advancing the fields, Gauss’s law and \( \nabla \cdot B = 0 \) are still enforced.

We emphasize that solving a particle-in-cell system is different from solving an N-body system, where every particle individually interacts with each others. It is, however, equivalent to solving the Vlasov equation using Monte Carlo method with each particle in the particle-in-cell approach corresponding to a sample point in phase space in the Monte-Carlo Vlasov approach.

### 2.2 The Normalized Equations

While applying Newton’s second law and Maxwell’s equations in the form shown in the previous section is possible, it is not the most convenient form to work with since there are many physical constant (e.g. \( c, q, m, 4\pi \)) floating around. Also, the values of the variables (e.g. \( \mathbf{x}, \mathbf{v}, \mathbf{E}, \mathbf{B} \)) can differ by many orders of magnitude. The large differences in scale can compound computational errors. We, therefore, need a way to rewrite the governing equations into a more amenable form with fewer constants and variables closer in size. One way of doing this is to change the units from \( cgs \) to a more appropriate set of units. This process is called normalization.
Instead of cgs units, magnetic field strengths in P3D are normalized to an arbitrary value $B_0$. Number densities are normalized to an arbitrary value $n_0$. Lengths are normalized to the ion inertial length $d_{i0} = c/\omega_{pi0}$, where $\omega_{pi0} = \sqrt{4\pi e^2 n_0/m_i}$ is the ion plasma frequency at the reference density. Times are all normalized to the ion cyclotron time $\Omega_c^{-1} = (eB_0/m_i c)^{-1}$, while speeds are normalized to the ion Alfvén speed $c_{A0} = B_0/(4\pi m_i n_0)^{1/2}$ at the reference magnetic field and density. Masses, in turn, are normalized to the ion mass $m_i$. Charges are normalized to the ion charge. Electric fields are normalized to $E_0 = cA_0 B_0/c$, and finally, temperatures (Boltzmann constant included) are normalized to $T_0 = m_i c_{A0}^2$ respectively. With this set of units, which we will later refer to as the code units, the evolution equations for particles become

\[
\frac{d\tilde{x}}{dt} = \tilde{v} \tag{2.7}
\]

\[
\frac{d}{dt}(\gamma \tilde{v}) = \pm \frac{1}{m} \left[ \tilde{E} + \left( \tilde{v} \times \tilde{B} \right) \right], \tag{2.8}
\]

where + is for ions, and − is for electrons. Maxwell’s equations will change to

\[
\frac{\partial \tilde{B}}{\partial t} = -\tilde{\nabla} \times \tilde{E} \tag{2.9}
\]

\[
\frac{\partial \tilde{E}}{\partial t} = \tilde{c}^2 (\tilde{\nabla} \times \tilde{B} - \tilde{J}) \tag{2.10}
\]

\[
\tilde{\nabla} \cdot \tilde{B} = 0 \tag{2.11}
\]

\[
\tilde{\nabla} \cdot \tilde{E} = \tilde{c}^2 \tilde{\rho}_e. \tag{2.12}
\]

Here, the tilde above the variables means that the variables are expressed in the code units. These normalized equations are the equations that will be discretized and then used in the code.

### 2.3 P3D as a Particle-In-Cell Code

Just like other finite difference schemes, particle-in-cell models discretize the space into grid points. At each grid point, the electric field and magnetic field are stored. The “cell” in the name “particle-in-cell” refers to the regions between grid
While the electric and magnetic fields are stored only at grid points, the particles are free to be anywhere in the simulation domain. Hence, the name, “particle-in-cell.” See Figure 2.1.

Note that the number density of these particles does not represent the actual number density of ions or electrons in the system. In the P3D code, there is a parameter called \(ppg\), which stands for “particle per grid-cell.” This determines how many sample particles are required in one cell to represent an actual density of 1. Setting higher values of \(ppg\) reduces the noise in the simulation. Unfortunately, this comes at the cost of increased computational time.

We now have almost all of the elements we need to paint the whole picture. However, some questions still remain. If there is no field data stored within the cells, how can we calculate the Lorentz force acting on particles inhabiting the cell-space? Also, if the particle density and velocity, which are required to calculate the current density, are not stored at grid points, how do we feed them into Faraday’s law in order to advance the on-grid magnetic field? The P3D code handles this issue in the following way.

**Fields off the Grid**

In order to calculate the electric field and the magnetic field acting on a particle located in a cell, P3D uses a bilinear interpolation, which averages the fields from the 4 nearest grid points weighted by the rectangular areas diagonally opposed to the grid points (see [2], p. 308-309). If we let \(F_i\) be a field component at grid point \(i\), the areas \(A_i\) are the diagonally opposed areas of grid points \(i\), where \(i\) can be \(a, b, c\) or \(d\), as shown in Figure 2.2, the value of the component of the field at the location of the particle \(F_p\) can be calculated by the following formula

\[
F_p = \frac{(F_a \times A_a) + (F_b \times A_b) + (F_c \times A_c) + (F_d \times A_d)}{(A_a + A_b + A_c + A_d)}. \tag{2.13}
\]
Figure 2.1: An illustration of how a particle-in-cell model looks like. The fields are stored only at grid points while particles can be anywhere in the simulation domain.
Figure 2.2: Each grid point (a, b, c, and d) and rectangular area (A_a, A_b, A_c, and A_d) are given a color. The contributing factor from each grid point to the particle at p (or from the particle to each grid point) is equal to the ratio of the area of the same color to the total area of a cell.
Velocity and Density on the Grid

In order to advance the electric and magnetic fields, we need to know the current density at all the grid points, which means we need to know the bulk velocities and densities of both ions and electrons at the grid points. This can be done in a similar way to the previous case where we summed over the weighted field values from the 4 nearest grid points. This time, however, we have to sum over the weighted densities and velocities of all of the particles in the 4 cells adjacent to the grid point. How much each particle contributes to a grid point is again weighted by the rectangular area diagonally opposed to the grid point.

2.4 Time Stepping

With the equations 2.7, 2.8, 2.9, and 2.10 and the initial values of \( x, v, E, \) and \( B \), we can calculate the values of \( x, v, E, \) and \( B \) at later times.

STEP 1: Advance the Fields for a Half Time Step

Before proceeding further, let us clarify that the fields use a time step different from the particle time step. This is due to the fact that the evolution of the fields, which includes evolution of the light wave, occurs at a faster time scale than the evolution of the particles. Therefore, a smaller time step is used for the fields. Within a single time step for the particles \( \Delta t \), there will be at least 2 time steps for the fields \( \Delta t_f \). The field time step can be made smaller by increasing a parameter called \( \text{substeps} \), which links \( \Delta t \) and \( \Delta t_f \) by the relation \( \Delta t_f = \Delta t / (2 \times \text{substeps}) \). When “time step” is later referenced under this advancing-the-field discussion, it will mean the field time step.

The code uses an explicit trapezoidal leapfrog scheme, which discretizes the time in a central difference way, to step forward the fields. This technique requires the field values to be calculated at half time steps. The half-step fields can also be referred to as auxiliary fields.

Assuming \( x(t_0), v(t_0), E(t_0), \) and \( B(t_0) \) are known at \( t = t_0 \), and \( E(t_0 - \Delta t_f / 2), \) and \( B(t_0 - \Delta t_f / 2) \) have been already calculated by the previous evolution round, the
code will calculate the electric field and magnetic field at the next full step $\mathbf{E}(t_0 + \Delta t_f)$, and $\mathbf{B}(t_0 + \Delta t_f)$ by

1. Calculate the auxiliary fields at $t = t + \Delta t_f/2$, using the normalized Ampere’s and Faraday’s laws,

$$\frac{\mathbf{E}(t_0 + \Delta t_f/2) - \mathbf{E}(t_0 - \Delta t_f/2)}{\Delta t_f} = -[\nabla \times \mathbf{B}(t_0)]$$

$$\frac{\mathbf{B}(t_0 + \Delta t_f/2) - \mathbf{B}(t_0 - \Delta t_f/2)}{\Delta t_f} = c^2[\nabla \times \mathbf{E}(t_0) - \mathbf{J}(t_0)].$$

**Note 1:** All the equations mentioned in the Time Stepping section are in their normalized form. We drop the tilde above the variables for convenience.

**Note 2:** Also for convenience, the space derivatives are not exactly written in a discretized form here. However, keep in mind that they are discretized using central difference in the code.

**Note 3:** At time $-\Delta t_f/2$, the auxiliary electric field $\mathbf{E}(-\Delta t_f/2)$ and the auxiliary magnetic field $\mathbf{B}(-\Delta t_f/2)$ are set to be equal to the electric field $\mathbf{E}(0)$ and magnetic field $\mathbf{B}(0)$ at $t = 0$.

2. Use the auxiliary half-step fields at $t = t + \Delta t_f/2$, to calculate the full-step fields at $t = t + \Delta t_f$,

$$\frac{\mathbf{E}(t_0 + \Delta t_f) - \mathbf{E}(t_0)}{\Delta t_f} = -[\nabla \times \mathbf{B}(t_0 + \Delta t_f/2)]$$

$$\frac{\mathbf{B}(t_0 + \Delta t_f) - \mathbf{B}(t_0)}{\Delta t_f} = c^2[\nabla \times \mathbf{E}(t_0 + \Delta t_f/2) - \mathbf{J}(t_0)].$$

On the right hand side of the equation 2.15 and 2.17, the current density $\mathbf{J}$ are of the same value because the particles have not been updated within one field time step $\Delta t_f$.

3. Recalculate the auxiliary fields at $t = t_0 + \Delta t_f/2$,

$$\mathbf{E}(t_0 + \Delta t_f/2) = \frac{\mathbf{E}(t_0) + \mathbf{E}(t_0 + \Delta t_f)}{2}$$

$$\mathbf{B}(t_0 + \Delta t_f/2) = \frac{\mathbf{B}(t_0) + \mathbf{B}(t_0 + \Delta t_f)}{2}.$$

Although repeating step (1) and step (2) is enough to step forward in time, there is no guarantee that the auxiliary half-step fields and the full-step fields will not diverge. Step (3) assures that the two will converge, which helps stabilize the code.
The three steps will be repeated until the electric field and the magnetic field at the half particle time step, \( E(t_0 + \Delta t/2) \) and \( B(t_0 + \Delta t/2) \), are known. These fields will then be used to advance the positions and velocities of the particles.

**STEP 2: Advance Particles for Full Time Step**

To advance the particles, the code uses a Boris algorithm. Note that Newton’s second law actually updates \( v^* \equiv \gamma v \). However, the transformation between \( v \) and \( v^* \) can be done easily enough through the relation \( (1 - (v/c)^2)^{-1/2} = (1 + (v^*/c)^2)^{1/2} = \gamma \).

1. Advance the position for a half time step using the velocity at \( t_0 \),

\[
\frac{x(t_0 + \Delta t/2) - x(t_0)}{\Delta t/2} = v(t_0). \quad (2.20)
\]

2. Advance the velocity for a full time step \( \Delta t \) with the following substeps:
   - Step forward the velocity due to the electric force for a half time step,
     \[
     \frac{v^*_{\text{new}} - v^*_{\text{old}}}{\Delta t/2} = \pm \frac{1}{m} E(t_0 + \Delta t/2). \quad (2.21)
     \]
   - Step forward the velocity due to the magnetic force for a full time step,
     \[
     \frac{v^*_{\text{new}} - v^*_{\text{old}}}{\Delta t} = \pm \frac{1}{m} \left( \frac{v_{\text{new}} + v_{\text{old}}}{2} \right) \times B(t_0 + \Delta t/2). \quad (2.22)
     \]
   - Step forward the velocity due to the electric force for the second half time step to get the final velocity of the next time step,
     \[
     \frac{v^*_{\text{new}} - v^*_{\text{old}}}{\Delta t/2} = \pm \frac{1}{m} E(t_0 + \Delta t/2). \quad (2.23)
     \]
3. Advance the position for the second half time step using the velocity at \( t_0 + \Delta t \),

\[
\frac{x(t_0 + \Delta t) - x(t_0 + \Delta t/2)}{\Delta t/2} = v(t_0 + \Delta t). \quad (2.24)
\]
STEP 3: Advance the Fields for the Second Half Step

This step is essentially the same as STEP 1. The only difference is that on the right hand side of Ampere’s law, we use the new current density \( J(t_0 + \Delta t) \), derived from the position and the velocity of the particles obtained by STEP 2.

Going from STEP 1 to STEP 3 completes one round of evolution, advancing time by \( \Delta t \). The code will iterate this procedure until the final time has been reached.

2.5 Enforcing Gauss’s Law and \( \nabla \cdot B = 0 \)

Ideally, stepping forward the fields using Ampere and Faraday’s law should not make the fields lose their consistency with the divergence laws in Maxwell’s equations (Gauss’s law and \( \nabla \cdot B = 0 \)). When numerical errors are introduced, however, there is no guarantee that the consistency with the divergence laws will be retained. Therefore, instead of letting the value of \( \nabla \cdot E \) and \( \nabla \cdot B \) being passive, P3D actively enforces their values to be consistent with the divergence laws by calculating the needed correction in every time step.

The enforcing process is done at half steps before using the fields to step forward the velocity of the particle, ensuring the forces acting on the particle are from well-behaved fields. After being stepped forward with Ampere’s law, the electric field might not be consistent with the Gauss’s law as discretization error comes into play, \( \nabla \cdot E \neq c^2 \rho_c \). So, we need a correction term \( E_{\text{cor}} \), which allows

\[
\nabla \cdot (E + E_{\text{cor}}) = c^2 \rho_c.
\]

(2.25)

If we define the correction potential as \( E_{\text{cor}} = -\nabla \phi_{E,\text{cor}} \), we can rewrite Equation 2.25 as a Poisson equation with all the known quantities on the right hand side,

\[
\nabla^2 \phi_{E,\text{cor}} = \nabla \cdot E - c^2 \rho_c.
\]

(2.26)

The code solves for \( \phi_{E,\text{cor}} \) in Equation 2.26 with a multigrid method, which solves the problem using relaxation approach at multiple scales. Substituting \( \phi_{E,\text{cor}} \) back in to
\( \mathbf{E}_{\text{cor}} = -\nabla \phi_{E,\text{cor}} \) gives us the correction term needed to ensure that the electric field is consistent with Gauss’s law. Similarly, we can get

\[
\nabla^2 \phi_{B,\text{cor}} = \nabla \cdot \mathbf{B},
\]

(2.27)

where \( \phi_{B,\text{cor}} \) is the correction potential for the magnetic field, defined by \( \mathbf{B}_{\text{cor}} = -\nabla \phi_{B,\text{cor}} \) and \( \nabla \cdot (\mathbf{B} + \mathbf{B}_{\text{cor}}) = 0 \). It has been experienced by users of the code, however, that the code can preserve \( \nabla \cdot \mathbf{B} = 0 \) well, and the divergence-B enforcing step can be skipped, if the grid spacing is sufficiently small to resolve the current sheets.

### 2.6 The Numerical Conditions

The code needs appropriate step sizes in both space and time (\( \Delta x \) and \( \Delta t \)) in order to resolve the physical scales of interest and to run stably. The following is the list of conditions P3D needs.

- \( \Delta t \) has to be smaller than \((\text{electron plasma frequency})^{-1}\) and \((\text{electron gyrofrequency})^{-1}\) of the plasma in the system.
- \( \Delta x \) has to be smaller than the \( \pi \ast (\text{Debye length}) \), \( 2 \ast (\text{electron gyroradius}) \), and \((\text{electron inertial length})\) of the plasma in the system.
- \( \Delta x/\Delta t_f \) has to be greater than the light speed in the system.

### 2.7 The Boundary and Initial Conditions

All of the reconnection simulations are performed in 2.5D. Vector quantities in the third dimension are allowed to exist. However, there is no variation along the third dimension. The simulation domain is a rectangular box of size \( L_x \times L_y \), with \( L_x = 204.8d_{i0} \) and \( L_y = 102.4d_{i0} \). We use the periodic boundary condition in all directions.

The initial conditions our simulations use consist of a double asymmetric current sheets. The magnetic field has only an \( x \)-component. The magnetic field varies across the current sheets with a tanh profile, which is mathematically given by:

\[
B_x(y) = \left( \frac{B_2 + B_1}{2} \right) \left[ \tanh \left( \frac{y - 0.25L_y}{w_0} \right) \right]
\]
Figure 2.3: An initial magnetic field with a tanh profile across the simulation box in the $y$-direction. $B_1 = 1$ and $B_2 = 2$ are the asymptotic values.

$$
\begin{align*}
- \tanh \left( \frac{y - 0.75L_y}{w_0} \right) &+ \tanh \left( \frac{y - 1.25L_y}{w_0} \right) \\
- \tanh \left( \frac{y + 0.25L_y}{w_0} \right) + 1 &+ \left( \frac{B_2 - B_1}{2} \right),
\end{align*}
$$

where $B_1$ and $B_2$ are the asymptotic magnetic field strength on the “1” and “2” upstream sides, and $w_0$ is the initial width of the sheet, $L_y$ is the size of simulation box in the $y$-direction, which is across the current sheets. Figure 2.3 shows how the magnetic field with the profile given in Equation 2.28 varies in the $y$-direction across the simulation box. Figure 2.4 shows the double current sheet created by such magnetic field profile in a 2D plot.

The temperature varies in the $y$-direction across the current sheets with a tanh profile as well,

$$
T(y) = \left( \frac{T_2 - T_1}{2} \right) \left[ \tanh \left( \frac{y - 0.25L_y}{w_0} \right) \\
- \tanh \left( \frac{y - 0.75L_y}{w_0} \right) + \tanh \left( \frac{y - 1.25L_y}{w_0} \right) \\
- \tanh \left( \frac{y + 0.25L_y}{w_0} \right) + 1 \right] + \left( \frac{T_2 + T_1}{2} \right),
$$

Figure 2.3 shows how the magnetic field with the profile given in Equation 2.28 varies in the $y$-direction across the simulation box. Figure 2.4 shows the double current sheet created by such magnetic field profile in a 2D plot.
where $T_1$ and $T_2$ are the asymptotic temperatures. Note that Equation 2.29 are applicable for both ion and electron temperatures.

Once the magnetic field and the temperature are specified, the number density profile of particles $n(y)$ will be determined using total pressure constant,

\[
\frac{B^2_2(y)}{2} + n(y)T_{\text{tot}}(y) = \frac{B^2_1}{2} + n_1T_{\text{tot}1} = \frac{B^2_2}{2} + n_2T_{\text{tot}2},
\]

where $n_1$ and $n_2$ are the asymptotic particle number density and $T_{\text{tot}}(y)$ is the total temperature with $T_{\text{tot}1}$ and $T_{\text{tot}2}$ being its asymptotic values.

At last, it is time to give the particles their positions and velocities to realize the predefined temperature and density while keeping the consistency with the magnetic field. This can be done by first giving a random position to a particle. This particle can be either accepted or rejected. The probability of being accepted is controlled by the density profile - a particles with a given position in a high-density area is more probable to be accepted than one with a position in a low-density area. If rejected,
the particle will be given another random position. The code repeats this process until
the particle is no longer rejected. A similar procedure is also used in the process of
assigning the velocity. Once the particle is accepted, it will be given a random speed,
which has the approval chance regulated by the relativistically-corrected Maxwellian
speed distribution associated with the local temperature. The code then appoints a
random direction to the particle after the speed is approved. Finally, the velocity
is added in the out-of-plane direction by an appropriate value to create the current
consistent with $\nabla \times \mathbf{B}$.

We note that this initialization is not a strict 1D kinetic equilibrium (see [1]
for one example). However, any initial transient effects have diminished by the time
relatively steady reconnection is occurring. A small magnetic perturbation is used to
initiate magnetic reconnection. The reconnection is antiparallel, i.e., there is no guide
magnetic field ($B_z = 0$ at $t = 0$).

2.8 Unrealistic Parameters

In all of our simulations, the speed of light $c$ is set to be $15c_{A0}$. Also, the electron
to ion mass ratio is $m_e/m_i = 1/25$. Why do we use these unrealistic parameters? Are
the results from the simulations really valid?

The reason we use these unrealistic parameters is because they allow the simu-
lation to be performed with reasonable resource consumption. According to the code
units, the electron plasma frequency ($\omega_{pe}$), electron cyclotron frequency ($\omega_{ce}$), electron
inertial length ($d_e$), electron Larmor radius ($\rho_e$), and Debye length ($\lambda_D$) can be
expressed (with the tilde being dropped for convenience) as

$$\omega_{pe} = \sqrt{\frac{n_e}{m_e}} c \quad \text{(2.31)}$$

$$\omega_{ce} = \frac{B}{m_e} \quad \text{(2.32)}$$

$$d_e = \sqrt{\frac{m_e}{n_e}} \quad \text{(2.33)}$$

$$\rho_e = \frac{\sqrt{2T_e m_e}}{B} \quad \text{(2.34)}$$
\[
\lambda_D = \frac{1}{c} \sqrt{\frac{T_e}{n_e}},
\]

(2.35)

We can see that comparing to the mass ratio of \(m_e/m_i = 1/25\), using the realistic mass ratio of \(m_e/m_i = 1/1836\) will increase the electron plasma frequency and electron cyclotron frequency by the factors of about 8 and 70 respectively. It also causes the electron inertial length and the electron Larmor radius to be about 1/8 of their counterparts based on the mass ratio of 1/25. To resolve such a realistic system, it requires much smaller \(\Delta x\) and \(\Delta t\) than when we use the unrealistic mass ratio. Similarly, using the realistic light speed will increase the electron plasma frequency and decrease the Debye length significantly. For example, the magnetospheric plasma has the Alfvén speed in the order of 1000 km/s. The light speed is 300,000 km/s. So, the realistic velocity ratio \(c/c_A\) is in the order of 300. For the case of the realistic velocity ratio, we then require about 20 times smaller \(\Delta x\) and \(\Delta t\) in order to resolve the Debye length and electron plasma frequency compared to the case of velocity ratio of 15.

How about the validity of the results? The type of plasma being in our simulation is different from the magnetospheric plasma, which is the plasma of our interest. Can the results of our simulations make meaningful predictions for the magnetosphere? According to the purposes of our study, the answer is “yes.” It has been shown numerically by [57] that the reconnection rate is independent of the mass ratio. So, we can still get a realistic reconnection rate even with an unrealistic mass ratio. The outflow speed scales as the Alfvén speed, which is not electron mass dependent. So, the problem of obtaining an unrealistic outflow speed in our simulations is not likely either. The structure of the diffusion region, however, will not be realistic as the electron length scales are now much larger than they should. Nevertheless, it is not difficult distinguish which structures are derived from the electron dynamics and which structures are derived from the ion dynamics with the mass ratio of 1/25. The light wave does not play a noticeable role in mechanism of reconnection, so having unrealistic light speed should not significantly alter the results especially in terms of gross properties. However, we should not set the ratio of \(c/c_A\) to be too low either because the relativistic effects
might significantly come into play. The Lorentz factor $\gamma$ is about 1.15 if $v/c = 0.5$. So, as long as we do not have any flow in the system that exceeds 0.5$c$, the results can still be considered acceptable.
Chapter 3
THE GROSS PROPERTIES

3.1 Introduction

Conventional studies of magnetic reconnection have mainly focused on symmetric reconnection where the magnetic field and particle density are the same on both sides upstream of the diffusion region (e.g. [5] and references therein). However, in many physical situations the magnetic field and density in these two upstream regions can vary substantially. Day-side reconnection, where magnetosheath plasma (magnetic field $\approx 10$-$20$ nT, density $\approx 20$-$30$ cm$^{-3}$) reconnects with magnetospheric plasma (magnetic field $\approx 50$-$60$ nT, density $\approx 0.3$-$0.5$ cm$^{-3}$), is a good example of this so called “asymmetric reconnection” ([50, 37], and references therein). As another example, asymmetric reconnection has been observed in the magnetotail [43].

To understand asymmetric reconnection, [12] introduced a model of the asymmetric diffusion region on which they performed a Sweet-Parker-type scaling analysis. They then obtained scaling relations predicting asymmetric reconnection properties including reconnection rate, outflow speed, outflow density, x-line position, and stagnation point position as functions of upstream magnetic fields and particle densities (as discussed earlier in Chapter 1). This general theory was consistent with a previous scaling analysis for asymmetric density [9], and the reconnection rate and outflow speed predictions have been verified thoroughly with fluid simulations for both collisional [12, 10, 56] and collisionless reconnection [13]. A systematic study of the scaling results have not been carried out using more realistic particle-in-cell codes. However, various predictions have been borne out in PIC simulations [39, 51, 68]. The reconnection rate scaling was used to determine a physics-based solar wind/magnetosphere
coupling model which showed similar or better correlations with various empirical geomagnetic indices than previous coupling models [8]. We emphasize that the focus of this chapter is on the scaling of the diffusion region during asymmetric reconnection. See [12] for references on other aspects of asymmetric reconnection.

One important aspect of the theory, however, has not been born out by simulations. A basic assumption of the scaling study was that in the asymmetric density case, the plasmas with disparate densities in two recently reconnected flux tubes will quickly mix while at the same time conserving the total flux tube volume [12], which allows the determination of the outflow density and the location of the stagnation point. This assumption has been called into question by MHD and two-fluid simulations results [3, 14], where parallel pressure balance along flux tubes prevents mixing, leading to stagnation point locations that do not match predictions. Modification of the scaling theory to remove the mixing assumption, while predicting the fluid stagnation point, leads to erroneous predictions of the X-line location [14].

The failure of the stagnation point predictions in fluid simulations highlights the need for a systematic simulation study of asymmetric reconnection which includes kinetic effects such as parallel thermal diffusion [42]. Such a study can directly test the downstream mixing assumption for collisionless systems such as the Earth’s magnetosphere. In addition, there is the outstanding question of whether the inclusion of kinetic physics beyond the Hall term fundamentally alters the diffusion region structure and require a significant revision of the scaling theory [12, 14].

In this study, we perform a systematic set of electromagnetic kinetic particle-in-cell (PIC) simulations of collisionless asymmetric reconnection and compare various gross diffusion region properties with analytical scaling predictions. We find that the simulation results for reconnection rates and ion outflow speeds agree well with the predictions of a scaling theory [12], which suggests that the kinetic physics beyond the Hall term does not fundamentally alter ion scale properties of the diffusion region. Moreover, the density just downstream of the diffusion region is in good agreement with the prediction, implying that volume conserving particle mixing along newly reconnected
flux tubes is a good approximation.

### 3.2 A Review of the Theory

In order to understand the properties of asymmetric reconnection, [12] generalized the Sweet-Parker-type analysis to a case with asymmetric upstream conditions. In this analysis, the steady-state diffusion region is assumed to be roughly a rectangular region, outside of which single-fluid MHD is assumed to be applicable. Using conservation of mass and energy across the boundaries of the diffusion region and the sub boxes delineated by the X-line and stagnation point, they arrived at scaling relations of asymmetric reconnection as functions of the upstream magnetic fields and densities (More details are discussed in Chapter 1).

Two general properties of reconnection, namely, the reconnection electric field $E$ and the outflow speed $v_{out}$ are predicted to scale as,

\[
E \sim \left( \frac{2B_1B_2}{B_1 + B_2} \right) \left( \frac{v_{out}}{c} \right) \left( \frac{\delta}{L} \right) \tag{3.1}
\]

\[
v_{out}^2 \sim \frac{B_1B_2}{4\pi m n_{out}}, \tag{3.2}
\]

where $B$ is the magnetic field, $n$ is the number density, $c$ is the speed of light, $m$ is the particle mass, $\delta$ is the half width of the dissipation region, and $L$ is the half length of the dissipation region. The subscripts “1” and “2” are used to denote the two upstream regions. The outflow number density is $n_{out}$, which can be estimated by assuming that the two densities within a recently reconnected flux tube completely mix while maintaining the original flux tube volume (See Figure 2 of [12]):

\[
n_{out} \sim \frac{n_1B_2 + n_2B_1}{B_1 + B_2}. \tag{3.3}
\]

Equation 3.2 and 3.3 apply equally well at the ion and electron layers, but we focus on the ion outflow speed and density for the present study.

We emphasize that there are uncertainties in applying these scaling laws to simulations studies. One clear source of uncertainty is that magnetic fields, densities, and velocities can vary at the edges of the diffusion region, while the theory assumes
they are relatively uniform. In addition, the finite Larmor radius of particles in kinetic PIC simulations give them meandering orbits in the diffusion region. A single particle may therefore spend part of its orbit in the diffusion region, and part of its orbit outside, leading to only partial acceleration of the particle by diffusion region electric fields. Because of these uncertainties, the Sweet-Parker-type analysis can only be expected to hold up to a constant of order unity, and expressions are denoted with a $\sim$ instead of a $\approx$.

3.3 Simulations and Results

In order to test the scaling predictions of Equations 3.1-3.3, we perform simulations of symmetric and asymmetric reconnection with various initial upstream magnetic fields, temperatures, and densities. The parameters for each simulation are shown in Table 3.1.

Each simulation is evolved until the reconnection reaches a steady state where the reconnection rate $E$ is relatively constant in time, an example of which is given in Figure 3.1 for Run BN2b. $E$ is calculated as the temporal rate of change of magnetic flux between the X-line and the O-line. For this particular run, a steady period of reconnection occurs between $t = 160$ and 220. The oscillations in the reconnection rate during the quasi-steady period allow us to estimate that there is approximately a 10 to 15 percent error in the electric field measurements. During the quasi-steady period
Table 3.1: Simulation information. The data from the column of Run # through the column of \textit{ppg} are user defined. The last three columns come from the analysis. In the Run # column, S = symmetric, B = asymmetric upstream magnetic field, N = asymmetric upstream density, BN = asymmetric upstream magnetic field and density, a and b denote runs with high \textit{ppg} and low \textit{ppg} respectively.

<table>
<thead>
<tr>
<th>Run #</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$T_{i1}$</th>
<th>$T_{i2}$</th>
<th>$T_{e1}$</th>
<th>$T_{e2}$</th>
<th>$\Delta x$</th>
<th>$\Delta t$</th>
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<th>$v_{i,out}$</th>
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of reconnection for Run BN2b, the reconnection exhibits typical ion scale signatures of asymmetric reconnection, as seen in Figure 3.2, which plots the time averages over 2 $\Omega_{ci0}^{-1}$ of $n_i$, $J_z$, and the Hall magnetic field $B_z$ overlaid with magnetic field lines (lines of constant magnetic flux). The magnetic island preferentially grows into the region with lower magnetic field as has been seen many times before. There is a strong current sheet on the high magnetic field side of the diffusion region, and the quadrupolar magnetic field structure is highly distorted [26, 37, 68], exhibiting more of a bipolar structure. This distorted $B_z$ structure does not mean that Hall physics is not playing a role. The existence of a significant $B_z$ requires currents in the $xy$ plane, which implies decoupled electron-ion motion and the importance of Hall physics. The ion density in the downstream diffusion region is clearly a hybrid of $n_1$ and $n_2$ [12].

Once the system has reached the quasi-steady state, the reconnection rate, outflow speed, and outflow density are measured. In typical symmetric reconnection, downstream properties of the diffusion region (outflow density, outflow speed) can be easily determined by taking a cut along $x$ at the symmetry line. However, the center of the magnetic islands in asymmetric magnetic reconnection are shifted along the inflow ($y$) direction, and the outflow velocities have a substantial component along $y$. In addition, kinetic PIC simulations are inherently noisy, which can make simulation data analysis difficult. To determine the downstream diffusion region properties, we first time average the simulation data over 1 or 2 $\Omega_{ci0}^{-1}$ of the quasi-steady time period. At each $x$-value, the $y$ location of the peak in-plane ion flux $n_iu_{i\perp}$ with $u_{i\perp} = \sqrt{u_{ix}^2 + u_{iy}^2}$ is determined, as shown in Figure 3.3(a) for Run BN2b. The trajectory of these values defines the outflow trajectory away from the diffusion region, which is denoted by the black line in Figure 3.3(a). Plotting $u_{i\perp}$, $u_{e\perp} = \sqrt{u_{ex}^2 + u_{ey}^2}$ and $n_i$ along this outflow trajectory allows the determination of downstream diffusion region properties, as shown in Figure 3.3(b). Within a few ion inertial lengths of the X-line the outflow values are not plotted because of spurious jumps of the outflow trajectory owing to the very small values of $n_iu_{i\perp}$. As expected near the electron diffusion region, the electrons are decoupled from the ions and thus are super-Alfvénic. There is very little
Figure 3.2: Asymmetric reconnection signatures during quasi-steady reconnection of Run BN2b. Contours of constant flux (magnetic field lines) overlaid with: (a) ion density \( n_i \), (b) out-of-plane current \( J_z \), and (c) out-of-plane magnetic field \( B_z \). The data for these plots are time averaged from \( t = 180 \) to 182.
Figure 3.3: Determination of outflow velocities and densities. (a) In-plane ion flux \( n_i u_{i\perp} \), where the in-plane ion speed \( u_{i\perp} = \sqrt{u_{ix}^2 + u_{iy}^2} \), showing the path of maximum outflow flux (black line). (b) In-plane ion speed (solid line), in-plane electron speed \( u_{e\perp} = \sqrt{u_{ex}^2 + u_{ey}^2} \) (dotted line), and downstream density \( n_i \) (dashed line) along the path of maximum ion flux. The data very close to X-line, which are not relevant to this study, are omitted because of spurious jumps in the outflow trajectory owing to the very small values of \( n_i u_{i\perp} \).
ion flow here. The electrons reach a peak velocity which can roughly be thought of as the downstream edge of the electron diffusion region \cite{60}. Their velocity then decreases as they decelerate to join the much slower ions \cite{59, 62}. The location where the electron and ion velocities cross is roughly the location where the ion diffusion region ends, and is denoted by vertical dotted lines in Figure 3.3b. The ion outflow speed and outflow density are determined by averaging the values at these two locations, as has been done in past two-fluid studies \cite{62}. The left-right asymmetry of the diffusion region, as well as scatter in the electron and ion velocities leads to uncertainties in both the location where the electron flow ultimately rejoins the ion flows and the value for $v_{i, out}$. We estimate an error of 10 to 15 percent for the ion outflow velocity and the downstream density. The resulting measured reconnection rate, ion outflow speed, and outflow density of each simulation are shown in the last three columns of Table 3.1.

The measured reconnection rates, outflow speeds, and outflow density from the simulations compare well with the theoretical predictions Equations 3.1-3.3, as can be seen in Figure 3.4. The theoretical values for this figure were determined by using the asymptotic upstream values of the magnetic field and density, a reasonable assumption owing to the initially thin equilibrium current sheets. It was shown in two-fluid studies \cite{14} that $\delta/L$ is approximately 0.1 independent of asymmetries. We therefore make an assumption of $\delta/L$ being 0.1 here. If the PIC results here agree, we would conclude that the assumption is reasonable. If the data do not fall on a line, we would conclude that kinetic effects beyond the Hall term are altering the structure of the dissipation region. The other reason to assume $\delta/L$ to be 0.1 is that it is prohibitively difficult to determine $\delta$ from the simulations because of the noise inherent in kinetic PIC simulations. The absolute magnitude of the theoretical and simulation values for $E$ and $v_{out}$ differ by a factor of around 2, though the data falls approximately on a line. The absolute magnitude of the outflow densities show very good agreement between theory and simulations. For all three quantities, the scaling is consistent with the theory \cite{12}. The disparity between simulations and theory of around a factor of two is not a concern for the present study. As discussed earlier, scaling analyses can only be expected to
Figure 3.4: Simulation results vs. theoretical predictions for (a) reconnection rate $E$, (b) outflow speed $v_{i,\text{out}}$, and (c) outflow density $n_{i,\text{out}}$. Circles (black) represent symmetric runs. Diamonds (red) represent asymmetric upstream density runs. Squares (blue) represent asymmetric upstream magnetic field runs. Asterisks (green) represent asymmetric upstream density and magnetic field runs.
be correct up to a factor of order unity. The fact that all three quantities in Figure 3 approximately lie on a line gives evidence that the scaling laws are consistent with simulation results. Clearly, however, there is scatter in the data, which is a result of the uncertainties (of 10 or 15 percent) in determining quantities from the simulations.

3.4 Discussion and Conclusion

A careful analysis of zero guide field, asymmetric reconnection has been performed using the fully electromagnetic kinetic PIC code P3D. The reconnection rate, ion outflow velocity, and ion outflow density scale as predicted by a Sweet-Parker-like scaling analysis \[12\], assuming the ratio \(\delta/L = 0.1\) within the estimated 10-15% uncertainty in the simulations. This implies that: (1) kinetic physics beyond the Hall term does not fundamentally change the ion scales of the asymmetric diffusion region as understood in the fluid sense \[12\] near the X-line, and (2) the assumption that particles on newly reconnected field line fully mix without changing the flux tube volume \[12\] is valid to lowest order, i.e., plasma from just reconnected flux tubes mixes together while preserving the total volume. However, modifications such as including the effects of compressibility and enthalpy flux might improve the accuracy of the predictions \[4\].

As the mixing assumption is valid, we expect the stagnation point locations to match the predictions. However, determining the stagnation point location requires a careful determination of the relatively small inflow velocity which is extremely difficult because of (1) the inherent random noise present in PIC simulations, and (2) the tendency of the X-line to propagate along the inflow direction \[23, 70, 31, 12\]. Point (2) requires a shift of reference frames to a frame moving with the X-line. This kind of analysis is planned for the future.

This work has limitations which should be addressed in future studies. First, for the dayside magnetosphere, the incoming solar wind often has a significant \(B_y\) component in Geocentric Solar Magnetospheric (GSM) coordinates, which is the equivalent of including a guide magnetic field along \(z\) in this study. This guide field can substantially alter some of the signatures of asymmetric reconnection \[39, 38, 52\], although
a systematic analysis of its effect on the outflow velocity and reconnection rate has not been performed. In addition, with a guide field the orientation of the X-line is uncertain and is the topic of current study [66]. The presence of a pressure gradient across the X-line can lead to propagation of the X-line due to the diamagnetic drift [67, 51]. Second, the analytical scaling study predictions tested here ignore plasma compressibility in the diffusion region, which could lead to a plasma $\beta$ dependence [4]. This $\beta$ dependence should be considered for future studies, although the effect at typical magnetospheric values may be modest. Finally, the day-side magnetosphere is fundamentally a three-dimensional system, with curvature and three-dimensional stagnation flow effects. Global magnetohydrodynamic simulations of day-side reconnection have found that these 3D effects can significantly alter the structure of reconnection ([17], and references therein).

Understanding the physics controlling asymmetric magnetic reconnection will ultimately allow much more realistic predictions for the reconnection signatures in the magnetosphere. These predictions play a critical role in the implementation of satellite missions such as the Magnetospheric Multiscale Mission (MMS).
Chapter 4
THE SIGNATURES AND DIFFUSION REGION STRUCTURE

4.1 Introduction

Owing to the upcoming Magnetospheric Multiscale (MMS) mission whose primary mission is to study magnetic reconnection in the magnetosphere, it is important to be able to pinpoint whether the satellites are in the vicinity of a reconnection site. The understanding of the reconnection signatures under asymmetric inflow conditions is therefore needed to increase the chance of locating reconnection sites. Once a site is located, the MMS satellites will provide unprecedented measurements of the diffusion region. This is a great opportunity to put our theoretical understanding of the asymmetric diffusion region into test. Recently, some attempts have been made to gain more understanding on both the signatures and the structure of the diffusion region of asymmetric reconnection.

The signatures of asymmetric reconnection is a topic that has been put under scrutiny. Using fully kinetic particle-in-cell (PIC) simulations with inflow parameter sets resembling typical conditions of the day-side reconnection, Mozer [39], Pritchett [51], and Tanaka [68] show that the quadrupolar Hall magnetic field becomes bipolar under asymmetric inflow conditions. Furthermore, the bipolar Hall electric field is shown to appear unipolar. Nevertheless, some other fully kinetic studies [67, 33] find that although skewed, the Hall magnetic field and electric field still retain the quadrupolar and bipolar structures. Satellite observations show that both bipolar [37, 39, 68] and quadrupolar [40, 43] magnetic field structure can be found at magnetopause and magnetotail reconnections. One reason for the disagreement is that even though all these studies have focussed on asymmetric reconnection, they do not have the same
set of inflow conditions. This disagreement can be settled if we understand how and, more importantly, why the Hall electric and magnetic fields are altered when the inflow conditions vary. However, such a systematic study of the effect of inflow conditions has not been performed. Another important question is whether or not the asymmetry introduces additional signatures, beside the Hall electric and magnetic fields, that may help locate reconnection diffusion regions?

Although not as numerous as the studies of Hall signatures, studies of the asymmetric diffusion-region structure have also been carried out. Using a Sweet-Parker-type analysis, Cassak and Shay [12, 14] are able to predict the size of the diffusion region as well as the locations of the X-line and stagnation point. Confirmations with simulations, however, were restricted to resistive MHD [12] and Hall MHD models[14]. What if kinetic particle effects are included? Would the predictions still work? These questions are still left unanswered.

The study in this chapter is intended to fill some of the voids discussed above. Using the fully kinetic PIC code, P3D [75], we are able to capture all the physics in the vicinity of the diffusion region, and so are able to self-consistently simulate the structure of the diffusion region. Furthermore, the signatures obtained will not be limited to the signatures due to the Hall physics. We show in this study that there are signatures that are spawned from kinetic effects, which include the kinetic electric field and the electron anisotropy. The inflow conditions are varied systematically instead of choosing just one particular set of parameters. This allow us to understand the trend of how the signatures and diffusion region structure change as we alter the upstream conditions.

4.2 Simulations

The study in this chapter uses the fully kinetic particle-in-cell code P3D [75] to perform simulations of collisionless anti-parallel asymmetric reconnection with different sets of upstream conditions. Simulation parameters are summarized in Table 4.1. Each
simulation is evolved until the reconnection reaches a quasi-steady state. Detailed explanation of the simulation is covered in Chapter 2.

4.3 Results and Discussion

4.3.1 The Hall Signatures

In the Hall model of reconnection, the Hall electric field and the Hall magnetic field are created due to the out-of-plane frozen-in flow of the electrons [34]. This electron flow can be thought of as the negative flow of the current because the ions are relatively stationary in the Hall region. In the symmetric case, the current is symmetrically located around the X-line, leading to the symmetric bipolar and quadrupolar structures of the Hall electric and magnetic fields. In asymmetric reconnection, the current is no longer symmetric in general.

The fact that the current can be skewed, having the center not co-located with the X-line, can be understood by considering the diffusion region as it is in 1D equilibrium along the inflow direction. This 1D equilibrium is necessary owing to slow inflow speeds relative to the Alfvén speed, as can be seen by examining the ion force equation in the context of a Sweet-Parker-like analysis of the diffusion region [65, 46]. The diffusion region has a width along the inflow direction (\(\hat{y}\)) of \(\delta\) and a length along the outflow direction (\(\hat{x}\)) of \(L\). By mass continuity, \(u_y/u_x \sim \delta/L\), and \(\nabla \cdot \mathbf{B} = 0\) requires that \(B_y/B_x \sim \delta/L\). Using these relations, the force equation along the inflow direction can be scaled in the following way:

\[
\begin{align*}
    mn \frac{\partial u_y}{\partial t} + mn (\mathbf{u} \cdot \nabla) u_y &= -\frac{\partial P}{\partial y} - \frac{\partial}{\partial y} \left( \frac{B^2}{8\pi} \right) + \left( \frac{\mathbf{B} \cdot \nabla}{4\pi} \right) B_y, \\
    \text{small} & : \frac{\delta^2}{L^2} u_x^2 : c_s^2 : c_A^2 : \frac{\delta^2}{L^2} c_A^2, \quad (4.2)
\end{align*}
\]

where \(c_s\) and \(c_A\) is the sound and Alfvén speeds respectively. The assumption of quasi-steady inherent in a Sweet-Parker-like analysis removes time dependence. Because \(\delta/L\) is at most of order 0.1 ([73], and references therein) and the speed in the outflow direction \(u_x\) scales as \(c_A\), only the two pressure terms survive, giving \(P + B^2/8\pi \approx \text{constant}\). Unlike the symmetric case, the kinetic pressures of the two asymmetric upstream sides
Table 4.1: Simulation information. $B_a$ is the magnetic field strength on side $a$. $n_a$ is the number density on side $a$. $T_{ea}$ is the electron temperature on side $a$. $T_{ia}$ is the ion temperature on side $a$, where $a$ can be either 1 or 2. $\Delta x$ is the spatial grid size of the run. $\Delta t$ is the time step size of the run. $ppg$ is the number of ions/electrons representing density of 1.0. In the Run # column, N = asymmetric density, B = asymmetric magnetic field, and BN, bN, BN0, Bn are all asymmetric density and magnetic field. These 4 different types are categorized by where the stagnation point is theoretically located compared to the X-line. The theory [12] says that using the X-line as the reference, the stagnation point will be located on the inflow side with low-density and high-magnetic-field. Depending on the predicted location of the stagnation point, the simulations are labeled as BN, bN, Bn, and BN0, as described in the table.

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<th>$B_2$</th>
<th>$n_1$</th>
<th>$n_2$</th>
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<td>1.33</td>
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do not have to be equal, and so the transition of the kinetic pressure has to occur at some point within the diffusion region. When there is a kinetic pressure transition, there has to be a magnetic pressure transition to keep the total pressure unchanged. The magnetic pressure transition, in turn, creates current. Since, in general, the transition does not generally occur at the X-line, the current is not required to be symmetric around it. Figure 4.1 provides an example of how a change in magnetic pressure is needed where there is a kinetic pressure transition for an asymmetric reconnection with typical day-side conditions (Run BN-2). The figure also demonstrates that consequently, the off-the-X-line magnetic pressure transition leads to a current profile asymmetrically situated around the X-line.

An important question, then, is where does the transition occur? As is evident in Figure 4.1(c), the transition denotes the change of plasma conditions from one inflow side to another. Since the stagnation point (defined where $u_{iy} = 0$) represents that location along $y$ where the two inflowing plasmas can no longer flow inwards, it is expected that the transition between the two distinct inflowing plasmas should occur here. Therefore, we expect the current to be the most intense in the vicinity of the stagnation point.

Unfortunately, testing this idea directly is problematic because of the difficulty of directly determining the location of the stagnation point from the point where $u_{iy}$ goes to zero. In asymmetric magnetic reconnection, there is a tendency of the x-line to drift along the inflow direction, so $u_{iy}$ must be shifted into the frame moving with the x-line. Furthermore, the noise in the simulation due to the finite number of particles tends to swamp the relatively small signal of $u_{iy}$, especially on the low density inflow sides where the temperature tends to be high.

To test the idea, we assume that the location of the current is the same as the location of the actual stagnation point. Then we use the location of the current as a proxy of the actual stagnation point. However, unlike the stagnation point which is just a point, the current varies in space. We, therefore, have to find a way to represent the location of the current with one value. In this study, we choose the mid-point
Figure 4.1: (a) 2D plot of the out-of-plane current $J_z$ of Run BN-2, over-plotted with the magnetic field lines in $xy$-plane. The colors red/blue means the field is pointing in the positive/negative $z$ direction (pointing out-of-the-page/into-the-page). White means zero. (b) 1D cut of the out-of-plane current at the location shown by the vertical dashed line in (a). (c) 1D cut of the magnetic pressure (green) and kinetic pressure (red). The solid vertical lines in (b) and (c) represent the location of the X-line.
Figure 4.2: A 1D cut of the out-of-plane current of Run BN-2 at an $x$ value shown by the vertical dashed line in Figure 4.1(a). The horizontal dashed line represents the half-maximum value of the current. The vertical dashed lines represent the locations of the two points with the half-maximum value. The vertical red solid line is the line in the middle of the two vertical dashed lines. This solid red line represents the mid-point half-maximum (MPHM). The vertical blue solid line denotes the position of the X-line. The MPHM of $J_z$ and the X-line are generally not co-located. $\delta_{X-J}$ is defined as the distance from MPHM to the X-line as shown by the arrow. If the X-line is on the left of MPHM, leading to the arrow pointing in the negative $y$-direction, $\delta_{X-J}$ will be taken to be negative.

Given the magnetic field strengths and the densities of plasmas on side 1 and side 2 ($B_1, B_2, \rho_1,$ and $\rho_2$ respectively) of the upstream, we can theoretically predict the locations of the X-line and the stagnation point using the fluid-physics-based relations proposed by [12] and [14]:

\[
\frac{\delta_{X2}}{\delta_{X1}} \sim \frac{B_2}{B_1} \tag{4.3}
\]

\[
\frac{\delta_{S2}}{\delta_{S1}} \sim \frac{\rho_2 B_1}{\rho_1 B_2} \tag{4.4}
\]

\[
\delta \sim \left( \frac{B_1 + B_2}{2\sqrt{B_1 B_2}} \right) \left[ \left( \frac{m_i c^2}{4\pi e^2} \right) \left( \frac{\rho_1 B_2 + \rho_2 B_1}{B_1 + B_2} \right) \right]^{1/2} \tag{4.5}
\]
where $\delta_{X1}$ and $\delta_{X2}$ are the distances from the edges of the diffusion region of side 1 and side 2 to the X-line, $\delta_{S1}$, and $\delta_{S2}$ are the distances from the edges of the diffusion region of side1 and side2 to the stagnation point, $\delta$ is the half width of the diffusion region, and

$$2\delta = \delta_{X1} + \delta_{X2} = \delta_{S1} + \delta_{S2}.$$  \hspace{1cm} (4.6)

Using Equation 4.3, 4.3, and 4.6 and we can write the distance between the X-line and stagnation point as

$$\delta_{X-S} \equiv \delta_{S1} - \delta_{X1} = \delta_{X2} - \delta_{S2}$$  \hspace{1cm} (4.7)

$$\delta_{X-S} \sim \left[ \frac{B_2^2 \rho_1 - B_1^2 \rho_2}{(B_1 + B_2)(\rho_1 B_2 + \rho_2 B_2)} \right] 2\delta.$$  \hspace{1cm} (4.8)

The assumption that the stagnation point and MPHM of the current are co-located requires that $\delta_{X-J} \sim \delta_{X-S}$. Measuring $\delta_{X-J}$ as shown in Figure 4.2, a comparison of $\delta_{X-S}$ and $\delta_{X-J}$ are shown in Figure 4.3. Note that $\delta_{X-J}$ is taken to be negative when the X-line appears on the left-hand side of the stagnation point. The comparison shows a very rough linear relationship with $\delta_{X-J} \simeq 0.5\delta_{X-S}$. What is striking is the near perfect correlation between the signs of $\delta_{X-S}$ and $\delta_{X-J}$, meaning that the stagnation point is always on the predicted side of the x-line. Therefore, given a set of parameters $[B_1, B_2, \rho_1, \rho_2]$, we can roughly predict what the current and Hall electric field profiles will be like. At least, we can be quite certain that which side of the X-line the center of the profiles will be on. Note that there is the case of asymmetric density but symmetric magnetic field (the 3 plus signs in Figure 4.3) in which the transition of the magnetic pressure was not expected to occur away from the X-line. Due to some physics of the kinetic equilibrium, however, the system readjusts itself and yields a magnetic pressure transition where the stagnation point is expected. Understanding this case requires the knowledge of kinetic equilibrium in addition to the total pressure balance, which is not in the scope of this study. Nevertheless, no matter whether we thoroughly understand this case, it does not change the results that the current is roughly correlated with the location of the stagnation point.
Figure 4.3: $\delta_{X-J}$ (defined by [the location of the X-line] - [location of the MPHM of the out-of-plane current]) vs. $\delta_{X-S}$ (defined by [theoretical location of the X-line] - [theoretical location of the stagnation point]). The runs of type $BN$ are represented by the asterisks, $N$ represented by the pluses, $bN$ represented by the diamonds, $BN0$ represented by the triangles, $Bn$ represented by the squares, and $B$ represented by the crosses.
The spread of points away from a straight line in Figure 4.3 is to be expected since what we are measuring is a fine structure. So, the uncertainty involved can be quite high. We also do not expect that the prediction from the fluid picture to yield exact results either. Nevertheless, to have such a trend with well placed zero points (the 3 triangles in Figure 4.3) is enough to infer that (1) our understanding of the diffusion structure gained from analyzing the fluid picture (Equations (4.3)-(4.5)) is valid (though not complete) and (2) the reconnection signatures and the diffusion region structure are fundamentally linked.

In the Hall region of the diffusion region, where the electrons are frozen-in but the ions are not, this current is generated by frozen-in electrons: \( J_z \approx ne u_{ez} \approx ne c E_y / B_x \), which generates a strong normal (to the magnetic field) electric field \( E_{\text{normal}} = E_y \) in reconnection where Hall physics is playing a role \[58\]. In the symmetric case, the out-of-plane current and the density are symmetric around the X-line while the reconnection magnetic field is anti-symmetric. Hence, \( E_{\text{normal}} \) will have an anti-symmetric bipolar structure. With the day-side asymmetric upstream conditions, however, the out-of-plane current does not have to be symmetric around the X-line anymore (Figure 4.1(b)). This results in \( E_{\text{normal}} \) losing the anti-symmetric property (Figure 4.4(a)), having one pole so much stronger than the other that it looks unipolar. The blue line in Figure 4.4(b) is a plot of the Hall normal electric field \( (1/nec)(J \times B)_y \), which reveals the expected bipolar structure.

However, from the same plot, we can see some disagreement between the normal electric field and the Hall term. Does this mean the Hall model no longer works in an asymmetric set up? Not quite. The Hall model still works. It is just not the whole story. To understand the disagreement, we break the electric field into 2 parts: (1) the negative peak on the left of the X-line and (2) the positive peak and the small negative on the right of the X-line. The first one is where the Hall model does not apply. This structure is created by kinetic effects and will be discussed in more detail later. At the moment, we focus on the second part where the Hall model still works. Since this is a region of strong current where a pressure transition occurs, there are some contribution
Figure 4.4: (a) 2D plot of the normal electric field $E_y$ of Run BN-2, overplotted with the magnetic field lines in $xy$-plane. The colors red/blue means the field is pointing in the positive/negative $y$-direction (pointing up/down). White means zero. The vertical dashed line designates the $x$-position where 1D cut is made in (b) and (c). (b) 1D cut of the normal electric field $E_y$ (black) and its contributing terms from generalized Ohm's law: the ion frozen-in term $-(1/c)(u_i \times B)_y$ (red), Hall term $(1/\nu e c)(J \times B)_y$ (blue), pressure gradient term $-(1/\nu e)(\nabla \cdot P_e)_y$ (green), and electron inertia term $-(m_e/e)(du_e/dt)_y$ (yellow) at the location denoted by the vertical dashed line in (a). (c) 1D cut of the ion temperature $T_{i,yy}$. (d) 1D cut of the ion density $\rho_i$. The vertical solid lines in (b) and (c) and (d) demonstrate the $y$-position of the X-line.
to the normal electric field from the pressure gradient term in the generalized Ohm’s law. This makes the normal electric field not perfectly match with the Hall term. In few cases, the pressure gradient can be so strong that the Hall signature becomes significantly less distinguishable (see Figure 4.5 for an example). Nevertheless, in most cases, the Hall term still dominates and controls the shape of the normal electric field in this region especially the region of the large peak of the Hall term. All in all, though not perfectly, we can roughly predict the Hall structure of the normal electric field once we have the inflow conditions.

Tanaka [68] explains that given the Hall electric field is shifted toward the magnetospheric side of the X-line, the electron flow along ẑ will be asymmetrically created. This flow, then, pulls the magnetic field out of plane preferentially on the magnetospheric side, leading to the bipolar structure in the out-of-plane Hall magnetic field. Using the preferential-pull reasoning along with the current leaning toward the stagnation point, we are able to predict in most cases that using the X-line as the reference,
the out-of-plane magnetic field will span greater area on the side opposite to the stagnation point. (see Figure 4.6(a)-(e)). This is especially true when the stagnation point is far from the X-line and the current is distinctly skewed. This again shows that the Hall signatures of the asymmetric reconnection are fundamentally linked to its diffusion region structure. Nevertheless, in the case where the X-line and the stagnation point are co-located, some other factors might become noticeably important. Both Figure 4.6(c) and (f) feature a co-location of the X-line and stagnation point. However, only Figure 4.6(c) has a symmetric magnetic field structure.

4.3.2 The Kinetic Signatures

Not all of the diffusion region signatures during asymmetric reconnection are controlled by Hall physics, however. One such example is the electric field with the negative peak left of the X-line in Figure 4.4(b). This electric field does not require strong current. So, the Hall physics is not the gist of it. What causes this electric field structure then? We attribute its foundation to the physics of finite Larmor radius. It is common for asymmetric reconnection that the Larmor radius for one of the inflowing plasmas is quite large owing to the high temperature required for the pressure balance on the side of low density and/or low magnetic field strength. Often, it can be larger than the width of the diffusion region. However, to maintain the equilibrium, the Larmor radius of the particles must be less than or comparable with the width of the diffusion region. The high radius population has to be screened out before it reaches the diffusion region. The system maintains the equilibrium by generating an electric field pointing away from the X-line, which acts as a barrier, keeping the high temperature population further from the diffusion region. Since this structure of electric field originates from the kinetic physics, we will refer to it as “kinetic” electric field. Figure 4.4(b) and (c) shows how the kinetic electric field structure relates with the ion temperature gradient in the same region. The ion temperature does not start dropping where the Hall current is strong. Instead, it starts dropping where the kinetic electric field starts appearing.
Figure 4.6: 2D plots of the out-of-plane magnetic field, overplotted with the magnetic field lines in $xy$-plane of (a) Run BN-2, (b) Run bN-2, (c) Run BN0-1, (d) Run Bn-2, (e) Run B-5, and (f) Run BN0-3. The numbers in the square bracket represent the theoretical percentage distances between the top edge, or side 1 edge, of the diffusion region (left bracket), the X-line (X), the stagnation point (S), and the bottom edge, or side 2 edge, of the diffusion region (right bracket).
From Figure 4.4(b), one might wonder why do we have the frozen-in out-of-plane flow of ions if the kinetic electric field is derived from kinetic physics? The reason is that besides the high-temperature population, there is also a low-temperature population bleeding from the other inflow side and mixing in that region. Only the low-temperature population is under the frozen-in condition and flows out-of-the page in the $z$-direction. The high-temperature population still has seemingly zero average velocity (Figure 4.7(b) and (c)). Again, this bleeding and mixing of particles can not be explained using the fluid description.

To verify that both the “Hall” and “kinetic” $E_{\text{normal}}$ are in fact fundamentally 1D electrostatic equilibrium structures, we perform 2.5D kinetic-PIC simulations with 1D conditions of asymmetric current sheets, letting them evolve into self-consistent equilibrium structures. To maintain periodic boundary conditions, we simulate a double current sheet system similar to the initial conditions used for run BN-2 shown in Table 4.1. The major difference is that this initial condition does not have a magnetic reversal. The magnetic field changes only its strength at the current sheet. The domain of simulation $[L_x \times L_y]$ is $[3.2d_{i0} \times 51.2d_{i0}]$, and $ppg$ is 800. The initial width $w_0$ is 1. There is no perturbation to initiate reconnection. Figure 4.8 (left column) shows the initial normal electric field, magnetic field, ion density, electron temperature, and ion temperature averaged over all $x$ values of one of the two current sheets.

Although these initial conditions are in fact a two-fluid equilibrium, they are not a fully kinetic equilibrium. The system evolves self consistently to generate such a kinetic equilibrium, as shown in Figure 4.8 (right column). As a result, the electric field $E_y$ has grown from zero to generate the same $E_{\text{normal}}$ structures observed in Figure 4.4(b). Both the Hall electric field and the kinetic electric field are present.

The link between the kinetic electric field and Larmor radius is made evident by examining the effect of inflow temperature on the physical size of the kinetic electric field. In Figure 4.9, the case with higher temperature exhibits a much more pronounced kinetic electric field. Note that these are 1D kinetic equilibrium of tangential discontinuities where the magnetic field only changes its strength not its direction. Being
Figure 4.7: The 2D distribution function of ion velocity from the region of the kinetic electric field: (a) $v_{i,x}$ vs. $v_{i,y}$, (b) $v_{i,x}$ vs $v_{i,z}$, (c) $v_{i,y}$ vs $v_{i,z}$. The distribution function is created by sampling all of the particles in the physical domain $x = [158, 159]$, and $y = [20, 21]$. The larger-headed arrow represents the magnetic field. The smaller-headed arrow represents the electric field.
Figure 4.8: Initial conditions, averaged over all $x$ values, for 1D kinetic-PIC simulation of one of the two current sheets (a) normal electric field $E_y$, (b) magnetic fields $B_x$, (c) ion density $\rho_i$, (d) electron temperature $T_{e,yy}$ (blue) and ion temperatures $T_{i,yy}$ (red). Left column: at the initial time $t = 0\Omega_{ci0}^{-1}$. Right column: at an equilibrium $t = 80\Omega_{ci0}^{-1}$.
Figure 4.9: The electric field $E_y$ and the Hall term in 1D kinetic equilibrium of 2 tangential discontinuities. The asymptotic magnetic field strength and density of the 2 systems are the same - $B= 2$ and $\rho=0.1$ on the left edge of the simulation and $B=1$ and $\rho=1$ on the right edge. The asymptotic temperatures, however, are different - [red] $T_e=1.67$, $T_i=3.33$ on the left edge and $T_e=0.67$, $T_i=1.33$ on the right edge and [blue] $T_e=8.33$, $T_i=16.67$ on the left edge and $T_e=1.33$, $T_i=2.67$ on the right edge.

able to observe the kinetic structure in these systems rules out the possibilities that this structure is associated with the reversal of the magnetic field required in magnetic reconnection process and suggests that the structure is derived from the physics of 1D equilibrium.

Although it is clear that the kinetic electric field is generated to contain, or hold back, high Larmor radius inflowing plasma, the question becomes over what distance must it be contained? Since the definition of the stagnation point is predicated on the assumption that inflowing plasma cannot simply free stream beyond this point, we examine the dependence of the kinetic electric field on this distance. We define $\delta_{SE}$ as the absolute value of the distance between the measured stagnation point (MPHM of $J_z$) to the end point of the kinetic electric field where the field becomes zero. Therefore, $\delta_{SE}$ is effectively a measure of the existence and strength of the kinetic electric field. If the kinetic electric field does not exist, then $\delta_{SE} = 0$. We define $\rho_{i,th}$ as the ion Larmor radius based on the asymptotic ion temperature and magnetic field, $\delta_{SD,th}$ as
the theoretical distance between the stagnation point and the diffusion region edge, and $\delta_{SD,m}$ as the distance from the measured stagnation point to measured diffusion-region edge (half-maximum point on the side of the kinetic electric field). Both $\delta_{SD,th}$ and $\delta_{SD,m}$ are defined to be negative if there is the X-line between the stagnation point and the diffusion-region edge. Note that if $|\rho_{i,th}/\delta_{SD,th}| < 1$, it would be expected that the kinetic electric field is not necessary because the plasma can be contained by the magnetic field only, giving $\delta_{SE} = 0$. On the contrary, if $|\rho_{i,th}/\delta_{SD,th}| > 1$, the kinetic electric field would be expected to be present.

Normalizing $\delta_{SE}$ to $\delta_{SD,m}$, we plot $\delta_{SE}/\delta_{SD,m}$ versus $\rho_{i,th}/\delta_{SD,th}$ in Figure 4.10. The dashed lines represent those regions where $|\rho_{i,th}/\delta_{SD,th}| < 1$. As expected, there is no kinetic electric field ($|\delta_{SE}/\delta_{SD,m}| = 0$) for points in this region. Furthermore, for those instances where the kinetic electric field is expected, it occurs on the predicted side of the stagnation point, as is evidenced by the fact that all data points with $|\delta_{SE}/\delta_{SD,m}| > 1$ lie in the upper right or upper left quadrants. It is clear that increasing $\rho_{i,th}/\delta_{SD,th}$ leads to an increase in $\delta_{SE}/\delta_{SD,m}$. However, there is substantial spread in the data points, so it seems doubtful that physically $\rho_{i,th}/\delta_{SD,th} \propto \delta_{SE}/\delta_{SD,m}$. One clear uncertainty in this analysis lies in the determination of the edge of the diffusion region edge using the half-maximum width of the current. Unfortunately, as opposed to the case of symmetric reconnection where we simply take the location at which the Hall term (or the current) becomes insignificant to be the edge of the diffusion region, there is not yet a good way to define the edge of the diffusion region in the asymmetric case.

The 1D quasi-static model of the structure of the diffusion region allows significant physical insight and predictability. However, not all properties are amenable to a 1D treatment. In our simulation with the day-side parameters, we observe a strong electron anisotropy - the parallel temperature is hotter than its perpendicular counterpart. This is not observable in 1D equilibrium (Figure 4.11). This suggests that this kinetic signature is borne out from a 2D effect. This kind of electron anisotropy has been theoretically investigated by Egedal [20]. However, we hope that the simple
Figure 4.10: The theoretical ratio $\rho_{i,\text{th}}/\delta_{SD,\text{th}}$ vs. the measured ratio $\delta_{SE}/\delta_{SD,m}$. $\rho_{i,\text{th}}$ is the ion Larmor radius based on the asymptotic ion temperature and magnetic field. $\delta_{SD,\text{th}}$ is the theoretical distance between the stagnation point and the diffusion-region edge. $\delta_{SE}$ is the absolute value of distance between the measured stagnation point (MPHM of $J_z$) to the end point of the kinetic electric field where the field becomes zero. $\delta_{SD,m}$ is the distance from the measured stagnation point to measured diffusion-region edge (half-maximum point on the side of the kinetic electric field). $\delta_{SD,m}$ is taken to be negative if there is the X-line in between the measured stagnation point an the measured diffusion-region edge. The negativity of $\delta_{SD,\text{th}}$ is also defined in a similar way. The only difference is $\delta_{SD,\text{th}}$ concerns the theoretical values instead of the measured ones. For each run, there are two values of $\rho_{i,\text{th}}/\delta_{SD,\text{th}}$. The greater ratio is plotted here. The theoretical value of $|\rho_{i,\text{th}}/\delta_{SD,\text{th}}| > 1$ is expected to yield the kinetic electric field structure. $\delta_{SE}$ is set to be zero if there is no kinetic electric field.
kinetic picture demonstrated below can provide another perspective to the mechanism. Due to the 2D bending of magnetic field lines in the inflowing region during reconnection, the charge density accumulated to satisfy the divergence of the normal electric field can have variation along the magnetic field lines. We can see that there is some amount of positive charge density in the area captured by the bright-green box in Figure 4.12(a). Tracing along the magnetic field line that passes through the area either to the right or to the left, the charge density decreases. This variation of the charge density along the field lines leads to the parallel electric field pointing away from the X-line (Figure 4.12(b)). As a result, the electrons, which is more mobile than the ions, are accelerated parallel to the field toward the X-line. The counter streaming of the electrons then causes high parallel temperature leading to an electron anisotropy (Figure 4.12(c) and (d)).

4.4 Summary

The structure of the diffusion region can be fundamentally altered when an asymmetry in the inflow conditions are introduced to the system. Using fully kinetic particle-in-cell simulations with different sets of inflow parameters, we arrive at key conclusions as follows.

1. The structure of the diffusion region can be roughly considered to follow a quasi-static 1D equilibrium condition along the inflow direction due to the fact that the inflow speed is much less than the magnetosonic speed.

2. At the stagnation point, the transition between inflow plasma species occurs. To remain in an equilibrium, the system requires a strong magnetic field gradient at the stagnation point, creating the out-of-plane current that is asymmetrically situated around the X-line in the way that it leans toward the stagnation point.

3. Once we have the inflow conditions to predict the asymmetry of current profile, the prediction of how the normal electric field and out-of-plane magnetic field look can generally be done using Hall physics. This shows that the Hall signatures are fundamentally linked with the diffusion region structure. However, the Hall physics is
not the only governing factor. In fact, under some types of inflow asymmetries, the Hall signatures can be significantly modified by other physics.

(4) Not only can the introduction of asymmetry lead to physics modifying the Hall signatures, but it can also lead to physics that creates new signatures, which include

- Kinetic electric field - To maintain equilibrium, this electric field is required to keep the ions with Larmor radii larger than the natural width of the diffusion region away from the diffusion region.

- Electron anisotropy - The charge density required to satisfy the divergence of the normal electric field is generally not constant along the field lines. This creates the parallel electric field pointing away from the X-line, which accelerates the electrons toward the X-line. As a result, the electrons form counter-streaming populations, leading to high parallel temperature and so anisotropy, near the X-line.
Figure 4.11: The electron parallel temperature $T_{e,xx}$ (green) and the electron perpendicular temperature $T_{e,yy}$ (blue) of (a) 1D cut through the X-line along the inflow direction (in the same fashion as Figure 4.4(b)) of Run BN-2. (b) 1D equilibrium of a tangential discontinuity with the same asymptotic magnetic field strength, density, and temperature as Run BN-2. The vertical dashed line in (a) indicates the $y$-position of the X-line.
Figure 4.12: 2D plots near the X-line of (a) charge density, (b) the x component of the parallel electric field, and (c) $(T_{e,\parallel}/T_{e,\perp}) - 1$. Red means positive (or pointing in the +x direction). Blue means negative (or pointing in the −x direction). (d) 2D electron velocity distribution in x−y plane taken from the area of $x=[158,159]$ and $y=[21,22]$, shown as the area in the bright-green box in (a), (b), and (c), displaying an electron anisotropy of BN-2. The larger-headed arrow represents the magnetic field. The smaller-headed arrow represents the electric field.
Chapter 5
CONCLUSION

Due to the success of the Sweet-Parker and Hall models in explaining the mechanism of magnetic reconnection for two dimensional systems with inflow symmetries, scientists have started to move on to explore some other territories of magnetic reconnection - 3D reconnection, reconnection in a turbulent environment, and particle acceleration due to reconnection, to name a few. This work is an exploration of another interesting area of reconnection research, namely, asymmetric reconnection - the reconnection with non-identical inflows.

This research area is not totally uncharted, however. Cassak and Shay [12, 14] have made an important contribution by providing the scaling relations for both gross properties (reconnection rate, outflow speed, and outflow density) and diffusion region structure (locations of the X-line and stagnation point and width of the diffusion region) of asymmetric reconnection. These scaling relations were obtained by a Sweet-Parker-type analysis, more specifically, by considering the conservation of mass, momentum, and energy of single-fluid plasmas coming in and going out of the diffusion region. Furthermore, they agree quite well with resistive-MHD and Hall-MHD simulations except for the particle-mixing aspect. Fully kinetic simulations have also been shown to be generally in agreement with them. However, no systematic comparison has been performed.

This work utilizes the fully kinetic particle-in-cell code P3D to perform simulations of asymmetric reconnection. With inflow parameters varied systematically, it is able to show that the gross properties scale as theoretically predicted, which implies that the fluid physics considered in the Sweet-Parker-type analysis for asymmetric reconnection plays a significant role in governing the reconnection. It also shows that
the asymmetry introduced does not crucially affect the ratio of the width $\delta$ to the length $L$ of the diffusion region, as the ratio $\delta/L$ is still found to be in the order of 0.1. Nevertheless, we have to note that the simulation results and the predictions are not exactly identical. This suggests that some other physics might also be playing a role, and some improvement to the theoretical prediction can be made once the additional physics is identified and understood.

Beside confirming the validity of the scaling relations for the gross properties of the diffusion region, we have made progress in terms of understanding the reconnection Hall signatures. We are able to demonstrate for the first time that the signatures and the inner structure of the diffusion region are fundamentally related. One key discovery is that the current is asymmetrically situated around the X-line with its physical location being correlated with the stagnation point. In the Hall picture, the current can be thought of as the source of the Hall electric and magnetic field. So, being able to predict how the current behaves (by predicting the distance between the X-line and stagnation point), we can predict how the Hall signatures behave as well. Again, although the prediction works most of the time, there are discrepancies. In some cases, other physics can become so important that it significantly alters what we expect from the Hall physics alone.

In addition to the Hall electric and magnetic fields, there are some structures borne out from kinetic physics that can be used as signatures to help satellites locate the diffusion region as well. These new signatures include the kinetic electric field and the electron anisotropy. The kinetic electric field is a normal electric field structure required in order to prevent the high Larmor radii ions from getting too close to the width-specified diffusion region and disrupt the equilibrium. The electrons anisotropy is derived from the fact that the field lines that form an X at the diffusion region is essentially a 2D structure, with bending of the magnetic field lines in the inflow region. The normal electric field required to satisfy 1D equilibrium therefore varies along the magnetic field lines, as does the charge density. The charge density variation along the field lines then creates the parallel electric field, which, in turn, accelerates the
electrons toward the X-line creating counter-streaming populations of electrons, and therefore an electron temperature anisotropy with $T_{e\parallel} > T_{e\perp}$.

During the course of this exploration, some questions have been answered, and some discoveries have been made. However, many questions still remains. Actually, we end up having more questions than when we started as we keep having unexpected observations. Some of the more interesting questions are listed below.

- Where is the edge of the diffusion region?
  In the symmetric case, we can determine the edge of the diffusion region by looking for the location where the Hall term becomes insignificant. However, this simple method does not appear to be satisfactory under asymmetric inflow conditions. Unlocking this key will probably be helpful in improving our understanding of the asymmetric reconnection mechanism.

- Can we better understand the effects of the pressure gradient?
  We have shown qualitatively that the pressure gradient, which is common for asymmetric reconnection, plays some role in controlling the behavior of the normal electric field. Can we make another step forward by understanding it quantitatively? Also, we would also want to know whether this pressure gradient is related to the reconnection rate and the outflow speed. If so, understanding it will lead us to better prediction of the reconnection gross properties.

- What is the effect of high temperature?
  In asymmetric reconnection, it is normal to have a high temperature on one side of the inflowing plasmas required for pressure balance. We have learned from this work that this high temperature plasma can cause the kinetic electric field, a structure never seen before by symmetric studies. Nonetheless, a question remains - how does this high temperature affect the gross properties? In fact, studying the high temperature effect in symmetric case, where all the complications of inflow asymmetry are taken away, should be very helpful on this front.

- Does each type of asymmetry have different fundamental physics controlling it?
  As you have seen in this work, we made an attempt to explain asymmetric reconnection by linking its signatures to the inner structure such as the X-line and stagnation point, which is general to all asymmetric reconnection. At the same time, however, we also categorized asymmetric reconnection into many types. This is to see whether each type of reconnection will have its own characteristics. So far, we have not yet been able to clearly identify those characteristics. Nevertheless, looking at the Hall magnetic field from many different runs suggests that each type has its own traits.
Apparently, this is still far from an end. Even though an important step has been made by Cassak and Shay [12], there seem to be much more to it before we get a reasonably complete picture of asymmetric reconnection. In fact, some of the questions posted here make us think how much do we really understand the symmetric case, which we thought was well understood. So, we might even have to do some more work on symmetric reconnection to gain more understanding on asymmetric cases. In many ways, however, this is good, as it means there are many things await to discover. We can enjoy this trip at least a little longer.

In this dissertation, we have focused on a type of reconnection that is complicated than the conventional symmetric case. We hope that the contributions from this dissertation help us better understand reconnection in a more general point of view. However, it is important to note that what this dissertation has covered is only one class of generalization. Other classes of generalization include asymmetric outflows, non-steady inflow and outflow conditions, and reconnection in a turbulent plasma.


A.1 Introduction

The first self-consistent model of magnetic reconnection was the Sweet-Parker model [65, 46], in which electron-ion collisions break the frozen-in constraint. Laboratory experiments in collisional plasmas provide observational support for this model [69, 22]. However, the Sweet-Parker reconnection is slow, meaning the reconnection rate decreases when the system size increases, providing not fast enough reconnection rate to explain observed energy release in large-scale reconnection events of space physics interest such as solar eruptions. Furthermore, the Sweet-Parker model cannot explain how reconnection occurs in a collisionless plasma such as the magnetosphere.

Understanding the physics allowing reconnection to be fast is crucial. It has been suggested [34, 59] that the Hall term is critical to make reconnection fast as the rate stays in the order of 0.1 when the system size increases. The Hall term operates at sub-ion gyroradius scales and describes the decoupling of ions from the magnetic field when their gyro-orbit is comparable to gradient scales in the magnetic field. [34] and [54] argued that collisionless (Hall) reconnection is fast because of the dispersive nature of the whistler and kinetic Alfvén waves introduced by the Hall term.

The importance of the Hall effect was shown in the GEM Challenge study ([5] and references therein), which compared fluid, two-fluid, hybrid, and particle-in-cell (PIC) simulations. All simulations containing Hall physics had a similar (fast) reconnection rate, while the simulation without the Hall effect was much slower. The Hall model has had wide success explaining observations in the magnetosphere [41,
and laboratory experiments \cite{44, 40, 55, 7, 49}, and there is indirect evidence that the Hall effect is important in solar and stellar coronae \cite{11}.

There have been significant challenges to the Hall reconnection model and specifically to the argument that dispersive waves shorten the current layer and facilitate fast reconnection. In one case, \cite{25} performed hybrid simulations in which the Hall term was manually removed from the generalized Ohm’s law. In these so-called Hall-less hybrid simulations, the rates of reconnection were found to be fast, calling the Hall model into question.

In a second challenge, PIC simulations with open boundary conditions were performed, and it was found that the electron current layer continuously increased in length and was limited only by the formation of secondary magnetic islands \cite{16}. Thus, dispersive waves were insufficient to limit the length of the electron current layer but fast reconnection was possible due to the periodic ejection of secondary islands. This model is reminiscent of earlier MHD models of turbulent reconnection where secondary islands break up the Sweet-Parker current layer and facilitate fast reconnection \cite{36, 27, 30, 29}. Subsequent large scale periodic simulations \cite{61} and larger open boundary conditions \cite{24, 28} found long time periods with steady fast reconnection rates and steady electron current layer lengths. \cite{61} concluded that secondary magnetic islands are not necessary for fast reconnection, while \cite{24} viewed the steady reconnection periods as transient and concluded that magnetic reconnection as a whole is time dependent due to secondary island formation \cite{36}. Although it is clear that secondary islands often form during magnetic reconnection, there is currently no consensus on the role they play in setting the reconnection rate.

A surprising result of these simulations \cite{61, 24} is that the electron diffusion region exhibits a two-scale structure along the outflow direction, an inner diffusion layer synonymous with previous two-fluid and hybrid results, and an extremely long outer layer (10s of $c/\omega_{pi}$ in length) containing a super-Alfvénic jet of electrons which are not frozen-in. Signatures of this extremely long outer electron diffusion region (greater than 60 $c/\omega_{pi}$) have been observed in satellite measurements of reconnection.
in the Earth’s magnetosphere [49]. We emphasize that these new features are due to kinetic electron physics, which are not present in hybrid simulations.

In this study, we focus on the first challenge, that “ion kinetics” alone facilitate fast reconnection [25]. We perform a study of the role of the Hall effect by comparing hybrid and Hall-less hybrid simulations, similar to part of the study by [25]. In contrast to [25], we find that reconnection is slow (Sweet-Parker) in the absence of the Hall effect. The removal of the Hall effect during fast reconnection leads immediately to a collapsing of the open outflow to a Sweet-Parker layer. This reconfirms the importance of the Hall effect in producing fast reconnection in the hybrid model. Potential reasons for the discrepancy with [25] are discussed.

A.2 Hall-less Reconnection Physics

There are fundamental physics differences between the standard hybrid system (with particle ions and fluid electrons) and the Hall-less hybrid system, much more disparate than the differences between MHD and Hall-MHD. In a standard system, ions and electrons are frozen-in far from the neutral line. Ions within a gyroradius of the neutral line undergo stochastic orbits, decoupling from the magnetic field and electrons. Since \( \mathbf{E} + \mathbf{u}_e \times \mathbf{B}/c \simeq 0 \) in the region between the ion and electron gyroradii (with \( \mathbf{u}_e \) the electron fluid velocity), the electrons remain frozen to the magnetic field. The electrons decouple only at electron gyroradius scales, and the magnetic topology can change in this region.

In the Hall-less hybrid system, the ions undergo stochastic trajectories within a gyroradius of the neutral line, as in the standard system. However, in the absence of the Hall term, \( \mathbf{E} + \mathbf{u}_i \times \mathbf{B}/c \simeq 0 \) and the magnetic field remains frozen to the average ion velocity \( \mathbf{u}_i \) until resistive scales are reached.

A.3 Simulations and Results

We use the hybrid code P3D [75] in 2 1/2 dimensions. The evolution equations are normalized to a length of the ion inertial scale \( d_i = c/\omega_{pi} \), a velocity of the Alfvén
speed \( c_A = B_0/(4\pi m_i n_0)^{1/2} \), and a time of the ion cyclotron time \( \Omega_{ci}^{-1} = (eB_0/m_i c)^{-1} \), where \( n_0 \) is the initial density outside the current sheet, \( B_0 \) is the asymptotic magnetic field strength, and \( m_i \) is the ion mass. This gives normalizations of electric fields, temperature, resistivity, and hyperviscosity as follows: \( E_0 = c_A B_0/c, T_0 = m c^2 A, \eta_0 = 4\pi d_i c A / c^2, \) and \( \eta_4 = c A d_i^3 \) The ions are evolved in time \( t \) using

\[
\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \quad (A.1)
\]

\[
\frac{d\mathbf{v}_i}{dt} = \mathbf{E}_{ION} + \mathbf{v}_i \times \mathbf{B} \quad (A.2)
\]

where \( \mathbf{x}_i \) and \( \mathbf{v}_i \) are the positions and velocities of the individual protons and \( \mathbf{B} \) is the magnetic field. The electric field used to update the ions \( \mathbf{E}_{ION} \) is given by

\[
\mathbf{E}_{ION} = -\left( \mathbf{u}_i - \frac{\mathbf{J}}{n} \right) \times \mathbf{B} \quad (A.3)
\]

where \( n \) is the density, \( \mathbf{J} = \nabla \times \mathbf{B} \) is the current density, and \( \mathbf{u}_i \) is the ion bulk flow velocity. The quantity in parentheses is the electron bulk flow velocity \( \mathbf{u}_e \). The magnetic field is updated using

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}_{OHM} \quad (A.4)
\]

\[
\mathbf{E}_{OHM} = -\mathbf{u}_i \times \mathbf{B} + \frac{\mathbf{J}}{n} \times \mathbf{B} + \eta \mathbf{J} - \eta_4 \nabla^2 \mathbf{J} \quad (A.5)
\]

where \( \eta \) is the resistivity and \( \eta_4 \) is a hyperviscosity. We emphasize that both explicit dissipation terms in the code have constant and uniform coefficients (i.e., they are not spatially localized). The contributions to the generalized Ohm’s law [Eq. (A.5)] from electron inertia \( (m_e = 0) \) and the electron pressure gradient (the electron temperature \( T_e = 0) \) are omitted to isolate the effect of the Hall term. The code assumes quasi-neutrality. We refer to simulations using Eqs. (A.1) - (A.5) as “standard” hybrid simulations.

The Hall-less hybrid system is defined by removing the Hall term from Eq. (A.5), giving

\[
\mathbf{E}_{OHM} = -\mathbf{u}_i \times \mathbf{B} + \eta \mathbf{J} - \eta_4 \nabla^2 \mathbf{J} \quad (A.6)
\]
It is important to note that the $\mathbf{J} \times \mathbf{B}$ term is not removed from the electric field used to step forward the ions $\mathbf{E}_{ION}$ because doing so would eliminate bulk forces on the ion fluid [see [25] for a thorough discussion]. It has been shown that this prescription for removing the Hall term removes dispersive waves from the system [25]. The simulations using Eq. (A.6) instead of Eq. (A.5) are referred to as Hall-less hybrid simulations.

The simulated domain is $204.8 \times 102.4$ with a $1024 \times 512$ grid (i.e., the grid scale is 0.2). There are initially 100 particles per cell loaded with an initial Maxwellian distribution having a uniform temperature 0.5. The background density is initially 1.0. We use $\eta = 0.015$ and $\eta_4 = 10^{-3}$. There is no initial out-of-plane (guide) magnetic field. The domain has periodic boundary conditions in all directions. The initial equilibrium is a double Harris sheet with an initial current sheet width of 1. A coherent perturbation in the $y$-direction to the magnetic field of amplitude 0.3 is used to initiate reconnection.

First, we perform a standard hybrid simulation, which reveals known properties of reconnection. In the steady-state, the out-of-plane current $J_z$ is opened out into a Petschek-type outflow jet, as shown at $t = 300$ in Fig. A.1(a), an indication of fast reconnection. The reconnection rate $E$, computed as the time rate of change of magnetic flux between the X-line and the O-line, is shown in Fig. A.2(a) as a function of time. The steady value is $E \approx 0.03$, which is fast, but somewhat slower than typical values closer to 0.1 seen in collisionless reconnection simulations. This is due to the large value of the resistivity $\eta$ employed in the simulations. Indeed, the transit time across the dissipation region is $\tau_{tr} \sim \delta/v_{in} \approx 1/0.03 \approx 30$, while the diffusion time across the layer is $\tau_d \sim \delta^2/\eta \approx 1/0.015 \approx 60$ (where $\delta$ is the thickness of the ion dissipation region and $v_{in}$ is the ion inflow speed), revealing that diffusion is playing a non-negligible role during the reconnection process. When the same simulation is performed with $\eta = 0$, the reconnection rate is closer to 0.045. The hyperviscosity $\eta_4$ still plays the main role, however, for setting $\eta_4 = 0$ in the Hall runs leads to the current sheet width collapsing down to grid scale lengths.

Second, we perform a Hall-less hybrid simulation with the same parameters as the previous simulation. In this case, $J_z$ has the signatures of slow (Sweet-Parker
Figure A.1:  (Color online.) Grayscale plots of the out-of-plane current $J_z$ during the transition from standard to Hall-less hybrid reconnection at (a) $t=300$, (b) $t=310$, (c) $t=330$, and (d) $t=400$. 
like) reconnection. [very similar to Fig. A.1(d), which is discussed later]. The current sheet elongates to the system size with no Petschek-type open outflow region. The reconnection rate is plotted in Fig. A.2(b), and the steady-state value is $E \sim 0.015$, with a horizontal line marking the prediction from a standard Sweet-Parker analysis using parameters measured near the end of the simulation. The rate is considerably lower than in the standard hybrid run. We emphasize that the only difference between this simulation and the previous one is the omission of the Hall term in the Ohm’s Law, which shows the importance of the Hall term in enabling fast reconnection.

Note that from the parameters chosen, the predicted half-width $\delta_{SP}$ of a Sweet-Parker current layer is $\delta_{SP} \sim (\eta L/c_{Aup})^{1/2} \approx 0.77$, where $L$ is the length of the current sheet from the X-line to one end of the current sheet and $c_{Aup}$ is the Alfvén speed based on the upstream reconnecting magnetic field strength $B_{up}$. This thickness is smaller than the ion inertial scale, so if kinetic effects of the ions alone were sufficient for fast reconnection, as has been previously suggested [25], we presumably would not have seen slow reconnection. Setting $\eta = 0$ in the Hall-less case leads to the current sheet width collapsing to the grid scale.

Finally, to emphasize the role of the Hall term, we perform a standard hybrid simulation until the system reaches a steady state, then suddenly turn off the Hall term at $t = 300$ and continue the simulation with the Hall term disabled. The current sheet $J_z$ after turning off the Hall term is plotted in Fig. A.1, with plots at $t = 310$ [plot (b)], $t = 330$ [plot (c)], and $t = 400$ [plot (d)]. The transition from the Petschek-type open outflow region to a Sweet-Parker type layer on a very fast time scale is clearly seen.

The reconnection rate as a function of time for this run is plotted in Fig. A.2(c). As soon as the Hall term is turned off, $E$ drops to the Sweet-Parker value in less than 10 ion cyclotron times. The reconnection rate then gradually increases and finally levels off. The reason for the gradual increase is as follows: When we suddenly turn the Hall term off, the reconnection suddenly changes from fast to slow. However, the plasma upstream of the dissipation region continues to flow toward the X-line at a fast rate. This causes an accumulation of the magnetic field upstream, leading to a higher
reconnection rate. To remove the effect of the changing upstream magnetic field $B_{up}$ during Hall-less reconnection, Fig. A.2(d) shows the reconnection rates from all three simulations normalized to the $B_{up}^{3/2}$ as a function of time. If the length of the diffusion region is not changing in time, this normalization should give a constant reconnection rate during Hall-less reconnection [46]. The upstream density changes very little during this time. When the reconnection rate is normalized to the upstream magnetic field, the reconnection rate becomes rather steady. The plot clearly shows that the reconnection rate after turning the Hall term off is in agreement with the Sweet-Parker value. The system remains in a steady-state of Sweet-Parker type reconnection for over 300 ion cyclotron times.

A.4 Discussion

Our simulations reveal that while reconnection in the standard hybrid model is fast, reconnection in the Hall-less hybrid system is slow when a constant and uniform resistivity is employed. This clearly suggests that the Hall term plays a fundamental role in limiting the length of the current layer and facilitating reconnection in the hybrid model.

We can also address the role of secondary island formation in our simulations. In the Hall-less hybrid simulations, a secondary island forms and is ejected out of the dissipation region. The production of the secondary island does not significantly disrupt the structure of the Sweet-Parker current sheet, nor does it greatly affect the reconnection rate. Thus, we see no evidence in this system that the formation of secondary islands facilitates fast reconnection, as has been suggested previously in [16, 24] based on collisionless PIC simulations. However, as has been postulated with current sheets in resistive MHD simulations [36] and Hall MHD simulations [63], perhaps secondary island formation in extremely large Hall-less hybrid simulations would anomalously increase the reconnection rate. Resolving this issue will require extremely large amount of computational resource.
Figure A.2: The reconnection rate $E$ as a function of time $t$ for various simulations: (a) a standard hybrid simulation, (b) a Hall-less hybrid simulation, with the horizontal line indicating the predicted Sweet-Parker value for this system, and (c) a standard hybrid simulation in which the Hall term is removed at $t = 300$ (denoted by the vertical line), with the gray dashed line denoting $E$ for the standard hybrid simulation [from (a)]. (d) $E$ normalized to $B_{sp}^{3/2}$ in the standard hybrid (gray dashed line), Hall-less hybrid (gray solid line), and standard hybrid with the Hall term removed mid-run (black solid line) simulations. The time scale for the Hall-less data is rescaled in (d) so that the final time of $t = 1700$ is shown at $t = 300$. 

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We also would like to emphasize that numerical resolution can significantly affect these simulations. In particular, we find that if the Hall-less current layer is not sufficiently resolved (with at least 4 grid cells across the dissipation region for the algorithm in use for these simulations), the current layer is unstable to secondary islands at the grid scale and seemingly substantially increase the reconnection rate. This is tested with simulations using the same grid scale (0.2) as the simulations described in the previous section but with a lower resistivity ($\eta = 0.007$). With the lower $\eta$, the predicted $\delta_{SP} \approx 0.52$, leaving only 2.5 grid cells per $\delta_{SP}$. Simulations with this $\eta$ in which the Hall effect is turned off mid-run reveal that a Sweet-Parker type current layer forms after the Hall term is disabled, but grid scale instabilities soon form. When the $\eta = 0.007$ simulation is redone with a grid scale of 0.1 (giving 5 cells across $\delta_{SP}$), no grid scale effects occur and the reconnection remains steady at its slow Sweet-Parker rate. As such, ostensibly fast reconnection can occur if numerical resolution is insufficient, and care is needed to ensure that fast reconnection is not being caused by numerical effects (see e.g. [72]).

The present results disagree with [25], who claimed that ion kinetics alone are sufficient to make reconnection fast (i.e., reconnection in the Hall-less hybrid system is fast even in the limit of a uniform resistivity). We attribute the discrepancy to the fact that all of the simulations of reconnection performed in that study employed a localized resistivity, which itself is sufficient to produce fast reconnection even in MHD [6]. To confirm this interpretation, we perform a Hall-less hybrid simulation with an ad hoc cosh-profile resistivity similar to that used in [25], and find that the reconnection is fast. As such, the hybrid system behaves much like the fluid system in that either the Hall term or an ad hoc localized resistivity is sufficient to make reconnection fast.

In conclusion, consistent with the GEM Challenge result [5], we find in hybrid simulations that the Hall term is required to produce fast reconnection when the resistivity is uniform. The kinetic dynamics of ions alone are insufficient to produce fast reconnection. We emphasize that hybrid simulations do not contain 3D effects, which may be playing a critical role during reconnection, nor kinetic electron physics, which
has been shown to modify substantially the electron diffusion region.
Appendix B

PERMISSION LETTER

Michael Connolly <MConnolly@agu.org> Fri, Jun 29, 2012 at 11:41 AM
To: Kittipat Malakit <kmalakit@udel.edu>

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