

# Reversibility and irreversibility in the packing of vibrated granular material

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## Abstract

We report on the settling of loosely packed, cohesionless granular material under mechanical vibrations. Monodisperse spherical beads were confined to a long vertical cylinder that was driven by an electromagnetic vibration exciter. Under vibrations the bead packing evolves from an initial, low-density configuration towards higher density. Ramping the vibration intensity repeatedly up and back down again reveals the existence of both an irreversible and a reversible branch in the response. The reversible branch represents a steady state in which the packing density depends monotonically on the vibration intensity. We have investigated the bead size, depth, and ramp rate dependence of the compaction process. Our results indicate how the occupied volume fraction can be optimized by slowly reducing the vibration intensity along the reversible branch. © 1997 Elsevier Science S.A.

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## 1. Introduction

Loose, cohesionless granular material confined inside a container can settle into a more compact state when the system is vibrated. This settling is important to a wide range of industries and many technological processes in which the density of granular solids needs to be controlled [1]. For example, in the casting of metal parts, it is important to achieve a uniform, dense bed or mold in which to pour the molten metal. Moreover, the handling of granular material often subjects it to vibrations with a range of continuously varying intensities, and a direct correlation between the vibration parameters and the resulting density is highly desirable. It turns out, however, that the mechanisms underlying the settling of granular material are complicated and that the literature on granular compaction, specifically by vibratory means, is rather sparse [2–4]. Most available data [3,5–13] as well as theoretical models [14–24] concern the time dependence of the packing density for various materials under given vibration conditions. In addition, the influence of the grain-size distribution [2,8] and the grain shape [9,25,26] on the final packing density have been investigated as well the roles of shaking parameters such as amplitude and fre-

quency. From these studies two key results emerged, namely, that there is a nonlinear time dependence to the settling process so that the rate of compaction slows down as the material becomes denser [3,11,13], and that there may be an optimum shaking acceleration that allows for the material to settle quickly without becoming stirred up too much [5,11]. In this paper we revisit the acceleration conditions for optimum vibrational compaction. In particular, we show that the rate of compaction and the final density depend sensitively on the *history* of vibration intensities that the system experiences. Our work identifies a reversible regime of steady-state behavior in the density of vibrated granular material.

In order to make contact with results from the extensive literature on sphere packings [2,7,11,27–31], our experiments focus on the simplest case of a granular material, namely, monodisperse spherical beads. For such systems, packing densities (i.e. occupied volume fractions)  $\rho$  up to 61–62% are obtained after several minutes of shaking at moderate intensities (e.g. see Refs. [7,13]). These values, however, do not represent the densest possible configurations as they fall significantly below both the crystalline close-packed ( $\rho=0.74$ ) and random close-packed ( $\rho=0.64$ ) limits [27]. One reason is that the density increases extremely slowly (essentially logarithmically) in time towards the final packing state, indicating a complicated, cooperative relaxation process that requires very long shaking times [13,16,23].

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Higher vibration intensities speed up the settling process but, beyond a certain ‘optimum’ value of acceleration, also lead to lower final densities [3,5]. This nonmonotonic dependence of packing density on applied vibration intensity was found to exhibit a maximum in the vicinity of 2–3 times the Earth’s gravitational acceleration. Here, we demonstrate that this apparent maximum is due to an irreversible portion of the initial compaction process. We find that even higher densities can be achieved by ramping the applied acceleration first up past the density maximum and then back down again in a prescribed manner. This procedure results in a reversible and monotonic dependence of density on acceleration.

## 2. Experimental

A brief description of the experimental apparatus and measurement technique follows (see Ref. [13] for details). Monodisperse, spherical soda-lime glass beads (of diameter  $d=1, 2, \text{ or } 3 \text{ mm} \pm 3\%$ ) were confined to a Pyrex tube of diameter  $D=1.88 \text{ cm}$  and length  $1 \text{ m}$ . The tube was mounted vertically on an electromagnetic Brüel and Kjaer 4808 vibration exciter and was subjected to discrete vertical shakes (‘taps’) each consisting of one complete cycle of a sine-wave of frequency  $f$ . The vibration intensity was parameterized by  $\Gamma$ , the ratio of the recorded peak acceleration during a single tap to the gravitational acceleration,  $g=9.81 \text{ m/s}^2$  [5,11]. In our experiments,  $\Gamma$  was varied by changing the amplitude of the excitation,  $A$ , at fixed frequency  $f=30 \text{ Hz}$ . Individual taps were spaced sufficiently far apart in time to allow the system to come to complete rest between taps. In this way no spurious effects from continuous vibrations, such as period doubling or surface waves [32], would affect the measurements. By using a tall container with smooth, low-friction interior walls, granular convection was suppressed [33]. Such vibration-induced convection, as well as axial rotation of horizontal containers, can lead to size separation in granular mixtures [32,34–36]. Apparatus similar to ours, albeit using mechanical shakers, have been described in Refs. [7,9].

The packing density was determined in two ways. The average density for the whole container was obtained from a direct measurement of the total filling height. Capacitive probes mounted on the outside wall of the cylinder allowed for measurements of the local density within three 15 cm tall sections of the cylinder: near the top, middle, and bottom of the pile of beads (distance from the pile’s top surface to the center of the probe: 32, 49, and 68 cm, respectively). Prior to loading the cylinder, all beads were cleaned and baked, and precautions were taken to minimize complications resulting from electrostatic charging. The initial filling height of the beads before tapping commenced was typically 83 cm, corresponding to a volume packing fraction  $\rho=0.58$ . This low-density initial state could be attained reproducibly by flowing high-pressure, dry nitrogen gas through the bottom of the cylinder. After this loose packing state was established, the cylinder was evacuated and kept under vacuum for the

duration of the experiment in order to mitigate the effect of external humidity fluctuations. After a preset number of taps,  $t$ , the packing density  $\rho(\Gamma)$  was recorded.  $\Gamma$  was then changed and the compaction continued for the next  $t$  taps. The value of  $t$  was kept constant during each run and ranged from  $10^2$  to  $10^5$ . For comparison, this range corresponds to continuous shaking at 30 Hz for time intervals ranging from 3.3 s to 55 min between successive data points.

## 3. Results and discussion

The three panels in Fig. 1 show  $\rho(\Gamma)$  for 1, 2, and 3 mm diameter beads. In this figure,  $t=10^5$  between successive data points. Closed (open) symbols indicate data for increasing

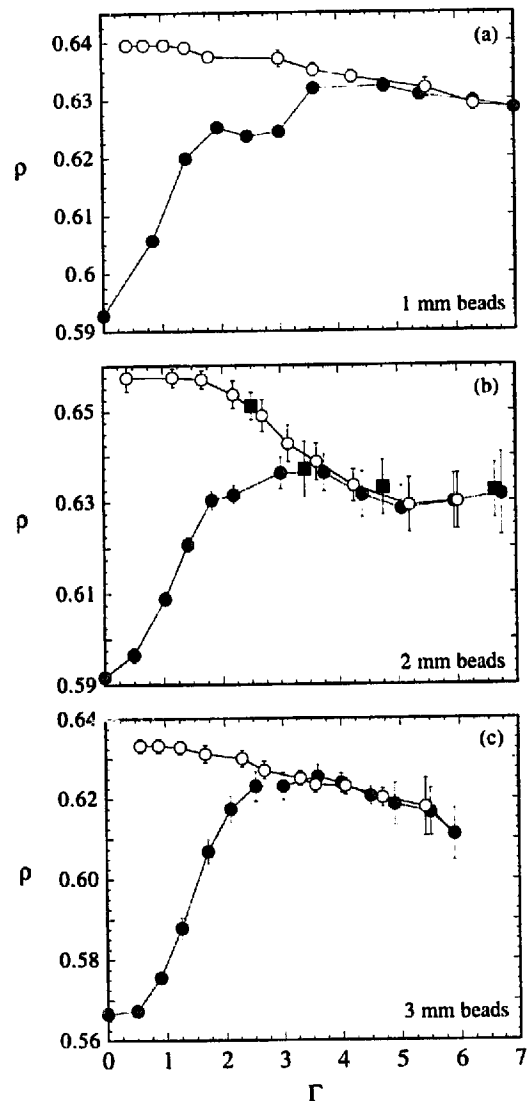


Fig. 1. The dependence of the volume packing fraction  $\rho$  on the vibration history for various bead sizes. The beads are first prepared in a low initial density state. The acceleration amplitude was then slowly, and successively, first increased (solid symbols) and then decreased (open symbols). The packing fraction was recorded after  $10^5$  discrete taps at each value of  $\Gamma$ . For each curve, the upper branch that has the higher density is reversible, and, upon subsequently raising the value of  $\Gamma$  again,  $\rho$  retraces the values obtained on the downward trajectory (see e.g. the square symbols in (b)).

(decreasing)  $\Gamma$ . Starting from a low initial packing density at  $\Gamma=0$ ,  $\rho(\Gamma)$  increases with increasing  $\Gamma$  as voids are eliminated. For sufficiently large  $\Gamma$ , however,  $\rho(\Gamma)$  eventually begins to slowly decrease since at higher accelerations void ‘annealing’ competes with void creation during each tap. (In two-dimensional packings this competition can be directly and easily observed by image analysis of snapshots of the bead arrangements from successive taps: see, e.g., Ref. [37].) Similar behavior was observed in Refs. [3,5,6]. If in this regime  $\Gamma$  is now reduced (open symbols), we find that  $\rho(\Gamma)$ , rather than decreasing back to the initial density, continues to increase until it reaches a maximum at  $\Gamma=0$ . Subsequent changes of  $\Gamma$  trace out a reversible, upper branch of the  $\rho(\Gamma)$  curve. This is shown in a representative manner for the 2 mm beads in Fig. 1(b) (squares). Thus, a loosely packed bead assembly first undergoes *irreversible* compaction, corresponding to the lower branch of  $\rho(\Gamma)$ . The settling behavior becomes reversible only once a characteristic acceleration,  $\Gamma^*$ , has been exceeded ( $\Gamma^* \approx 3$  for  $t = 10^5$  in Fig. 1).

In previous work [13], we have shown that granular assemblies exhibit logarithmically slow relaxation towards higher packing densities, asymptotically approaching a steady state at the longest times. In particular, for acceleration values below  $\Gamma \approx 3$ , vibrations are rather ineffective in bringing the packing from its initially ‘fluffed up’ low-density configuration to the final, steady-state density within any reasonable experimental time scale; in excess of 100 000 taps may be required because the bead configurations can become trapped in numerous metastable arrangements. We believe that this behavior leads to the pronounced history dependence of the achievable packing fraction in Fig. 1 as  $\Gamma$  is varied.

While the overall behavior is qualitatively similar for all three bead sizes shown in Fig. 1, there are quantitative differences that depend on the aspect ratio,  $\delta = D/d$ , of container diameter to bead diameter. Both the maximum achievable density on ramping  $\Gamma$  down the reversible branch and the maximum slope  $\Delta\rho/\Delta\Gamma$  along the same path are affected by  $\delta$ . From Fig. 1 we find that this variation in density is greater for larger diameter beads, suggesting that complex, void-generating structures (e.g. arches) are more readily created and sustained for smaller  $\delta$ . In addition, for small  $\delta$ , ordering (crystallization) induced by the container walls [11] will increase the measured volume fraction over its bulk value, leading to densities that can exceed the random close-packed limit. Finally, for certain values of  $\delta$  in cylindrical containers, the additional possibility becomes significant of settling into nested, shell-like bead arrangements that optimally fill the container cross-section [2,28]. This commensurability may be responsible for the particularly high maximum densities seen in Fig. 1(b). Overall, we expect the results for the 1 mm beads (Fig. 1(a)), for which  $\delta \approx 19$ , to best reflect the compaction properties of a bulk system.

We note that compared with the reversible branch, where data are well reproducible from run to run within the indicated spread, the detailed shape of the irreversible branch is generally more variable. Occasionally, a kink or plateau will

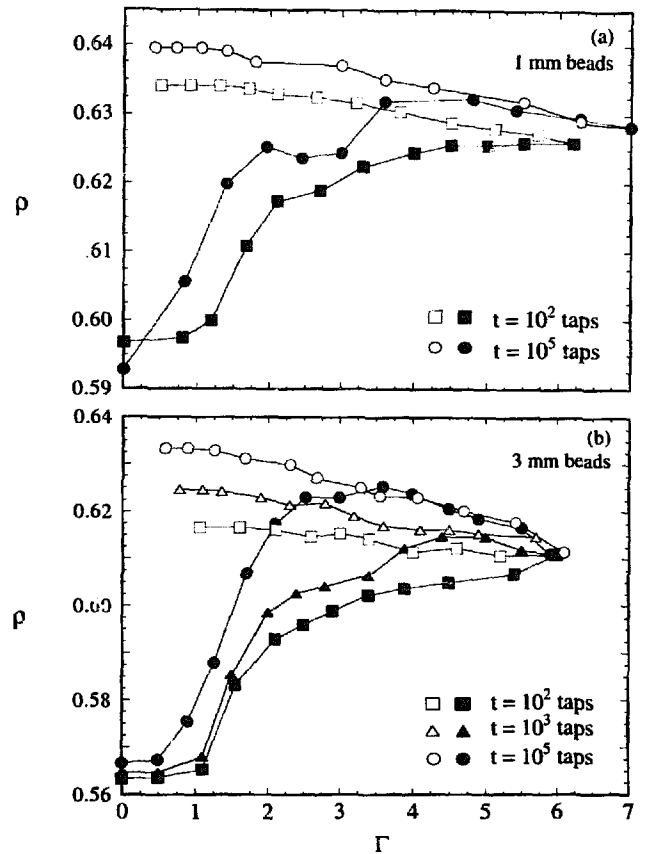


Fig. 2. The dependence of the packing behavior  $\rho(\Gamma)$  on the number of discrete taps,  $t$ , undergone at each value of acceleration. The step change in  $\Gamma$  between data points was kept nominally the same for all curves. Solid (open) symbols represent the dependence for increasing (decreasing)  $\Gamma$ . Error bars were omitted for clarity. For small values of  $t$ , the irreversible behavior extends over a larger range of  $\Gamma$  and the maximum final density is reduced, particularly for the larger beads in (b).

appear on the irreversible branch which is not present when the experiment is repeated. This behavior is indicative of the highly metastable nature of the settling process in this low-density region in which the granular system explores a multitude of bead configurations corresponding to different densities at similar values of  $\Gamma$ .

Our results clearly show that if one is interested in obtaining the highest packing density in a cohesionless granular material, one must follow the reversible branch downwards after first subjecting the material to large vibration intensities. This procedure is analogous to slowly cooling a thermal system (i.e. one in which thermal energies,  $k_B T$ , are significant) in order to best anneal out structural defects. Analogous to the cooling rate in a thermal system, a key parameter that controls the packing density is the rate,  $\Delta\Gamma/t$ , at which the vibration intensity is varied. Since the density relaxes exceedingly slowly, the compaction behavior  $\rho(\Gamma)$  will depend on the length of time spent at each value of  $\Gamma$ . (The change in  $\Gamma$  between successive data points was kept relatively constant at  $\Delta\Gamma \approx 0.5$ ). This ramp-rate dependence is shown in Fig. 2 where we plot results for 1 and 3 mm beads for three different values of  $t$ . The irreversible regime, i.e. the value of  $\Gamma^*$ ,

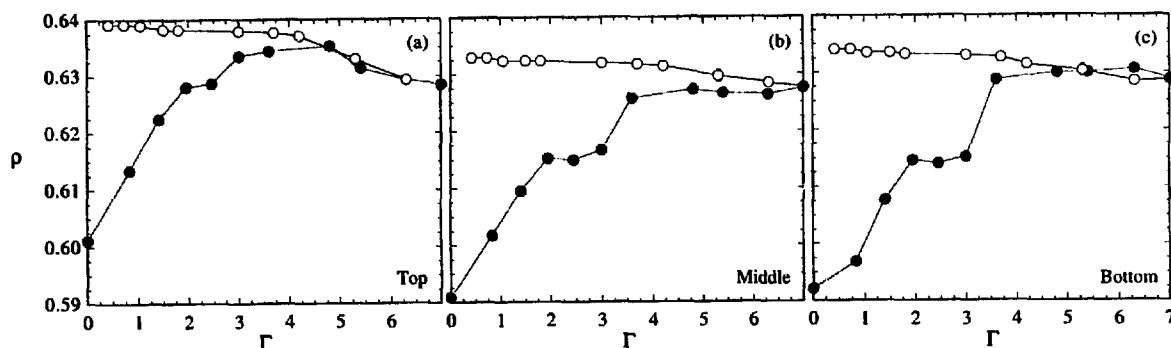


Fig. 3. The depth dependence of the packing behavior  $\rho(\Gamma)$  for the 1 mm beads and the slowest ramp rate, i.e.  $t = 10^5$  taps. Solid (open) symbols represent the dependence for increasing (decreasing)  $\Gamma$ . The curves show qualitatively the same behavior, indicating that the assembly of beads is responding homogeneously.

increases dramatically for faster ramp rates (smaller values for  $t$ ). It is largest for the curve with 100 taps per point. This is to be expected since the shorter the time that the system is allowed to relax, the larger will be the deviation from its final steady-state behavior.

The ramp rate also affects the maximum final density after ramping back down to  $\Gamma = 0$ . Increasing the time spent at each value of acceleration, increases the maximum final density, as seen for  $t = 10^2$  and  $10^5$  in Fig. 2. A similar effect is typical of disordered thermal systems such as glasses, spin glasses, or magnets. It has also recently been predicted for granular systems by Coniglio and Herrmann [24]. By analogy,  $\Gamma^*$  then defines a point on what in thermal systems would be called the 'irreversibility line' in the phase diagram. As in real glass-forming liquids, the final density is higher for slower cooling rates.

The upper branches of curves as in Fig. 2 are not expected to be reversible for fast ramp rates. Here, changes in the vibration intensity generally lead to *increases* in packing density until the  $\rho$  reaches the reversible steady-state behavior represented by the  $t = 10^5$  curve. Thus, rapid changes in vibration intensity can easily 'supercool' or 'superheat' the system into a metastable configuration that is far from the optimally packed state for that vibration intensity and persists indefinitely. This effect becomes particularly acute as the random close-packing limit is approached. Near  $\rho \approx 0.64$ , the free volume per particle becomes very small. In order for sufficient particles to cooperatively rearrange and fill in any remaining voids, excessively long tapping times are required. If the resulting time scale for reaching steady-state behavior exceeds our experimental waiting time, then this can lead to a leveling off of  $\rho(\Gamma)$  at low  $\Gamma$ , as seen in Figs. 1 and 2.

It is interesting to consider whether longer times spent at a given acceleration on the irreversible branch would eventually bring the system to the reversible branch. Examination of the density relaxation as a function of time on the irreversible branch indicates that often a steady state is not reached for low  $\Gamma$  because the packing's evolution becomes stuck in a low-density configuration after only a few hundred taps. This brings out one fundamental complication for any attempt to directly identify  $\Gamma$  with an effective temperature of the granular system: in thermal systems there always is a

finite probability for a large enough fluctuation to occur and drive the system out of a metastable state and bring it closer to equilibrium. However, the spectrum of our applied vibration intensity is sharply cut off above the peak acceleration, and thus lacks the exponential tail of its statistical mechanical counterpart, temperature. Consequently, once the configurations giving rise to the largest voids (which are presumably the least stable to vibration) have been eliminated, the system may still find itself in a metastable, low-density configuration indefinitely unless the vibration intensity is increased.

We have also investigated whether the compaction behavior is homogeneous throughout the container or if it occurs predominately in certain regions, e.g. near the top of the pile. In Fig. 3, the local packing fraction is shown at three different depths within the pile of beads. For these data  $\rho$  was measured capacitively and represents a spatial average over volumes containing roughly 50 000, 6000, and 1900 particles for runs with 1, 2, or 3 mm beads, respectively. (This is distinct from Figs. 1 and 2 where the average behavior over the whole column was determined from measurements of the total height of the beads.) Fig. 3 indicates that all regions within the container are behaving in a similar manner with the very top portion having a somewhat more pronounced variation with acceleration on the reversible branch. The average packing density for the whole cylinder, estimated from an average of local density data at the three different depths, was in good agreement with that determined from direct height measurements. Moreover, at each depth the ramp-rate dependence of the data was qualitatively similar to that shown in Fig. 2.

Finally, we note that data qualitatively similar to those shown in Fig. 1(a) were obtained in experiments using poly-disperse aluminum oxide particles (platelets) having rough, irregular shapes. We therefore expect that the understanding obtained from this simple model system consisting of monodisperse spherical particles should be generically applicable to more complicated granular materials.

#### 4. Conclusions

These results have importance for situations where one wishes to produce the most compact material possible by

means of vertical vibration. It is clear that the highest densities are obtained by first increasing the magnitude of the acceleration and then *slowly* decreasing it to a much lower value. For practical applications it may be sufficient to start shaking immediately at large  $\Gamma$  before ramping down. This acceleration, however, must be large enough so that the system can eliminate metastable low-density configurations and reach the steady-state density on the reversible branch. For our system the minimum acceleration corresponds to roughly  $\Gamma^* = 3$  for  $t = 10^5$ . Fewer taps may be necessary at larger values of  $\Gamma$ . Finally, our data provides a direct, monotonic correspondence between  $\rho$  and  $\Gamma$  along the reversible branch. This might provide the experimental basis for recent theoretical approaches [14,15,17,18,38] that link the steady-state density to an effective temperature. For applications, this provides a means to reproducibly, and reversibly, change the packing density by varying the applied acceleration.

## 5. List of symbols

$A$	shaking amplitude
$d$	bead diameter
$D$	inner diameter of cylindrical tube
$f$	shaking frequency
$g$	Earth's gravitational acceleration
$k_B$	Boltzmann factor
$t$	number of taps (individual shaking cycles) before acceleration was varied
$T$	temperature

## Greek letters

$\delta$	$= D/d$
$\Gamma$	ratio of peak acceleration applied during a tap to gravitational acceleration $g$
$\Gamma^*$	value of $\Gamma$ at which reversible and irreversible branches meet
$\Delta\Gamma$	change in $\Gamma$ between successive data points
$\rho$	volume packing fraction
$\Delta\rho$	change in $\rho$ between successive data points

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