Phys207: Lecture 04

Reminders

- All Discussion and Lab sections start meeting this week
- Homework 2 is posted on course website
- Solutions to previous hwks will be posted Thursday mornings

Today's Agenda

- 3-D Kinematics
  - Independence of *x* and *y* components
  - Baseball projectile
  - Shoot the monkey
- Uniform circular motion

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3-D Kinematics

- The position, velocity, and acceleration of a particle in 3 dimensions can be expressed as:

$$\boldsymbol{r} = x\,\boldsymbol{i} + y\,\boldsymbol{j} + z\,\boldsymbol{k}$$
$$\boldsymbol{v} = v_x\,\boldsymbol{i} + v_y\,\boldsymbol{j} + v_z\,\boldsymbol{k} \qquad (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k} \text{ unit vectors})$$
$$\boldsymbol{a} = a_x\,\boldsymbol{i} + a_y\,\boldsymbol{j} + a_z\,\boldsymbol{k}$$

- We have already seen the 1-D kinematics equations:

$$x = x(t) \qquad v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

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3-D Kinematics

- For 3-D, we simply apply the 1-D equations to each of the component equations.

$$x = x(t) \qquad y = y(t) \qquad z = z(t)$$

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2}$$

- Which can be combined into the vector equations:

$$\boldsymbol{r} = \boldsymbol{r}(t) \qquad \boldsymbol{v} = d\boldsymbol{r} / dt \qquad \boldsymbol{a} = d^2\boldsymbol{r} / dt^2$$

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3-D Kinematics

- So for constant acceleration we can integrate to get:

🡐 $\boldsymbol{a} = const$
🡐 $\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{a}\, t$
🡐 $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}_0\, t + {}^1/_2\, \boldsymbol{a}\, t^2$

(where $\boldsymbol{a}$, $\boldsymbol{v}$, $\boldsymbol{v}_0$, $\boldsymbol{r}$, $\boldsymbol{r}_0$, are all vectors)

Link to Active Figure 4-5

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2-D Kinematics

- Most 3-D problems can be reduced to 2-D problems when acceleration is constant:
  - 🡐Choose ***y*** axis to be along direction of acceleration
  - 🡐Choose ***x*** axis to be along the “other” direction of motion
- **Example**: Throwing a baseball (neglecting air resistance)
  - 🡐Acceleration is constant (gravity)
  - 🡐Choose ***y*** axis up: $a_y = -g$
  - 🡐Choose ***x*** axis along the ground in the direction of the throw

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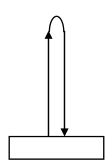
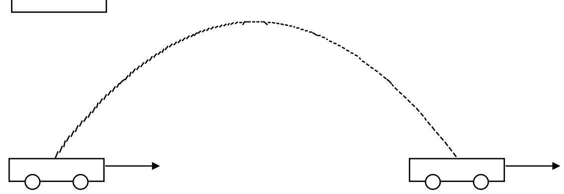
“x” and “y” components of motion are independent.

- A man on a train tosses a ball straight up in the air.
  - 🡐View this from two reference frames:

Reference frame
on the moving train.



Reference frame
on the ground.



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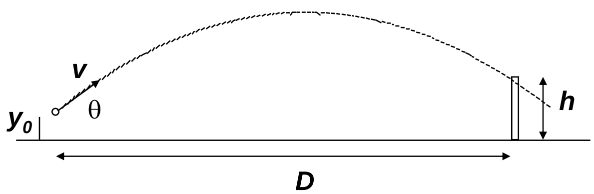
Problem:

- Mark McGwire clobbers a fastball toward center-field. The ball is hit 1 m (***y_o***) above the plate, and its initial velocity is 36.5 m/s (***v***) at an angle of 30º (θ) above horizontal. The center-field wall is 113 m (***D***) from the plate and is 3 m (**h**) high.

  - ➔What time does the ball reach the fence?
  - ➔Does Mark get a home run?



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Problem...

- First let's examine the motion Link to Active Figure 4-7
- Choose ***y*** axis up.
- Choose ***x*** axis along the ground in the direction of the hit.
- Choose the origin *(0,0)* to be at the plate.
- Say that the ball is hit at $t = 0$, $x = x_0 = 0$

  - ➔Equations of motion are:

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$
$$x = v_x t \qquad y = y_0 + v_{0y}\, t - {}^{1}/_{2}\, gt^2$$

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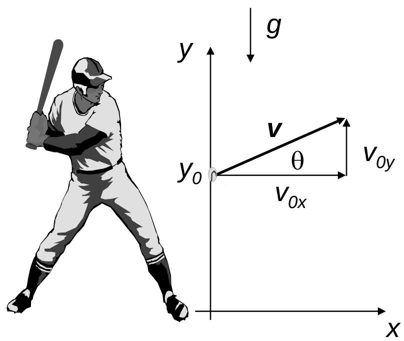
Problem...

- Use geometry to figure out v_{0x} and v_{0y} :



Find $v_{0x} = |\mathbf{v}| \cos\theta$.
and $v_{0y} = |\mathbf{v}| \sin\theta$.

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Problem...

- The time to reach the wall is: $t = D / v_x$ (easy!)
- We have an equation that tell us $y(t) = y_0 + v_{0y} t + a t^2 / 2$
- So, we're done....now we just plug in the numbers:

- Find:
  - ← v_x = 36.5 cos(30) m/s = 31.6 m/s
  - ← v_y = 36.5 sin(30) m/s = 18.25 m/s
  - ← t = (113 m) / (31.6 m/s) = 3.58 s
  - ← $y(t)$ = (1.0 m) + (18.25 m/s)(3.58 s) - (0.5)(9.8 m/s^2)(3.58 s)^2

    = (1.0 + 65.3 - 62.8) m = **3.5 m**

  - ← Since the wall is **3 m** high, Mark gets the homer!!

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Lecture 4, *Act 1*
Motion in 2D

- Two footballs are thrown from the same point on a flat field. Both are thrown at an angle of 30^o above the horizontal. *Ball 2* has twice the initial speed of *ball 1*. If *ball 1* is caught a distance D_1 from the thrower, how far away from the thrower D_2 will the receiver of *ball 2* be when he catches it?

  (a) $D_2 = 2D_1$ (b) $D_2 = 4D_1$ (c) $D_2 = 8D_1$

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Lecture 4, *Act 1*
Solution

- The distance a ball will go is simply
  x = *(horizontal speed)* x *(time in air)* = $v_{0x}\, t$
- To figure out "time in air", consider the equation for the height of the ball: $y = y_0 + v_{0y}\, t - \frac{1}{2} g\, t^2$
- When the ball is caught, $y = y_0$ ⇨ $v_{0y}\, t - \frac{1}{2} g\, t^2 = 0$

⇨ $t\left(v_{0y} - \frac{1}{2} g\, t\right) = 0$ ⇨ two solutions

$t = 2\frac{v_{0y}}{g}$ (time of catch)

$t = 0$ (time of throw)

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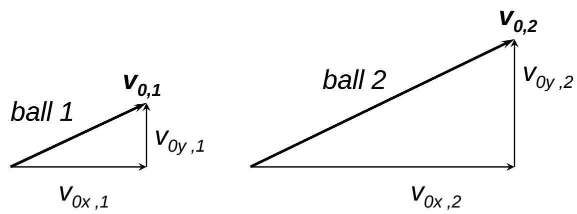
$x = v_{0x}\, t$

Lecture 4, *Act 1* Solution

- So the time spent in the air is proportional to v_{0y}: $t = 2\dfrac{v_{0y}}{g}$
- Since the angles are the same, both v_{0y} and v_{0x} for *ball 2* are twice those of *ball 1*.



- Ball 2 is in the air *twice* as long as ball 1, but it also has *twice* the horizontal speed, so it will go **4** times as far!!

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Trajectory of projectile (<u>special case</u> of no air resistance)

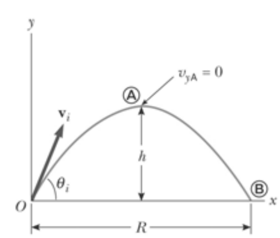
- Decompose into *x* and *y* motion

$x = x_0 + v_{0x}t + \tfrac{1}{2} at^2 = (v_0 \cos\theta)t$ No horizontal forces

$y = y_0 + v_{0y}t + \tfrac{1}{2} at^2 = (v_0 \sin\theta)t - \tfrac{1}{2} gt^2$

- Eliminate t using $t = x / (v_0 \cos\theta)$

$$y = (\tan\theta)\, x - \left(\frac{g}{2v_0^2 \cos^2\theta}\right) x^2$$



Trajectory is a parabola
Specified by only initial speed and launch angle

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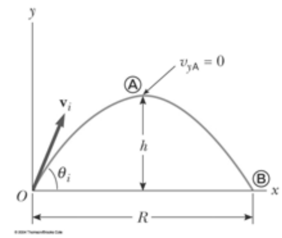
Maximum height of projectile
(<u>special case</u> of symmetric trajectory)

- At peak $v_y = 0$, so $v_y = 0 = v_{0y} - gt_A$

$$0 = v_0 \sin\theta - gt_A$$
$$t_A = \frac{v_0 \sin\theta}{g}$$



- $h_{max} = 0 + v_{0y}t_A - ½\, gt_A$

$$= (v_0 \sin\theta)\frac{v_0 \sin\theta}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$
$$= \frac{v_0^2 \sin^2\theta}{2g}$$

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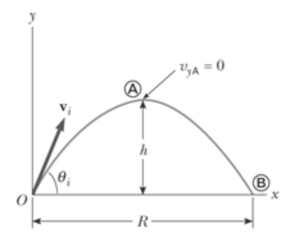
Horizontal range of projectile
(<u>special case</u> of symmetric trajectory)

- Total time in air is $2t_A$
- Range $R = v_{0x} \times$ total time in air

$$R = (v_0 \cos\theta) 2t_A$$
$$= (v_0 \cos\theta)\frac{2v_0 \sin\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$



(recall $\sin 2\theta = 2 \sin\theta \cos\theta$)

$$R_{\max} = \frac{v_0^2}{g} \quad \text{at } \theta = 45^0$$

Let's examine some trajectories – <u>Link to Active Figure 4-11</u>

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Shooting the Monkey
(tranquilizer gun)

- Where does the zookeeper aim if he wants to hit the monkey?

  (He knows the monkey will let go as soon as he shoots !)

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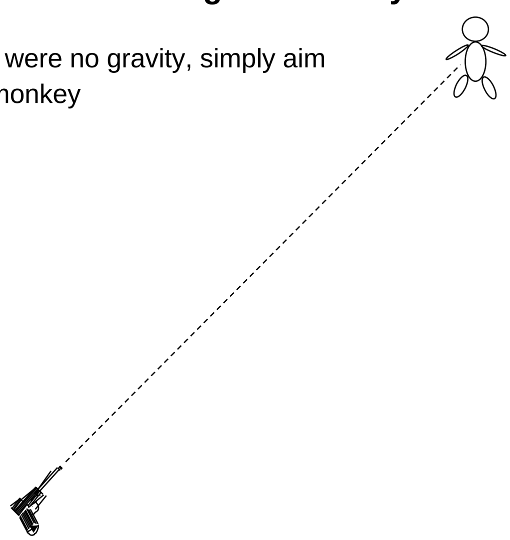
Shooting the Monkey...

- If there were no gravity, simply aim at the monkey



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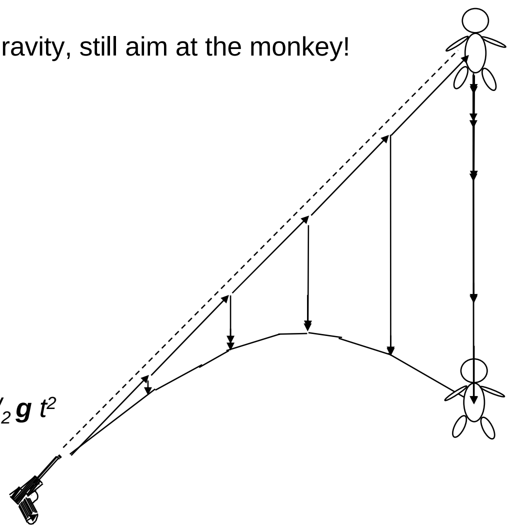
Shooting the Monkey...

- With gravity, still aim at the monkey!

$\boldsymbol{r} = \boldsymbol{r}_0 - {}^1/_2\, \boldsymbol{g}\, t^2$



$\boldsymbol{r} = \boldsymbol{v}_0\, t - {}^1/_2\, \boldsymbol{g}\, t^2$

Dart hits the monkey!

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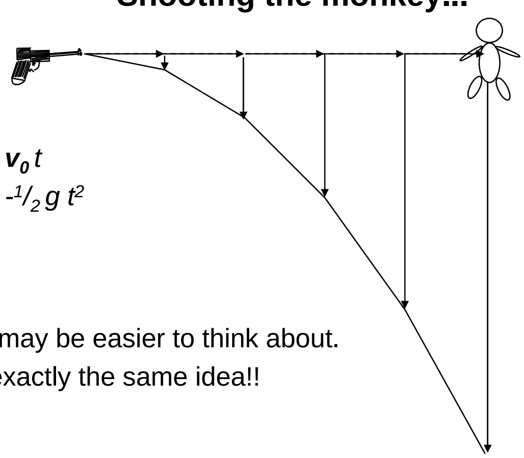
Recap: Shooting the monkey...



$x = \boldsymbol{v_0}\, t$
$y = -{}^1/_2\, g\, t^2$

- This may be easier to think about.
  It's exactly the same idea!!

$x = x_0$
$y = -{}^1/_2\, g\, t^2$

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