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Relativistic many-body calculations of the oscillator strengths, transition rates and polarizabilities in Zn-like ions

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Abstract

Transition rates, oscillator strengths and line strengths are calculated for electric-dipole (E1) transitions between even-parity 4s\textsuperscript{2}, 4p\textsuperscript{2}, 4s4d, 4d\textsuperscript{2}, 4p4f and 4f\textsuperscript{2} states and odd-parity 4s4p, 4s4f, 4p4d and 4d4f states in Zn-like ions with the nuclear charges ranging from Z = 32 to 100. Relativistic many-body perturbation theory (RMBPT), including the Breit interaction, is used to evaluate retarded E1 matrix elements in length and velocity forms. The calculations start from a 1s\textsuperscript{2}2s\textsuperscript{2}2p\textsuperscript{6}3s\textsuperscript{2}3p\textsuperscript{6}3d\textsuperscript{10} Dirac–Fock potential. First-order RMBPT is used to obtain intermediate coupling coefficients and second-order RMBPT is used to calculate transition matrix elements. Contributions from negative-energy states are included in the second-order E1 matrix elements to ensure the gauge independence of transition amplitudes. Transition energies used in the calculation of oscillator strengths and transition rates are from second-order RMBPT. Ground state scalar \(\alpha_0(4s^2 1S_0)\) polarizabilities are calculated for Zn-like ions from Z = 33 to 100. To evaluate the \(\alpha_0(4s^2 1S_0)\) polarizabilities, we calculate RMBPT energies for the odd-parity 4\(l_5\)\(l'_5\) complex with \(J = 1\) and line strengths between the even-parity 4\(l_4\)\(l'_4\) complex with \(J = 0\) and the odd-parity 4\(l_5\)\(l'_4\), 4\(l_6\)\(l'_4\) complexes with \(J = 1\).

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The group II-like elements are presently of significant interest to various AMO fields owing to the development of atomic clocks with various group II atoms (Sr, Hg, Yb, etc). In fact, 5s\textsuperscript{2}1\(S_0\)–5s5p 3\(P_0\) transition frequency in \(^{88}\text{Sr}\) has been recommended as the secondary representation of a SI second by the International Committee for Weights and Measures (CIPM) [1]. These atoms have also become of recent interest to quantum information [2]. These applications require good knowledge of atomic properties, such as transition rates and polarizabilities. For example, atomic clock schemes are subject to the blackbody radiation shift that is very hard to measure and needs to be calculated. Quantum information proposals require knowledge of wavelengths where atomic dynamic polarizabilities of group II atoms are zero. Study of degenerate quantum gases requires understanding of the long-range interaction coefficients. New methods for studying group II atoms are currently under development [3]. The main problem faced when developing a novel high-precision method for group II atoms is the lack of benchmark experimental data and comprehensive analysis and comparison of theory and experiment. While second-order relativistic many-body perturbation theory (RMBPT) is not sufficiently precise for accurate prediction of the required properties of neutral systems for some of the applications, it is quite precise for the ions of the corresponding isoelectronic sequence. The second-order calculations allow us to study possible issues that may arise when perturbation theory is combined with other methods, such as configuration interaction (CI) to produce more accurate results [4]. Second-order calculations also give the initial approximation for more accurate coupled-cluster approaches [5]. Zn-like systems are of particular interest owing to the convergence issues of the all-order methods [5].
due to large d-shell excitations. The goal of the present study is to fill gaps in our understanding of Zn-like ion properties and to analyse existing results for our better understanding of the properties of group II atoms in general. Our results are compared extensively with other calculations and experiment where available.

Recently, the relativistic configuration-interaction (CI) method with the numerical Dirac–Fock wavefunctions generated in the field of ab initio screened model potential was used by Glowacki and Migdalke [6] to compute oscillator strengths for the spin-allowed and spin-forbidden 4s21S0–4s4p1S2P1 transitions in neutral zinc. Lifetimes of the 4s4p1S2P1 levels in Ga II were evaluated using the CIV3 code by McElroy and Hibbert in [7]. The CIV3 code includes extensive CI calculations. Relativistic effects were introduced via the Breit–Pauli Hamiltonian. Core polarization effects were included by means of explicit CI [7]. Flexible Atomic Code (FAC) was used recently in [8] to calculate energy levels, oscillator strengths and electron impact collision strengths for Ge-, Ga-, Zn-, Cu-, Ni- and Co-like Au ions. The wavelengths and transition probabilities for EUV and x-ray lines in the spectra from Yb 40+ to U 62+ along the zinc isoelectronic sequence were calculated by Quinet et al [10]. The GRASP92 multi-configuration Dirac–Hartree–Fock (MCDF) model was used. Results were reported for the 4s3–4s4p, 4s4p–4s3p and 4s4p–4s4d transitions [9]. Relativistic many-body calculations of the n = 4 states along the zinc isoelectronic sequence were recently reported by Blundell et al [10]. The GRASP92 multi-configuration Dirac–Hartree–Fock package was recently [11] employed to calculate excitation energies, ionization potentials and oscillator strengths for all neutral and up to five times ionized species of element Cp. Z = 112. Weighted oscillator strengths in Coulomb and Babushkin gauges for EI transitions in Zn-like Ga II were recently reported by Jönsson et al [12]. The graspVU multiconfiguration Dirac–Fock–Fock package was used in [12] to evaluate energies, oscillator strengths and lifetime data in Zn-like Ga II. The same package was used in [13] to evaluate rates of forbidden lines within the 4s21S0 term and M2 transition rates were calculated for the 1S0–1P1 transitions [13]. The GRASP2k multiconfiguration Dirac–Hartree–Fock–Fock package was used recently in [14] to study the hyperfine interaction-dependent 4s4p1S2P1 lifetimes in Zn-like ions. A number of publications have been devoted to the investigation of hyperfine interactions in these ions [15–18] within the past 10 years. Hyperfine quenching of the 4s4p1S2P1 level in Zn-like ions was investigated by Marques et al in [17]. The graspVU multiconfiguration Dirac–Hartree–Fock package was used in [16] to calculate transition rates between the 4s4f3F2, 3 and 4s4d3D2 hyperfine levels in Ga II. It was pointed out in [16] that the hyperfine interaction redistributes the intensity among the hyperfine transitions. The authors underlined that these results have never been reported before and this could be of interest in the ongoing studies of the Ga abundance analysis of peculiar HgMn stars [16].

Oscillator strengths [19–49] and lifetimes [50–68] in Zn-like sequences have been studied in a number of works. Atomic spectra in Zn-like ions were studied in different scientific centres during the last 30 years [69–83]. However, very few papers contain any values for polarizabilities of Zn-like ions [84–89]. Recently, determination of polarizabilities and lifetimes for the Mg, Zn, Cd and Hg isoelectronic sequences was presented by Reshetnikov et al in [89]. It was underlined in that work that the measurement of the lowest resonance transition lifetime can be used to determine the polarizabilities and, alternatively, measurements of the polarizabilities can be used to deduce lifetimes. Additionally, isoelectronic regularities in line strengths can be used to obtain a comprehensive database from a small number of precision lifetime determinations. These methods were applied in [89] to produce values for polarizabilities and lifetimes for the Mg, Zn, Cd and Hg isoelectronic sequences.

In the present paper, an RMBPT method is used to calculate transition rates and oscillator strengths between the 4s21S0–4s4f3F2, 3 and 4s4d3D2 hyperfine levels in Zn-like ions [90–93]. Additionally, we calculate ground state scalar α(4s21S0) polarizabilities for Zn-like ions (Z = 33–100). These calculations involve new calculations of RMBPT energies for the odd-parity 4s21S1–4s4f3F2, 3 and 4s4d3D2 hyperfine levels in Zn-like ions [90–93]. The first-order reduced dipole matrix element \( J(vw) = \int (\mu_{jw} + \mu_{jw}) dJ(vw) \) for the transition between two states \( vw(J) → v′w′(J′) \) [90] is given by

\[
Z^{(1)}[v_{ij}w_{ij}(J) − v_{ij}w_{ij}(J')][J] = \sum_{vw} S^J(v_{ij}w_{ij}, vw) S^{J'}(v_{ij}w_{ij}, v′w′) × (-1)^{j′w′+j′w} \frac{J}{j′w} \frac{J′}{j′w} \frac{1}{j′w} Z^{vw}_{vw} δ_{vw},
\]

where \( [J] = 2J+1 \). The quantity \( S^J(v_{ij}w_{ij}, vw) \) is a symmetry coefficient defined by

\[
S^J(v_{ij}w_{ij}, vw) = δ_{v_{ij}w_{ij}} + (-1)^{j′w} \delta_{v_{ij}w_{ij}} δ_{vw},
\]

2
The second-order reduced matrix element $Z^{(2)}$ for the transition between two states $\nu\nu(J)\rightarrow \nu'\nu'(J')$ consists of four contributions: Dirac–Hartee–Fock $Z^{(HF)}$ term, random-phase approximation $Z^{(RPA)}$, term, correlation contribution $Z^{(corr)}$ term and derivative $P^{(deriv)}$ term \cite{90}. The $Z^{(HF)}$, $Z^{(RPA)}$, $Z^{(corr)}$ and $P^{(deriv)}$ contributions to second-order matrix elements in terms of Brueckner–Goldstone diagrams are illustrated in figure 1. The dashed lines indicate Coulomb + Breit interactions and the wavy lines indicate the interaction with the dipole field. Diagrams 'HF 1' and 'HF 2' as well as diagrams 'RPA 1' and 'RPA 2' represent direct and exchange contributions. These diagrams account for the shielding of the dipole field by the core electrons. Diagrams 'corr 1' and 'corr 2' are direct and exchange correlation contributions. These diagrams correct the matrix element to account for the interaction between the valence electrons. The 'deriv' diagram represents symbolically the second-order RMBPT correction from the derivative term \cite{90}. A detailed discussion of these diagrams for divalent systems was given by Safronova et al \cite{90}.

All of the second-order correlation corrections that we discussed above result from the residual Coulomb interaction. To include correlation corrections due to the Breit interaction, the Coulomb matrix element $X_1(abc)$ (see for detail \cite{94}) must be modified according to the rule

$$X_1(abc) \rightarrow X_1(abc) + M_2(abc) + N_2(abc),$$

where $M_2$ and $N_2$ are magnetic radial integrals defined by equations (A4) and (A5) in \cite{95}.

### 2.1. Uncoupled matrix elements

In table 1, we list values of the first- and second-order contributions to electric-dipole matrix elements $Z^{(1)}$, $Z^{(RPA)}$, $Z^{(corr)}$ and the matrix element of the derivative term $P^{(deriv)}$ for the $4s_{1/2}^24s_{1/2}(0)\rightarrow 4j_j^l(1)$ transitions in Zn-like silver, $Z = 47$. Both length and velocity forms of the matrix elements are given. The Coulomb second-order contribution $Z^{(HF)}$ vanishes in the present calculation since we use DF basis functions. We use symbol $B$ in table 1 to denote the Coulomb–Breit contributions to the second-order matrix elements, and we tabulate $100 \times B^{(HF)}$, $100 \times B^{(RPA)}$, $100 \times B^{(corr)}$ and the totals $100 \times B^{(2)}$. We multiply Coulomb–Breit values by 100 for more transparent comparison with Coulomb data. The ratios of the second-order $Z^{(RPA)}$ and the lowest $Z^{(1)}$ contribution are about 5\% for the $4s_{1/2}^24s_{1/2}(0)\rightarrow 4s_{1/2}^24p_{1/2}(1)$ transitions. However, the other second-order $Z^{(corr)}$ term decreases the ratios of the second and first orders to 3\%. The total second-order Breit corrections $B^{(2)}$ also decrease the value of the second-order contribution; however, the ratios of the $B^{(2)}$ and $Z^{(1)}$ terms are very small (about 0.1\%).

### Table 1. Contributions to El uncoupled reduced matrix elements (au) in length L and velocity V forms for transitions between excited states $\nu\nu(J)$ and $\nu'\nu'(J')$ in Ag$^{+7}$.

<table>
<thead>
<tr>
<th>$\nu\nu(J)$</th>
<th>$\nu'\nu'(J')$</th>
<th>$Z^{(1)}$</th>
<th>$P^{(deriv)}$</th>
<th>$Z^{(RPA)}$</th>
<th>$Z^{(corr)}$</th>
<th>$B^{(HF)}$</th>
<th>$B^{(RPA)}$</th>
<th>$B^{(corr)}$</th>
<th>$B^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4s_{1/2}^24s_{1/2}(0)$</td>
<td>$4s_{1/2}^24p_{1/2}(1)$</td>
<td>$L$</td>
<td>0.652 10</td>
<td>0.652 11</td>
<td>-0.032 63</td>
<td>0.009 89</td>
<td>0.050 32</td>
<td>-0.000 79</td>
<td>0.000 26</td>
</tr>
<tr>
<td>$V$</td>
<td>0.650 93</td>
<td>0.000 03</td>
<td>-0.028 80</td>
<td>0.004 05</td>
<td>-0.059 41</td>
<td>0.001 24</td>
<td>-0.000 56</td>
<td>-0.058 72</td>
<td>-0.058 00</td>
</tr>
<tr>
<td>$4s_{1/2}^24s_{1/2}(0)$</td>
<td>$4s_{1/2}^24p_{1/2}(1)$</td>
<td>$L$</td>
<td>-0.928 83</td>
<td>-0.928 78</td>
<td>0.045 84</td>
<td>-0.014 07</td>
<td>-0.058 41</td>
<td>0.001 24</td>
<td>-0.000 56</td>
</tr>
<tr>
<td>$V$</td>
<td>-0.922 02</td>
<td>0.000 08</td>
<td>0.035 75</td>
<td>-0.000 79</td>
<td>-0.058 41</td>
<td>0.001 24</td>
<td>-0.000 56</td>
<td>-0.058 72</td>
<td>-0.058 00</td>
</tr>
</tbody>
</table>

where \( \eta_{\nu\nu} \) is a normalization factor given by

\[
\eta_{\nu\nu} = \begin{cases} 
1 & \text{for } \nu \neq \nu \\
1/\sqrt{2} & \text{for } \nu = \nu.
\end{cases}
\]
The values of linear combinations of uncoupled two-particle states. For $4p_{1/2}/4s_{1/2}$ zero for two-particle transitions such as the $4s_{1/2}/4p_{3/2}$ electric-dipole matrix elements. The values of the $Z^{(\text{corr})}$ and $Z^{(\text{RPA})}$ terms have different signs and almost cancel each other for low-$Z$ ions. The ratio of these terms decreases very rapidly with $Z$ and becomes 50% for $Z = 37$ and 2% for $Z = 74$. The ratio of the $B^{(2)}$ and $Z^{(\text{RPA})}$ terms slowly increases with $Z$ from 1% for $Z = 37$ and 3% for $Z = 74$.

It should be noted that only the $Z^{(\text{corr})}$ terms are non-zero for two-particle transitions such as the $4s_{1/2}/4s_{1/2}(0) - 4p_{1/2}/4d_{5/2}(1)$ and $4s_{1/2}/4s_{1/2}(0) - 4d_{1/2}/4f_{7/2}(1)$ ones (see table 1). The values of $Z^{(\text{corr})}$ terms for two-particle transitions are of the same order of magnitude as for the one-particle transitions (for example, the $4s_{1/2}/4s_{1/2}(0) - 4p_{1/2}/4d_{5/2}(1)$ and $4s_{1/2}/4s_{1/2}(0) - 4p_{3/2}/4d_{3/2}(1)$ transitions).

2.2. Coupled matrix elements

As mentioned above, physical two-particle states are the linear combinations of uncoupled two-particle states. For the $\text{Ag}^{17+}$ example discussed above, the transition amplitudes between physical states are the linear combinations of the uncoupled transition matrix elements given in table 1. The mixing coefficients and energies are obtained by diagonalizing the first-order effective Hamiltonian which includes both Coulomb and Breit interactions. We let $C_{1}^{e}(\nu \nu)$ designate the $\lambda$th eigenvector of the first-order effective Hamiltonian and let $E_{1}^{e}$ be the corresponding eigenvalue. The coupled transition matrix element between the initial eigenstate $I$ with the angular momentum $J$ and the final state $F$ with the angular momentum $J'$ is given by

\[ Q^{(1+2)}(I \rightarrow F) = \frac{1}{E_{1}^{e} - E_{1}^{f}} \sum_{\nu \nu'} \sum_{\nu \nu'} C_{1}^{e}(\nu \nu) C_{1}^{f}(\nu' \nu') [\epsilon_{\nu \nu'} - \epsilon_{\nu' \nu'}] \]

\[ \times [Z^{(1+2)}[\nu \nu(J) - \nu \nu'(J')] + B^{(2)}[\nu \nu(J) - \nu \nu'(J')]] + [E_{1}^{e} - E_{1}^{f} - \epsilon_{\nu \nu'} + \epsilon_{\nu' \nu'}] p^{(\text{deriv})}[\nu \nu(J) - \nu \nu'(J')]. \]

\[ (4) \]

Here, $\epsilon_{\nu \nu'} = \epsilon_{\nu} + \epsilon_{\nu'}$, $Z^{(1+2)} = Z^{(1)} + Z^{(\text{RPA})} + Z^{(\text{corr})}$ and $B^{(2)} = B^{(\text{RF})} + B^{(\text{RPA})} + B^{(\text{corr})}$. Using these formulae together with the uncoupled reduced matrix elements given in table 1, we transform the uncoupled matrix elements to matrix elements between coupled (physical) states.

Values of coupled reduced matrix elements in length and velocity forms are given in table 2 for the transitions considered in table 1. Although we use an intermediate-coupling scheme, it is nevertheless convenient to label the physical states using the $LS$ scheme. Both designations are given in table 2. We see that $L$ and $V$ forms of the coupled matrix elements in table 2 differ by only 0.2–0.6%. These $L$–$V$ differences arise because we start our RMBPT calculations using a non-local Dirac–Fock (DF) potential. If we were to replace the DF potential by a local potential, the differences would disappear completely. The first two columns in table 2 show $L$ and $V$ values of coupled reduced matrix elements calculated without the second-order contribution. As we see from this table, removing the second-order contribution increases the $L$–$V$ differences up to 1–9%.

We used second-order RMBPT code to calculate uncoupled and coupled reduced matrix elements for $Zn$-like ions given in tables 1 and 2. Unfortunately, the implementation of the third-order RMBPT for heavier systems leads to many problems connected with the intruder states. Therefore, we studied the convergence of the RMBPT approach on the example of Cu-like ions using the RMBPT code developed by Johnson and Savukov [96]. In table 3, we illustrate our results for reduced matrix elements of Cu-like ions with nuclear charge $Z = 36–100$. The first three columns list values obtained in first-, second- and third-order RMBPT for the $4s$–$4p_{j}$ transitions in Cu-like ions. The fourth and fifth columns show ratios (in %) of the second- and first-order results and the third- and first-order results, respectively. We find that the third-order contribution is less than the second-order contribution by a factor of 5–10. The third-order contributes less than 1% for all of these ions confirming good convergence of RMBPT for Cu-like ions. We find that the ratios of second and first-order results ($Z^{(2)}/Z^{(1)}$) for the Zn-like ions are similar to the ratios shown in the table for Cu-like ions. Therefore, we expect that our conclusion regarding the convergence of MBPT holds for Zn-like ions as well.
2.3. Negative-energy contributions

It should be emphasized that we include negative energy state (NES) contributions into the sums over the intermediate states. Ignoring the NES contributions leads only to small changes in the L-form matrix elements but to substantial changes in some of the V-form matrix elements, with a consequent loss of gauge independence for a local potential.

The NES contributions to the second-order reduced matrix elements arise from the terms in the sums over states $i$ and $n$ in the $Z^{(\text{corr})}$ contributions [90] for which single-particle energy $\varepsilon_i < -mc^2$. The NES contributions for relativistically allowed transitions were discussed in [90, 91, 97] for Be-like and Mg-like ions, where they were found to be the most important for velocity-form matrix elements; they do not significantly modify length-form matrix elements. In [90], it was shown that NES contributions can be of the same order of magnitude as the ‘regular’ positive-energy contributions for certain non-relativistically forbidden transitions in Be-like ions. We observe similar large contributions for LS-forbidden transitions here. The matrix elements in tables 1 and 2 include NES contributions.

In figure 3, we illustrate the Z-dependence of the differences between line strengths calculated in length $S_L$ and velocity $S_V$ forms for the $4s^2\,^1S_0-4s4p\,^1P_1$ and $4s^2\,^1S_0-4s4p\,^3P_1$ transitions. We plot the ratio $(S_L-S_V)/S_L$ (in %)
calculated without (a) and with (b) negative-energy state contributions to the second-order reduced matrix elements. The ratio \( \left( S_{L} - S_{V} \right) / S_{L} \) for the \( 4s^{2} 1S_{0} - 4s4p^{3} 1P_{1} \) transition decreases from 1.2% for \( Z = 40 \) to 0.17% for \( Z = 70 \), respectively. The ratio \( \left( S_{L} - S_{V} \right) / S_{L} \) for the \( 4s^{2} 1S_{0} - 4s4p^{1} 1P_{1} \) transitions decreases from 4.7% for \( Z = 40 \) to 2.1% for \( Z = 70 \). However, this ratio decreases substantially (from 1.5% for \( Z = 40 \) to 0.37% for \( Z = 70 \)) when NES are included for the \( 4s^{2} 1S_{0} - 4s4p^{1} 1P_{1} \) transition. No large changes with including NES are observed in the \( \left( S_{L} - S_{V} \right) / S_{L} \) ratio for the \( 4s^{2} 1S_{0} - 4s4p^{3} 1P_{1} \) transition (see the right panel of figure 3).

In view of the gauge dependence issue discussed above, our results below are presented in the \( L \) form to decrease the volume of tabulated material. Uncertainties in the recommended values given in [98] were estimated to be less than 10% based on comparisons with experimental results from lifetime and emission measurements. The agreement between theoretical \( L \)-form and \( V \)-form results was also used in [98] as an indicator of accuracy. Since the present transition data are obtained using a single method for all \( Z \) and improves in accuracy with increasing \( Z \), owing to the decrease of relative importance of correlation correction, we expect our data for high \( Z \) to be very reliable.

3. Results and discussion

We calculate line strengths, oscillator strengths and transition probabilities for 851 \( 4l_{1}4l_{2}^{1,3}L_{J} - 4l_{3}4l_{4}^{1,3}L_{J}' \) lines for all ions with \( Z = 32-100 \). The results were calculated in both length and velocity forms, but only length-form results are presented in the following tables and figures for reasons discussed in the previous section. The theoretical energies used to evaluate oscillator strengths and transition probabilities are calculated using the second-order RMBPT formalism developed in [10].

3.1. Transition rates

The general trends of the \( Z \)-dependence of transition rates for the \( 4l_{1}4l_{2}^{1,3}L_{J} - 4l_{3}4l_{4}^{1,3}L_{J}' \) lines are presented in figures 4 and 5. In these figures, we show transitions to a fixed \( J \) state from states belonging to a limited set of the \( 4f4l^{1,3}L_{J} \) states, i.e. a complex of states. A complex includes all states of the same parity and \( J \) obtained from the combinations of the \( 4f4l^{1,3}L_{J} \) states. For example, the odd-parity complex with \( J = 1 \) includes the states \( 4s4p^{1}1P_{1}, 4p4d^{0}D_{1}, 4p4d^{1}P_{1}, 4f4d^{1}D_{1} \) and \( 4f4d^{1}P_{1} \) in \( LS \) coupling or \( 4s_{1/2}^{1/2}4p_{3/2}^{3/2}(1), 4s_{1/2}^{1/2}4p_{3/2}^{3/2}(1), 4p_{1/2}^{1/2}4d_{5/2}^{3/2}(1), 4p_{3/2}^{1/2}4d_{3/2}^{3/2}(1), 4p_{3/2}^{1/2}4d_{5/2}^{3/2}(1), 4d_{3/2}^{1/2}4f_{5/2}^{3/2}(1), 4d_{5/2}^{1/2}4f_{5/2}^{3/2}(1) \) and \( 4d_{5/2}^{1/2}4f_{7/2}^{3/2}(1) \) in \( j'j \) coupling. Later, we use the \( LS \) designations since they are more conventional.

In the top two panels of figure 4, we present a limited set (11 among 56 transitions included in the even-parity complex with \( J = 0 \) and odd-parity complexes with \( J = 1 \)) of transition rates for the \( 4s^{2} 1S_{0} - 4s4p^{3} 1P_{1} \), \( 4s^{2} 1S_{0} - 4p4d^{1}3P_{1} \), \( 4s^{2} 1S_{0} - 4d4f^{1}P_{1} \), \( 4p^{2} 1S_{0} - 4s4p^{1}1P_{1} \), \( 4p^{2} 1S_{0} - 4p4d^{1}3P_{1} \) and \( 4p^{2} 1S_{0} - 4d4f^{1}3P_{1} \) transitions. It should be noted that only two transitions shown in the left top panel of figure 4 (curves ‘1’ and ‘2’) are the 4s–4p electric-dipole one-particle transitions. Other three transitions (curves ‘3’, ‘4’ and ‘5’) are forbidden as electric-dipole one-particle transitions. The values of transition rates for these transitions are non-zero because of two-particle interaction between the \( [4s^{2} + 4p^{2} + 4d^{2} + 4f^{2}] \) and \( [4s4p + 4p4d + 4d4f] \) configurations as well as because of the second-order contribution from correlation diagrams \( Z^{(corr)} \) as demonstrated in table 1. As a result, the transition rates of these two-particle \( 4s^{2} 1S_{0} - 4p4d^{1}3P_{1} \) and \( 4s^{2} 1S_{0} - 4d4f^{1}P_{1} \) transitions presented in the left top panel of figure 4 are smaller (by two to four orders of magnitude) than the transition rates of one-particle \( 4s^{2} 1S_{0} - 4s4p^{1}1P_{1} \) lines for small \( Z \) but become even larger for high \( Z \). Similar ratios between the allowed \( 4p^{2} - 4p4d \) electric-dipole one-particle transitions and the forbidden \( 4p^{2} - 4d4f \) electric-dipole two-particle transitions are demonstrated by the top right panel of figure 4.

In the bottom two panels of figure 4, we present a limited set of transition rates for the \( 4s4d^{2} - 4p4d^{2} \), \( 4p^{2} - 4p4d \), \( 4s4d^{2} - 4s4f \) and \( 4p^{2} - 4s4f \) transitions (12 among 250 transitions between the states from the odd-parity complex with \( J = 2 \) and even parity complexes with \( J = 1, 2, \) and 3). The \( 4s4d - 4p4d \) and \( 4p^{2} - 4p4d \) transitions are illustrated by the \( 4s4d^{1,3}D_{J} - 4p4d^{1,3}D_{J} \) and by \( 4p^{2} 3P_{J} - 4p4d 1D_{J} \) transitions shown in the bottom left panel of figure 4. In the bottom right
Figure 4. Transition rates for even–odd transitions in Zn-like ions as a function of Z.

Panel of figure 4, we show Z dependence of transition rates for the 4s4d\(^{1,3}\)D\(_J\)–4s4f\(^3\)F\(_2\) and the 4p\(^2\)\(^3\)P\(_J\)–4s4f\(^3\)F\(_2\) transitions.

In the six panels of figure 5, we present all possible 4s4p–4s4d and 4s4p–4p\(^2\) electric-dipole one-particle transitions. The smallest values of transition rates are observed in figure 5 for singlet–triplet transitions: curves ‘2’ and ‘3’ (top left panel), curves ‘3’ and ‘4’ (top right panel), curves ‘2’ and ‘4’ (centre left panel), curves ‘1’ and ‘3’ (centre and bottom right panels) and curve ‘3’ (bottom left panel). It should be noted that in some cases this statement is not true for high-Z ions (see, for example, curves ‘2’, ‘4’ and ‘1’, ‘3’ on the centre left panel and curves ‘1’ and ‘2’ on the centre right panel of figure 5).

We see from the graphs that transitions with smooth Z dependences are rarer than transitions with sharp features but they still occur for all transition types: triplet–triplet, singlet–singlet and singlet–triplet, and include transitions with both small J and large J. One general conclusion that can be derived from those graphs is that the smooth Z-dependences occur more frequently for transitions with the largest values of transition rates among the transitions inside complexes.

Singularities in the transition-rate curves have three distinct origins: avoided level crossings, zeros in the dipole matrix elements and zeros in transition energies. Avoided level crossings result in changes of the dominant level configuration at a particular value of Z and lead to abrupt changes in the transition rate curves when the rates associated with the dominant configurations below and above the crossing point are significantly different. Zeros in transition matrix elements as functions of Z lead to cusp-like minima in the transition rate curves. Zeros in transition energies occur at high Z when levels of different parity cross.

Examples of each of these three singularity types are illustrated by figures 4 and 5. Dramatic example of the first type, avoided level crossings, is seen in the bottom right panel of figure 4 at Z = 68, corresponding to a change in the dominant configuration for the 4s4f\(^3\)F\(_2\) state, the 4p\(^2\)\(^3\)P\(_J\) instead of the 4s\(^1\)\(^3\)S\(_0\)\(→\)4p\(^2\)\(^3\)P\(_J\). Examples of the second type, zeros in matrix elements, are seen on the centre left panel of figure 5 at Z = 61–62 for the 4s4p\(^1\)\(^3\)P\(_J\)–4p\(^2\)\(^3\)P\(_J\) transition. Finally, singularity of the third type, corresponding to a very small (near zero) transition energy is seen at Z = 73 in the top-left panel of figure 5 for the 4s4p\(^1\)\(^3\)P\(_J\)–4p\(^2\)\(^3\)P\(_J\) transition. In this case, the level inversion occurs at the interface between the upper even- and odd-parity groups at high Z. The upper 4p\(^2\)\(^3\)P\(_J\) level becomes the lower 4p\(^2\)\(^3\)P\(_J\) level; however, the lower 4s4p\(^1\)\(^3\)P\(_J\) level becomes the upper level at Z = 73.

3.2. Wavelengths, transition rates and oscillator strengths

In tables 4 and 5, wavelengths and electric-dipole transition rates for the 16 4s\(^2\)–4s4p, 4s4p–4s4d, 4s4p–4p\(^2\) transitions in Zn-like ions with Z = 70–92 are presented. The RMBPT results are compared with experimental wavelengths from [9]. To save space, we did not include theoretical wavelengths and electric-dipole transition rates for the 4s\(^2\)–4s4p, 4s4p–4s4d, 4s4p–4p\(^2\) transitions calculated in [9]. These values were obtained using the fully relativistic...
multiconfiguration Dirac–Fock (MCDF) approach with the latest version of GRASP (General-purpose Relativistic Atomic Structure Package). The 4f/4f' model space was used for most of the ions, and 4f/5f' model space was added for Yb XLI and U LXIII [9]. This expansion of the model space resulted in only small differences in the results. The wavelengths were modified on average by only 0.012% and 0.009% while the changes of transition probabilities ranged from 0.60% to 0.70% in these two ions, respectively. We made detailed comparison with results from [9] and confirmed that our first-order RMBPT transition rates are in excellent agreement with transition rates from [9] (since similar model spaces are used). The second-order RMBPT includes additional correlation effects beyond the MCDF approach and is expected to produce more accurate results.

We find excellent agreement of our RMBPT values of wavelengths with experimental results taking into account experimental uncertainties. It should be noted that experimental values presented in [9] were taken from laboratories using different facilities, resulting in different uncertainties shown in experimental wavelengths listed in tables 4 and 5. The values for the 4s1/2→4p1/2(1)→4p1/2→4p3/2(2) and 4s1/2→4p3/2(2)→4p1/2→4p3/2(2) transitions in W44+ have the smallest uncertainties (0.0062 Å and 0.0040 Å). These measurements were done at the EBIT facility at the Lawrence Livermore National Laboratory [99]. Electron-beam energies of about 3 keV were sufficient to produce ions in all of the charge states of present interest [99]. For most of the wavelengths, the uncertainties were equal to 0.020 Å [79]. The spectra from laser-produced plasmas were recorded using a 3 m grazing incidence spectrograph. Thick planar targets were irradiated by one beam of the Nova laser at the Lawrence Livermore National Laboratory [79]. The uncertainties of

**Figure 5.** Transition rates for odd–even transitions in Zn-like ions as a function of Z.
Table 4. Wavelengths (\(\lambda\) in Å) and transition rates (\(A_r\) in s\(^{-1}\)) for the 4s\(^2\)–4s4p and 4s4p–4s4d transitions in Zn-like ions, \(Z = 70–92\). The RMBPT results (RMBPT) are compared with experimental (exp) wavelength results presented in [9]. Subscripts a, b, c and d indicate uncertainties of 0.020 Å, 0.005 Å, 0.050 Å and 0.100 Å, respectively.

<table>
<thead>
<tr>
<th>Z</th>
<th>RMBPT (\lambda) (Å)</th>
<th>exp (\lambda) (Å)</th>
<th>RMBPT (A_r) (s(^{-1}))</th>
<th>exp (A_r) (s(^{-1}))</th>
</tr>
</thead>
<tbody>
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<td>4s(<em>{1/2})24s(</em>{1/2})</td>
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<td>44.83(_{1/2})</td>
</tr>
<tr>
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<td>4s(<em>{3/2})24s(</em>{3/2})</td>
<td>45.07(_{3/2})</td>
<td>45.49(_{3/2})</td>
</tr>
<tr>
<td>72</td>
<td>4s(<em>{1/2})24p(</em>{1/2})</td>
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<td>45.78(_{1/2})</td>
<td>46.18(_{1/2})</td>
</tr>
<tr>
<td>73</td>
<td>4s(<em>{1/2})24p(</em>{3/2})</td>
<td>4s(<em>{3/2})24p(</em>{3/2})</td>
<td>46.49(_{3/2})</td>
<td>46.89(_{3/2})</td>
</tr>
<tr>
<td>74</td>
<td>4s(<em>{1/2})24d(</em>{3/2})</td>
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<td>47.20(_{3/2})</td>
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<td>81</td>
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<td>4s(<em>{3/2})24h(</em>{17/2})</td>
<td>52.17(_{3/2})</td>
<td>52.57(_{3/2})</td>
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<td>82</td>
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<td>4s(<em>{3/2})24i(</em>{19/2})</td>
<td>52.88(_{3/2})</td>
<td>53.28(_{3/2})</td>
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<td>83</td>
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<td>84</td>
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<td>4s(<em>{3/2})24j(</em>{23/2})</td>
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<td>55.41(_{3/2})</td>
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<td>55.72(_{3/2})</td>
<td>56.12(_{3/2})</td>
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<td>4s(<em>{3/2})24k(</em>{29/2})</td>
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<td>56.83(_{3/2})</td>
</tr>
<tr>
<td>88</td>
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<td>4s(<em>{3/2})24l(</em>{31/2})</td>
<td>57.14(_{3/2})</td>
<td>57.54(_{3/2})</td>
</tr>
<tr>
<td>89</td>
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<td>57.85(_{3/2})</td>
<td>58.25(_{3/2})</td>
</tr>
<tr>
<td>90</td>
<td>4s(<em>{1/2})24n(</em>{35/2})</td>
<td>4s(<em>{3/2})24n(</em>{35/2})</td>
<td>58.56(_{3/2})</td>
<td>58.96(_{3/2})</td>
</tr>
<tr>
<td>91</td>
<td>4s(<em>{1/2})24o(</em>{37/2})</td>
<td>4s(<em>{3/2})24o(</em>{37/2})</td>
<td>59.27(_{3/2})</td>
<td>59.67(_{3/2})</td>
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<tr>
<td>92</td>
<td>4s(<em>{1/2})24p(</em>{3/2})</td>
<td>4s(<em>{3/2})24p(</em>{3/2})</td>
<td>60.00(_{3/2})</td>
<td>60.40(_{3/2})</td>
</tr>
</tbody>
</table>

Some years later the same MCRRPA code was extended to evaluate values presented in the column ‘f’ [32] of table 6. Results for oscillator strengths are evaluated in first-order and second-order RMBPT. In two last columns of table 6, we show other theoretical values. The multiconfiguration Dirac–Fock (MCDF) technique (Grant code) was used by Biémont [36] to evaluate values presented in column ‘MCDF’. The multiconfiguration relativistic random-phase approximation (MCRRPA) approach developed by Huang and Johnson [32, 100, 101] was implemented to evaluate values presented in the column ‘f’ [32] of table 6. Some years later the same MCRRPA code was extended to calculate oscillator strengths for the 4s\(^2\)–4s4p1\(^3\)P transitions in Zn-like ions for high-Z ions up to \(Z = 92\) [39]. However, we can find only one additional result for Tb\(^{52+}\) to include in table 6 from [39]. There are no other differences in results given in columns ‘f’ [32] and ‘f’ [39] of table 6. In the last column of table 6, we list oscillator strengths for other ions up to Z = 92. Some years later the same MCRRPA code was extended to calculate oscillator strengths for the 4s\(^2\)–4s4p1\(^3\)P transitions in Zn-like ions for high-Z ions up to \(Z = 92\) [39]. However, we can find only one additional result for Tb\(^{52+}\) to include in table 6 from [39]. There are no other differences in results given in columns ‘f’ [32] and ‘f’ [39] of table 6. In the last column of table 6, we list oscillator strengths.
Table 5. Wavelengths ($\lambda$ in Å) and transition rates ($A_e$ in s$^{-1}$) for 4s$^3$–4p$^1$ transitions in Zn-like ions, Z = 70–92. The RMBPT results (RMBPT) are compared with experimental (expt) wavelength results presented in [9]. Subscripts a, b and c indicate uncertainties of 0.020 Å, 0.0062 Å and 0.0040 Å, respectively.

<table>
<thead>
<tr>
<th>Z</th>
<th>RMBPT</th>
<th>4s$^1$4p$^1$2(1)–4p$^1$4p$^1$(2)</th>
<th>RMBPT</th>
<th>4s$^1$4p$^1$2(0)–4p$^1$4p$^1$(1)</th>
<th>RMBPT</th>
<th>4s$^1$4p$^1$2(1)–4p$^1$4p$^1$(1)</th>
<th>RMBPT</th>
<th>4s$^1$4p$^1$2(1)–4p$^1$4p$^1$(0)</th>
</tr>
</thead>
</table>

Comparison of oscillator strengths shown in six columns of Table 6 shows that the MCDF (‘f [36]’) results are in good agreement with our first-order results. There is only a small difference between our first-order results and MCRRPA results given in columns ‘f [32]’ and ‘f [39]’. It should be noted that the RMBPT results are smaller than first-order results by 5–7%. Including core excitation channels in the MCRRPA approach decreases oscillator strengths for the 4s$^2$1S$^0$–4s4p$^1$P$^1$ transitions and brings those results closer to our RMBPT values with only 2–3% remaining difference. For the 4s$^2$1S$^0$–4s4p$^1$P$^1$ transition, the influence of including core excitation channels in MCRRPA approach appears to be very small. There is only 1% difference in all three MCRRPA f values for the 4s$^2$1S$^0$–4s4p$^1$P$^1$ transition.
Table 6. Wavelengths (λ in Å) and oscillator strengths for the 4s^2 1S_0–4s4p 1^3P_1 transitions in Zn-like ions, evaluated in first and second orders of RMBPT. The RMBPT values of oscillator strengths are compared with other theoretical results (MCDF method) [36] and (MCRRPA method) [32, 39, 40]. Experimental (expt) wavelengths are from [36] : a, and [79] : b.

<table>
<thead>
<tr>
<th>Z</th>
<th>RMBPT Wavelengths (Å)</th>
<th>expt</th>
<th>RMBPT First order</th>
<th>MCDF</th>
<th>MCRRPA</th>
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<tbody>
<tr>
<td>4s^2 1S_0–4s4p 1^3P_1 transition</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>47</td>
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<td>244.310^a</td>
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<td>1.334</td>
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<tr>
<td>48</td>
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<td>230.045^b</td>
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<td>51</td>
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<tr>
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<tr>
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<td>54</td>
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<td>1.114</td>
<td>1.226</td>
<td>1.225</td>
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<tr>
<td>55</td>
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<td>140.531^b</td>
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<td>77.460^a</td>
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<td>1.116</td>
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</table>

Wavelengths for the 4s^2 1S_0–4s4p 1^3P_1 transitions are compared in two columns of Table 6. Our RMBPT values are compared with experimental (expt) wavelengths from [36] and [79]. The uncertainties differ by a factor of 20 for some ions. No uncertainties were given in [36]. The difference between our RMBPT and experimental wavelengths is about 0.01–0.05% and below the uncertainties in experimental measurements. We note that the difference between MCDF [36] and experimental wavelengths is about 1%.

3.3. Ground state static polarizabilities in Zn-like ions

The electric-dipole static polarizability \( \alpha_0 \) of the level \( \|aJ \rangle \) is defined as [102]

\[
\alpha_0(aJ) = \frac{2}{3(2J + 1)} \sum_n \left| \langle aJ \| D \| nJ' \rangle \right|^2 \left( E(aJ) - E(nJ') \right).
\]

(5)

Here, \( \langle aJ \| D \| nJ' \rangle \) is the coupled electric-dipole matrix element defined by equation (4) and \( E(aJ) \) is the energy of the
The following designations are used in this table:

- First order RMBPT values are listed for comparison.

Since the value of \( \alpha \) to the polarizability of the \( 4s^{2}1S_{0} \) ground state in Zn-like krypton comes from the \( 4s^{2}1S_{0} \rightarrow 4s4p^{1}P_{1} \) transition, it was measured in the first-order approach. However, our final value, 2.56 au, is lower by 5%. More accurate configuration + all-order calculations [103] that include correlations in the more complete way are needed to resolve the discrepancy.

In table 8, we list dipole polarizabilities of the \( 4s^{2}1S_{0} \) ground state in Zn-like ions with \( Z = 33-47 \). We compare our RMBPT values with theoretical results given in [89]. In that paper, the polarizabilities were calculated using the lifetime measurements for the lowest resonance transition. It was underlined in [89] that, alternatively, measurements of the polarizabilities can be used to deduce lifetimes. Therefore, we include in table 8 our RMBPT results for the lifetimes of the \( 4s4p^{1}P_{1} \) level and oscillator strengths for the \( 4s^{2}1S_{0} \rightarrow 4s4p^{1}P_{1} \) transitions.
polarizability measurement of Lundeen and Fehrenbach [88], lifetime value used in [89] was obtained directly from the relatively low-$Z$ between the polarizability results for remaining relatively low-$Z$ ions. Since the lifetime value used in [89] was obtained directly from the polarizability measurement of Lundeen and Fehrenbach [88], it is expected that the polarizability value of [89] exactly agrees with the measurement of [88], since it is based directly on this measurement (see also [67]). We already addressed that discrepancy in the previous paragraph. The differences between the polarizability results for remaining relatively low-$Z$ ions ($Z = 33–47$) vary but either within or close to the uncertainties quoted in [89]. Therefore, the agreement for these ions is very good. For higher $Z$ ions, we observe a systematic increase between our polarizability values and that of [89]. We note that all of these higher-$Z$ values are based on extrapolated lifetime data owing to lack of measured data. It would be expected that the accuracy of the extrapolated data decreases for higher $Z$. However, we find excellent agreement (0–2%) between both results for the lifetime and oscillator strengths shown in this table for Zn-like ions with $Z = 37–47$. Therefore, the systematic increase of the differences between the polarizabilities may be due to the procedure used in [89] to deduce polarizabilities from the lifetime results. This difference may also be due to increased relativistic effects contributing to the $\alpha_0(4s^{2}1\text{S}_0)$ polarizability in high-$Z$ ions. Our calculations are intrinsically relativistic, and we expect the accuracy of our values to actually improve for higher $Z$ owing to decreased correlation corrections.

Contribution of relativistic effects is illustrated in figure 6 where we plot electric-dipole polarizabilities of the $4s^{2}1\text{S}_0$ ground state in Zn-like ions as functions of $Z$. Together with the $\alpha_0(4s^{2}1\text{S}_0)$ value (curve ‘3’), we illustrate contributions of two channels: the $4s^{2}1\text{S}_0-4snp\text{1P}_1$ and $4s^{2}1\text{S}_0-4snpp\text{1P}_1$ transitions, where $n = 4$ and 5. Those channels give numerical values for the following terms: 

\[
[J(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]
\]

(see equation (7)), described by curves ‘1’ and ‘2’, respectively. The contribution of $[I(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term increases with $Z$ and for $Z > 87$ even became larger than the $[J(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term. The $[I(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term needs to be included in calculation of the $4s^{2}1\text{S}_0$ ground state polarizability in Zn-like ions with $Z > 37$ if we want to guarantee 1% accuracy. That term was not considered in [89].

We already mentioned that the largest contribution to the ground state $4s^{2}$ polarizabilities in Zn-like ions comes from the $4s$-$4p$ one-electron transition $4s^{2}1\text{S}_0-4snp\text{1P}_1$. To estimate the effect of the continuum contributions, we carried out additional calculations of the ground state $4s$ polarizabilities in Cu-like ions. We find that the continuum contributes less than 0.2–0.3% to the value of the ground state $4s$ polarizabilities in Cu-like ions. We note that Sr $5s^2$ ground state polarizability has been studied in detail in [104, 105] using a combination of the configuration interaction and many-body perturbation theory. The continuum contribution has been included in those works. The same ($n = 5$) single transition was found to give overwhelmingly dominant contribution to the $4s^{2}1\text{S}_0$ polarizability. However, we find excellent agreement between the polarizability results for remaining relatively low-$Z$ ions ($Z = 33–47$) vary but either within or close to the uncertainties quoted in [89]. Therefore, the agreement for these ions is very good. For higher $Z$ ions, we observe a systematic increase between our polarizability values and that of [89]. We note that all of these higher-$Z$ values are based on extrapolated lifetime data owing to lack of measured data. It would be expected that the accuracy of the extrapolated data decreases for higher $Z$. However, we find excellent agreement (0–2%) between both results for the lifetime and oscillator strengths shown in this table for Zn-like ions with $Z = 37–47$. Therefore, the systematic increase of the differences between the polarizabilities may be due to the procedure used in [89] to deduce polarizabilities from the lifetime results. This difference may also be due to increased relativistic effects contributing to the $\alpha_0(4s^{2}1\text{S}_0)$ polarizability in high-$Z$ ions. Our calculations are intrinsically relativistic, and we expect the accuracy of our values to actually improve for higher $Z$ owing to decreased correlation corrections.

Contribution of relativistic effects is illustrated in figure 6 where we plot electric-dipole polarizabilities of the $4s^{2}1\text{S}_0$ ground state in Zn-like ions as functions of $Z$. Together with the $\alpha_0(4s^{2}1\text{S}_0)$ value (curve ‘3’), we illustrate contributions of two channels: the $4s^{2}1\text{S}_0-4snp\text{1P}_1$ and $4s^{2}1\text{S}_0-4snpp\text{1P}_1$ transitions, where $n = 4$ and 5. Those channels give numerical values for the following terms: 

\[
[J(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]
\]

(see equation (7)), described by curves ‘1’ and ‘2’, respectively. The contribution of $[I(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term increases with $Z$ and for $Z > 87$ even became larger than the $[J(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term. The $[I(4s4p\text{1P}_1) + I(4s4p\text{1P}_1)]$ term needs to be included in calculation of the $4s^{2}1\text{S}_0$ ground state polarizability in Zn-like ions with $Z > 37$ if we want to guarantee 1% accuracy. That term was not considered in [89].

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the polarizability. Replacement of its contribution by the value determined from the precision experiment modified the final result by about 2.5% which exceeds the continuum contribution. Therefore, the main uncertainty in the ground state polarizability calculation comes from the uncertainty in the $E1$ matrix element for the dominant transition.

4. Conclusion

We have presented a systematic second-order relativistic MBPT study of the reduced matrix elements, oscillator strengths and transition rates for the $4s-4p, 4p-4d, 4d-4f$ electric-dipole transitions in zinc-like ions with the nuclear charge $Z$ ranging from 33 to 100. Our retarded $E1$ matrix elements include correlation corrections from Coulomb and Breit interactions. Both length and velocity forms of the matrix elements were evaluated, and small differences, caused by the non-locality of the starting DF potential, were found between the two forms. Contributions from negative energy states were also included in order to improve the agreement between results calculated in length and velocity gauges. Second-order RMBPT transition energies were used in our evaluation of the oscillator strengths and transition rates. Ground state scalar $\alpha_0(4s^2 1S_0)$ polarizabilities were calculated for Zn-like ions ($Z = 33–100$). To evaluate the $\alpha_0(4s^2 1S_0)$ polarizabilities, we calculate RMBPT energies for the odd-parity $4f 5l'$ complex with $j = 1$ and line strengths between the even-parity $4f 4l'$ complex with $j = 0$ and the odd-parity $4f 5l', 4f 6l'$ complexes with $j = 1$. These calculations are compared with other calculations and with available experimental data. For $Z \geq 33$, our data give accurate benchmark values for transition properties of Zn-like ions.

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