Practical realization of Quantum Computation
Superconducting qubits

Superconductivity is a phenomenon occurring in certain materials at extremely low temperatures, characterized by exactly zero electrical resistance and the exclusion of the interior magnetic field (the Meissner effect).

A magnet levitating above a high-temperature superconductor, cooled with liquid nitrogen. Persistent electric current flows on the surface of the superconductor, acting to exclude the magnetic field of the magnet (the Meissner effect). This current effectively forms an electromagnet that repels the magnet.
Superconducting qubits – a timeline

1911: Heike Kamerlingh Onnes – Superconductivity in He

1933: Walter Meissner – Meissner effect

1957: Schnirman et al. – theoretical proposal for JJ qubits

1962: Supercurrent through a non-superconducting gap

1997: Bardeen, Cooper, Schrieffer – Theory of Superconductivity

1998: Devoret group (Saclay) – first Cooper Pair Box qubit

1999: Nakamura, Tsai (NEC) – Rabi oscillations in CPB

2000: Lukens, Han (SUNY SB) – Flux qubit

2002: Martinis (NIST) – phase qubit

2006: Martinis (UCSB) – two-qubit gate (87% fidelity)

Conductors

In a normal conductor, an electrical current may be visualized as a fluid of electrons moving across a heavy ionic lattice. The electrons are constantly colliding with the ions in the lattice. During each collision some of the energy carried by the current is absorbed by the lattice and converted into heat (which is essentially the vibrational kinetic energy of the lattice ions.) As a result, the energy carried by the current is constantly being dissipated. This is the phenomenon of electrical resistance.
Superconductivity

The electrical resistivity of a metallic conductor decreases gradually as the temperature is lowered. However, in ordinary conductors such as copper and silver, impurities and other defects impose a lower limit. Even near absolute zero a real sample of copper shows a non-zero resistance.

The resistance of a superconductor, on the other hand, drops abruptly to zero when the material is cooled below its "critical temperature", typically 20 kelvin or less. An electrical current flowing in a loop of superconducting wire can persist indefinitely with no power source. Like ferromagnetism and atomic spectral lines, superconductivity is a quantum mechanical phenomenon. It cannot be understood simply as the idealization of "perfect conductivity" in classical physics.

Superconductivity - the persistence of the resistantless electric currents.

In a conventional superconductor, the electronic fluid cannot be resolved into individual electrons. Instead, it consists of bound pairs of electrons known as Cooper pairs. This pairing is caused by an attractive force between electrons from the exchange of phonons.

- Singe electrons - the wave function is antisymmetric under exchange
- Cooper pairs - the wave function is symmetric under exchange
Superconductivity

Normally electrons do not form pairs as they repel each other. However, inside the material the electrons interact with ions of the crystal lattice. Very simply, the electron can interact with the positively charged background ions and create a local potential disturbance which can attract another electron.

The binding energy of the two electrons is very small, 1meV, and the pairs dissociate at higher temperatures.

At low temperatures, the electrons can exist in the bound states (from Cooper pairs).

From BCS theory we learn that the lowest state of the system is the one in which Cooper pairs are formed.

“Conventional” superconductivity is described by Bardeen-Cooper-Schrieffer (BCS) theory: in normal metals the electrons behave as fermions, while in superconducting state they form “Cooper pairs” and behave like bosons.

Fermions

\[ - \quad - \quad \text{Singe electrons- the wave function is antisymmetric under exchange} \]

Bosons

\[ - \quad - \quad \text{Cooper pairs - the wave function is symmetric under exchange} \]
Superconductivity

In a given superconductor all of the Cooper pairs of electrons (which have charge $2e$, mass $2m_e$ and spin zero, and are responsible for carrying a supercurrent) are condensed into a single macroscopic state described by a wave function $\Psi(r, t)$ (where $r$ is the spatial variable and $t$ is time.)

Due to quantum mechanics, the energy spectrum of this Cooper pair fluid possesses an energy gap, meaning there is a minimum amount of energy $\Delta E$ that must be supplied in order to excite the fluid. Therefore, if $\Delta E$ is larger than the thermal energy of the lattice (given by $kT$, where $k$ is Boltzmann's constant and $T$ is the temperature), the fluid will not be scattered by the lattice. The Cooper pair fluid is thus a superfluid, meaning it can flow without energy dissipation.

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Superconductivity

- Singe electrons- only one electron can occupy a particular state
- Cooper pairs – the above restriction no longer applies as electron pairs are bosons and very large number of pairs can occupy the same state

1. Therefore, the electron pairs do not have to move from an occupied state to unoccupied one to carry current.

2. The normal state is an excited state which is separated from the ground state (in which electrons form Cooper pairs) by an energy gap. Therefore, electrons do not suffer scattering which a source of resistance as there is an energy gap between their energy and the energies of the states to which they can scatter.
Superconductors and critical temperature

Flux quantization in superconductors

We consider a superconductor in form of a hollow cylinder which is placed in an external magnetic field, which is parallel to the axis of the cylinder. The magnetic field is expelled from the superconductor (Meissner effect) and vanishes within it. Therefore, Cooper pairs move in the region of $B=0$.

If the wave function of the Cooper pair in the absence of the field is $\psi^{(0)}$, then in the presence of the field we have

$$\psi'(r) = \psi^{(0)}(r)e^{i\int_{0}^{r} A(\tau) \, d\tau}$$
Flux quantization in superconductors

\[ \psi'(r) = \psi^{(0)}(r)e^{\frac{i2e\Phi}{\hbar}} \]

When we consider a closed path \( S \) around the cylinder which starts at point \( r_0 \) we get

\[ \psi'(r) = \psi^{(0)}(r)e^{\frac{i2e\Phi}{\hbar}} = \psi^{(0)}(r)e^{i2e\Phi/h} \]

As the electron wave function should not be multivalued as we go around the cylinder we get the condition

\[ \frac{2e\Phi}{\hbar} = 2n\pi \rightarrow \Phi = \frac{n\pi \hbar}{e}, \quad n = 0, 1, 2, \ldots \]

And the flux enclosed by the superconducting cylinder (or ring) is quantized!

This effect has been experimentally verified which confirmed that the current in superconductors is carried by the pair of the electrons and not the individual electrons.

How this effect can be used?

The main attraction of the Aharonov-Bohm effect is the possibility to use it in switching devices, i.e. to use the change in magnetic filed to change the state of the device from 0 to 1.

How much do we have to change the magnetic field to switch from the constructive to destructive electron interference?

\[ \Delta \Phi = \frac{\pi \hbar}{e} \]
\[ \Delta B = \frac{\pi \hbar}{eA} \approx \frac{\pi \times 1.05 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.6 \times 10^{-19} \text{ C})(20 \times 10^{-6} \text{ m}^2)} \approx 5.1 \times 10^{-6} \text{ T} \]

This is a very small field! The Earth’s magnetic field is about 40\( \mu \)T. It is very difficult to practically use.
Josephson junction

Josephson junction: a thin insulator sandwiched between two superconductors

\[ J = J_0 \sin \delta \]

phase difference \( \delta = \theta_2 - \theta_1 \)

Depends on the tunneling probability of the electron pairs

There is a current flow across the junction in the absence of an applied voltage!

Superconducting devices

Extremely interesting devices may be designed with a superconducting loop with two arms being formed by Josephson junctions.

The operation of such devices is based on the fact that the phase difference around the closed superconducting loop which encloses the magnetic flux \( \Phi \) is an integral product of \( \frac{2e \Phi}{\hbar} \).

The current will vary with \( \Phi \) and has maxima at \( \frac{e \Phi}{\hbar} = n\pi \).

The control of the current through the superconducting loop is the basis for many important devices. Such loops may be used in production of low power digital logic devices, detectors, signal processing devices, and extremely sensitive magnetic field measurement instruments.

SQUID magnetometer (Superconducting QUantum Interference Device)
This promising implementation of quantum information involves nanofabricated superconducting electrodes coupled through Josephson junctions. Possible qubits are charge qubits, flux qubits, and phase qubits.

Experiments on superconducting qubits are challenging!

Most superconducting qubits are:

1. created by using electron–beam lithography $0.1 \times 0.1 \, \mu m^2$
2. need millikelvin temperatures and an ultralow-noise environment to operate,
3. and can be studied only by using very sensitive measurement techniques.

Great efforts are made to attenuate external electrical and magnetic noise. The experiment is invariably enclosed in a Faraday cage — either a shielded room or the metal Dewar of the refrigerator with a contiguous metal box on top. The electrical leads that are connected to the qubits and their read-out devices are heavily filtered or attenuated.
JJ creates the potential of a tilted washboard. In the absence of fluctuations, for $I < I_0$ the particle remains trapped in one of the potential wells.

Flux qubit

The qubit representation is a quantum of current (flux) moving either clockwise or counter-clockwise around the loop.

Fig. 1 Three-junction flux qubit: (a) Schematics. The gray part is for the readout. (b) Scanning-electron micrograph. The larger loop with two big junctions is a SQUID for readout.
Flux qubits

In quantum computing, flux qubits (also known as persistent current qubits) are micrometer sized loops of superconducting metal interrupted by a number of Josephson junctions. The junction parameters are engineered during fabrication so that a persistent current will flow continuously when an external flux is applied.

The computational basis states of the qubit are defined by the circulating currents which can flow either clockwise or counterclockwise.

The state of the flux qubit is measured with a d.c. superconducting quantum interference device (SQUID).

Flux qubits

- Computational operations are performed by pulsing the qubit with microwave frequency radiation which has an energy comparable to that of the gap between the energy of the two basis states.
- Properly selected frequencies can put the qubit into a quantum superposition of the two basis states, subsequent pulses can manipulate the probability weighting that qubit will be measured in either of the two basis states, thus performing a computational operation.
In quantum computing, a charge qubit is a superconducting qubit whose basis states are charge states (i.e., states which represent the presence or absence of excess Cooper pairs in the island).

A charge qubit is formed by a tiny superconducting island (also known as a Cooper-pair box) coupled by a Josephson junction to a superconducting reservoir (see figure). The state of the qubit is determined by the number of Cooper pairs which have tunneled across the junction. In contrast with the charge state of an atomic or molecular ion, the charge states of such an "island" involve a macroscopic number of conduction electrons of the island. The quantum superposition of charge states can be achieved by tuning the gate voltage $U$ that controls the chemical potential of the island. The charge qubit is typically read-out by electrostatically coupling the island to an extremely sensitive electrometer such as the radio-frequency single-electron transistor.

(a) A single Cooper–pair–box (SCB) circuit is shown. The superconducting island is depicted in brown and the junction in blue. $E_J$ and $C_J$ are the Josephson coupling energy and self-capacitance, respectively, and $n$ is the number of Cooper pairs on the island, which is coupled to a voltage source with voltage $V_g$ by way of a capacitor with capacitance $C_g$.

(b) Micrograph of a Cooper–pair box coupled to a single–electron transistor (SET) is shown. Scale bar, 1 $\mu$m.
One-qubit device can control the number of Cooper pairs of electrons in the box, create superposition of states. Superconducting device, operates at low temperatures (30 mK).

Nakamura et al., Nature, 398(786), 1999

Two-qubit device

Pashkin et al., Nature, 421(823), 2003
Controllably coupled flux qubits

Two flux qubits are shown surrounded by a d.c. SQUID. The qubit coupling strength is controlled by the pulsed bias current \( I_{pb} \) that is applied to the d.c. SQUID before measuring the energy-level splitting between the states \(|0\rangle\) and \(|1\rangle\).

Because the flux qubit is a magnetic dipole, two neighboring flux qubits are coupled by magnetic dipole–dipole interactions.

Two flux qubits can be coupled by flux transformers — in essence a closed loop of superconductor surrounding the qubits — enabling their interaction to be mediated over longer distances. Because the superconducting loop conserves magnetic flux, a change in the state of one qubit induces a circulating current in the loop and hence a flux in the other qubit.

Scheme of nanowire-based qubits

Scheme of nanowire-based qubits. A nanowire bridging two superconductors substitutes the insulating barrier used in conventional qubits to form a Josephson Junction. These hybrid semiconducting-superconducting schemes could help researchers design flexible quantum computing architectures that combine multiple types of qubits.
"Scalable physical system with well-characterized qubits"

The system is physical – it is a microfabricated device with wires, capacitors and such. The system is in principle quite scalable. Multiple copies of a qubit can be easily fabricated using the same lithography, etc.

But, the qubits can never be made perfectly identical (unlike atoms). Each qubit will have slightly different energy levels; qubits must be characterized individually.

http://courses.washington.edu/bbbteach/576/

"ability to initialize qubit state"

Qubits are initialized by cooling to low temperatures (mK) in a dilution refrigerator. This is how:

Energy splittings between qubit states are of the order of $f = 1 - 10 \text{ GHz}$ (which corresponds to $T = hf/k_B = 50 - 500 \text{ mK}$)

If the system is cooled down to $T_0 = 10 \text{ mK}$, the ground state occupancy is, according to Boltzmann distribution:

$$P_{|0\rangle} = \exp(-hf/k_BT_0) = 0.82 - 0.98$$

Lower temperature dilution refrigerators mean better qubit initialization!
“(Relative) long coherence times

Many sources of noise (it’s solid state!)

Table 1 | Highest reported values of $T_1$, $T_2^*$ and $T_2$

<table>
<thead>
<tr>
<th>Qubit</th>
<th>$T_1$ (µs)</th>
<th>$T_2^*$ (µs)</th>
<th>$T_2$ (µs)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux</td>
<td>4.6</td>
<td>1.2</td>
<td>9.6</td>
<td>Y. Nakamura, personal communication</td>
</tr>
<tr>
<td>Charge</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>ref. 77</td>
</tr>
<tr>
<td>Phase</td>
<td>0.5</td>
<td>0.3</td>
<td>0.5</td>
<td>J. Martinis, personal communication</td>
</tr>
</tbody>
</table>

“universal set of quantum gates”

Single qubit gates: applying microwaves (1 – 10 GHz) for a prescribed period of time.

Two-qubit gates: via capacitive or inductive coupling of qubits.

*Science 313, 1432 (2006)* – entanglement of two phase qubits (Martinis’ group – UCSB)
“qubit-specific measurement”

Measurement depends on the type of qubit.

Charge qubit readout: amplifier with bimodal response corresponding to the state of the qubit.

Flux and phase qubits readout: built-in DC-SQUID that detects the change of flux.

Superconducting qubits – pros and cons

- Cleanest of all solid state qubits.
- Fabrication fairly straightforward, uses standard microfab techniques
- Gate times of the order of ns (doable!)
- Scaling seems straightforward

- Need dilution refrigerators (and not just for noise reduction)
- Difficult to couple to flying qubits
- Longer coherence needed, may be impossible
Superconducting qubits – what can we expect in near term?

• More research aimed at identifying and quantifying the major source(s) of decoherence.

• Improved control of the electromagnetic environment – sources, wires, capacitors, amplifiers.

• Integration of the qubit manipulation electronics (on the same chip as the qubits themselves).

• Connection to “flying” qubits