NMR
Quantum dots
Summary of trapped atom and ion schemes

Practical realization of Quantum Computation
Lecture 18
Back to the real world:

What do we need to build a quantum computer?

- **Qubits** which retain their properties. **Scalable** array of qubits.
- **Initialization**: ability to prepare one certain state repeatedly on demand. Need continuous supply of $|0\rangle$.
- **Universal set of quantum gates.** A system in which qubits can be made to evolve as desired.
- **Long relevant decoherence times.**
- Ability to efficiently **read out the result**.
QC implementation proposals

Bulk spin Resonance (NMR)
- Linear optics
- Cavity QED

Optical
- Atoms

Solid state

Trapped ions
- Optical lattices

Electrons on He
- Nuclear spin qubits
- Electron spin qubits
- Orbital state qubits

Semiconductors

Superconductors
- Flux qubits
- Charge qubits

http://courses.washington.edu/bbbteach/576/
Experimental proposals

- Liquid state NMR
- Trapped ions
- Cavity QED
- **Trapped atoms: summary**
- Solid state schemes
- And other ones …
1. A scalable physical system with well characterized qubits: memory

Internal atomic state qubits:
Ground hyperfine states of neutral trapped atoms
Well characterized
Very long lived!

\[ \begin{align*}
5s_{1/2} & \quad F = 2 \quad |1\rangle \\
 & \quad F = 1 \quad |0\rangle \\
6.8 \text{ GHz} & \\
M_F = -2, -1, 0, 1, 2 \\
87\text{Rb}: \text{Nuclear spin } I = 3/2 \\
M_F = -1, 0, 1
\end{align*} \]
2: Initialization

Internal state preparation: putting atoms in the ground hyperfine state using optical pumping technique.

- Very well understood (is in use since 1950)
- Very reliable (>0.9999 population may be achieved)

Loading with one atom per site: Mott insulator transition and other schemes.

3. Single-qubit gates:

- Just a population transfer between two levels using laser excitations. It is well understood and had been carried out in atomic spectroscopy since 1940’s.
3: Example of two-qubit gate Rydberg gate scheme

\[ \Delta \]

Rb

5s

40p

\[ |R\rangle \quad |1\rangle \quad |0\rangle \]

\[ |\pi[1]\rangle \quad |2\pi[2]\rangle \quad |\pi[1]\rangle \]

\[ |00\rangle \rightarrow |00\rangle \rightarrow -|00\rangle \rightarrow |00\rangle \]

\[ |01\rangle \rightarrow |01\rangle \rightarrow -|01\rangle \rightarrow -|01\rangle \]

\[ |10\rangle \rightarrow |R0\rangle \rightarrow |R0\rangle \rightarrow -|10\rangle \]

\[ |11\rangle \rightarrow |R1\rangle \rightarrow |R1\rangle \rightarrow -|11\rangle \]
4. Long relevant decoherence times

Memory: long-lived states.

Fundamental decoherence mechanism for optically trapped qubits: photon scattering.

Decoherence during gate operations: a serious issue.

5: Reading out a result

“Quantum jump” method via cycling transitions.

Advantages: standard atomic physics technique, well understood and reliable.
Experimental proposals

- Liquid state NMR
- **Trapped ions: summary**
- Cavity QED
- Trapped atoms
- Solid state schemes
- And other ones …
Encoding of quantum information requires long-lived atomic states:

- **Optical transitions**
  
  \[ \text{Ca}^+, \text{Sr}^+, \text{Ba}^+, \text{Ra}^+, \text{Yb}^+, \text{Hg}^+ \text{ etc.} \]

- **Microwave transitions**
  
  \[ 9\text{Be}^+, 25\text{Mg}^+, 43\text{Ca}^+, 87\text{Sr}^+, 137\text{Ba}^+, 111\text{Cd}^+, 171\text{Yb}^+ \]

\[ \begin{array}{c}
\text{P}_{1/2} \\
\text{D}_{5/2} \\
\text{S}_{1/2}
\end{array} \]

\[ \begin{array}{c}
\text{P}_{3/2} \\
\text{S}_{1/2}
\end{array} \]
A row of qubits in a linear Paul trap forms a quantum register.

\[ \omega_z \approx 0.7 - 2 \text{ MHz} \]
\[ \omega_{x,y} \approx 1.5 - 4 \text{ MHz} \]

String of Ca+ ions in linear Paul trap
2. Initialization: sideband cooling

External degree of freedom: ion motion

We can walk down the double ladder by exciting the red sideband and returning the ion dissipatively to the ground state. With this we can prepare the ions in the motional ground state with high probability, thereby initializing our quantum register.

Laser cooling to the motional ground state:

Cooling time: 5-10 ms
> 99% in motional ground state
Let's now begin to look at the coherent state manipulation. If we resonantly shine the light pulse at the carrier transition, the system evolves for a time $\tau$ with this Hamiltonian, where the coupling strength $\Omega$ depends on the square root of the intensity, and $\phi$ is the phase of the laser field with respect to the atomic polarization.

The effect of such a pulse is a rotation of the state vector on the Bloch sphere, where the poles represent the two states and the equator represents superposition states with different relative phases. The rotation axis is determined by the laser frequency and phase. The important message is here that we can position the state vector anywhere on the Bloch sphere, which is a way of saying that we can create arbitrary superposition states.

The same game works for sideband pulses. With a $\pi/2$ pulse, for example, we entangle the internal and the motional state! Since the motional state is shared by all ions, we can use the motional state as a kind of bus to mediate entanglement between different qubits in the ion chain.
Coherent excitation: Rabi oscillations

„Carrier“ pulses:

\[ |S\rangle \leftrightarrow |D\rangle \]

\[ \theta/2 = \Omega \tau \]

\[ \tau \geq \frac{2\pi}{\Omega} \]

D state population
3: Cirac - Zoller two-ion controlled-NOT operation

Ion 1 $|D\rangle$, $|S\rangle$

Motion $|0\rangle$

Ion 2 $|D\rangle$, $|S\rangle$

SWAP

SWAP$^{-1}$

Control qubit

Target qubit

Pulse sequence:

Laser frequency

Pulse length

Optical phase

Phase gate
## 4: Different decoherence mechanisms

<table>
<thead>
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<th>State Combination</th>
<th>Laser Frequency</th>
<th>Magnetic Field</th>
<th>Exc. State Lifetime</th>
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<td>$</td>
<td>S\rangle +</td>
<td>D\rangle$</td>
<td>×</td>
</tr>
<tr>
<td>$</td>
<td>SD\rangle +</td>
<td>DS\rangle$</td>
<td>×</td>
</tr>
<tr>
<td>$</td>
<td>SS'\rangle +</td>
<td>S'S\rangle$</td>
<td>×</td>
</tr>
<tr>
<td>$</td>
<td>SS'\rangle +</td>
<td>DD'\rangle$</td>
<td>×</td>
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5: Measurement

1. Initialization in a pure quantum state: Laser sideband cooling

2. Quantum state manipulation on $S_{1/2} - D_{5/2}$ transition

3. Quantum state measurement by fluorescence detection

Multiple ions:

Spatially resolved detection with CCD camera:

$|SSS\rangle$

$|SSD\rangle$

$|SDS\rangle$

... 50 experiments / s

Repeat experiments 100-200 times
What is NMR (Nuclear magnetic resonance)?

NMR: study of the transitions between the Zeeman levels of an atomic nucleus in a magnetic field.

- **If** put a system of nuclei with non-zero spin in static magnetic field, there will be different energy levels.

- **If** radio frequency wave with $\omega_{RF} = \omega_0$ is applied, transitions will occur.

\[
\hbar \omega_{RF} = \hbar \omega_0 = \Delta E
\]

\[
\hbar \omega_0 = E_2 - E_1
\]
Nuclear magnetic resonance

NMR is one of the most important spectroscopic techniques available in molecular sciences since the frequencies of the NMR signal allow study of molecular structure.

Basic idea: use **nuclear spins of molecules as qubits**

![Diagram of a molecule with nuclear spins indicated.](image)

Liquid NMR: rapid molecular motion in fluids greatly simplifies the NMR Hamiltonian as anisotropic interactions can be replaced by their isotropic averages, often zero. The signals are often narrow (high-resolution NMR).

It is impossible to pick individual molecule.

**The combined signal from all molecules** is detected ("bulk" NMR).
Billions of molecules are used, each one a separate quantum computer.

Most advanced experimental demonstrations to date, but poor scalability as molecule design gets difficult.

Qubits are stored in nuclear spin of molecules and controlled by different frequencies of magnetic pulses.

Used to factor 15 experimentally.  

Vandersypen, 2000
Liquid State NMR

Cory & Havel PNAS, 64, 1634, 1997
Gershenfeld & Chuang, Science 275, 350, 1997

- Larmor Frequency ~ 500MHz
- Single bit gate: 1/ ~ ms
- Two qubit gate: ~ 10ms
  - $z^1 \ z^2$ interaction
- $T_2 \sim 1s$
- $T_1 \sim 5-30s$
- $=e^{-} \ H \sim 1-\ H$

Bruker DRX-500
Choosing a molecule

- different chemical shifts
- large decoherence times
- strong couplings

Rudy Martinez, B-2
Stable Isotope Laboratory at Los Alamos

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<td>2.3</td>
<td>7.1</td>
<td>1.4</td>
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**Bulk NMR**

**Major problem:** it is very difficult to place an NMR quantum computer in a well defined state.

**Advantage:** Extensive technical development of NMR spectrometers (very complex but easily controlled devices).

**Quantum gates**

**One qubit gates:** rotation of a single spin within its own Hilbert space is done using RF (radio frequency) fields.

**Two-qubit gates:** use scalar coupling (J-coupling) between two nuclei

\[ H = J_{IS} I_Z S_Z \]
Typically, readout in quantum computing is reading values of one or more qubits which finish the calculation in terms of eigenstates $|0\rangle$ or $|1\rangle$.

It is not practical in NMR. Instead, 90° excitation pulse is applied to create superposition of $|0\rangle$ and $|1\rangle$ which then oscillates in a magnetic field. The relative population can then be determined by observing size and phase of the oscillatory signal. It is possible to include reference signal to include relative phases.

It is possible in certain cases to see result directly from NMR signal.
Quantum computation with chloroform NMR

Deutsch algorithm demonstrated.

Experimental realization of a quantum algorithm

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Quantum computers\(^1\text{-}^5\) can in principle exploit quantum-mechanical effects to perform computations (such as factoring large numbers or searching an unsorted database) more rapidly than classical computers\(^1\text{-}^8,^9\). But noise, loss of coherence, and manufacturing problems make constructing large-scale quantum computers difficult\(^9\text{-}^{13}\). Although ion traps and optical cavities offer promising experimental approaches\(^14,15\), no quantum algorithm has yet been implemented with these systems. Here we
5 qubit 215 Hz Q. Processor

- 5-spin molecule synthesized
- First demonstration of a fast 5-qubit algorithm
- Pathway to more qubits
Solid State Systems
$10^5$ $^{29}\text{Si}$ atomic chains in $^{28}\text{Si}$ matrix work like molecules in solution NMR QC. Many techniques used for solution NMR QC are available. No impurity dopants or electrical contacts are needed.

A large field gradient separates Larmor frequencies of the nuclei within each chain.

Fabrication: $^{29}\text{Si}$ Atomic Chain

Regular step arrays on slightly miscut $^{28}\text{Si}(111)7\times7$ surface ($\sim 1^\circ$ from normal)

Steps are straight, with about 1 kink in 2000 sites.

*STM image*

Donor electron spin in Si:P
Donor electron spin in Si:P

Structure

Natural Silicon:

$^{28}\text{Si} - 92\%$

$^{29}\text{Si} - 4.7\% \quad I=1/2$

$^{30}\text{Si} - 3.1\%$

Natural Phosphorus:

$^{31}\text{P} - 100\% \quad I=1/2$

In the effective mass approximation electron wave function is $s$-like:

$$F(r) = \frac{1}{\sqrt{\pi ab}} e^{-\sqrt{(x^2+y^2)/a^2+z^2/b^2}}$$
Recipe for Quantum Computing:

1. Do SWAPs and fractional SWAPs between electron and nuclear spins by accurately controlling the *time* electrons reside on donor states

2. Move electrons between nuclear sites to transmit quantum information.
Kane Quantum Computer

- Semiconductor substrate with embedded electron donors ($^{31}$P)
- Electron wave functions manipulated by changing gate voltages
- Most easily scalable

Potential wells in Kane Quantum Computer
MRS, February 2005, Kane

www.lanl.gov/physics/quantum/
The nuclear spin (I=1/2) of $^{31}$P is a natural two-level system embedded in a spin-free substrate of $^{28}$Si (I=0).

The nuclear spins of $^{31}$P donors are separated by approximately 20 nm and there is a hyperfine interaction between donor electron spin and nuclear spin (qubit).

Interaction between qubits is mediated through the donor-electron exchange interaction.

**Exchange coupling:** basic physical interaction between the spins of electrons whose wave functions overlap, arising from the Pauli exclusion principle.

The spins are maintained at millikelvin (mK) temperatures in an external magnetic field of several Tesla, perpendicular to the plane of the substrate.

Nanoscale surface A and J gates control the hyperfine and exchange interactions at qubit sites.
Nuclear spins alone will not interact significantly with other nuclear spins 20 nm away.

Nuclear spin is useful to perform single-qubit operations, but to make a quantum computer, two-qubit operations are also required.

This is the role of electron spin in this design.

Under A-gate control, the spin is transferred from the nucleus to the donor electron. Then, a potential is applied to the J gate, drawing adjacent donor electrons into a common region, greatly enhancing the interaction between the neighbouring spins. By controlling the J gate voltage, two-qubit operations are possible.
Kane Quantum Computer: qubits

P nucleus
- Spin mediated by electron spin through hyperfine interaction
- Controlled and measured by varying voltages in top gates
- Long decoherence times $\sim 10^{18}$ s
Si (-Si-Ge): P QC Architectures: Spin & Charge Qubits

(a) P

(b) Spin

(c) Charge
Kane Quantum Computer Future

• Further research into semiconductor materials
• Smaller technology while approaching limit by Moore’s law
Quantum dot:

A confining structure for electrons, which can be designed to stably hold a small number of electrons.
What is a Quantum Dot?

• A quantum dot is a structure in which the particles are confined in all spatial dimensions $L_x, L_y, L_z = \frac{1}{4} \lambda$ (DeBroglie wave length).

• The dimensions of the quantum dots range from 10nm to 1µm ($10^3$ to $10^9$ atoms).

• The energy levels in a quantum dot are quantized – they are called sometimes macroscopic atoms.
Depending on the fabrication technique there are different kinds of quantum dots:

- Self-assembled
- Vertical (etched)
- Lateral

A semiconductor heterostructure is a stack of layers made of different materials. Due to the mismatch of the Fermi levels, the electrons are confined at the interface of the two different semiconductors.
The vertical quantum dots can be fabricated by wet etching (chemically washing away the semiconductor layers) or dry etching (reactive ion etching).
Self-assembled QDs on the top of semiconductor layer
The 2DEG is confined by modifying the potential of the gates on the top of the heterostructure.
Qubits in QD

• Spin qubit – one dot

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + |\downarrow\rangle) \]

• Charge qubit – two dots

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \]

• Quantum Ground State qubit – 2 \((N + 1)\) dots for N-step evolution
DiVincenzo Criteria

- Scalable system with well-characterized qubits
- Initialization
- Long decoherence time w.r.t. the gate-operation time
- Universal set of gates
- Read-out of the result

- The qubit is one spin QD, or two charge QDs, or 2(N+1) QDs
- Spin/charge injection
- Decoherence time ~100μsec; gate-operation time ~1nsec
- Only single-qubit gates are demonstrated so far
- Depends on the system
The use of the spins of the electrons confined in quantum dots as qubits was motivated by the fact that spins are natural two-level systems and the possibility of building scalable arrays of quantum dots.

The interaction between neighboring electron spins is time dependent Heisenberg coupling. By controlling the strength of this interaction, we can implement the quantum computing operations.
Quantum-dot array proposal:


- quantum dots defined in 2DEG by side gates
- Coulomb blockade used to fix electron number at one per dot
- spin of electron is qubit
- gate operations: controllable coupling of dots by point-contact gate voltage
- readout by gatable magnetic barrier

/s/divine/lex/reviews/march2000/divines4
Charge density maps in solid state quantum computer
Loss- DiVincenzo proposal

Advantages

• The spins have long decoherence times (up to 1ms) compared to the gate operation times (of the order of 1ns).

• The quantum dot arrays can be fabricated with the current experimental techniques.

• Specific qubits can be addressed in order to perform quantum gate operations.

• Charge control and readout has been achieved in double dots, as well as single shot spin readout.
Disadvantages

- The generation of a local magnetic field for single spin rotations has not been achieved.
- Spin injection in semiconductors is not efficient with the current experimental techniques.
- Decoherence due to the hyperfine coupling with the atoms nuclei in semiconductor is unavoidable.
- The control over the Heisenberg interaction is not yet fully understood.
1. Semiconductor systems (GaAs, Si, SiGe,) offer inherent scalability. Established and new semiconductor patterning processes allow for the construction of submicron 2-D arrays of qubits.

2. Semiconductor systems have compatibility with existing microelectronics industry and have high potential for development of integrated on-chip devices.

3 Spin qubits in semiconductors are well-defined “native” qubits (two-level systems).

4. Spin qubits in semiconductors (single donor or QDs) can be decoupled from charge fluctuations, leading to long decoherence times (from s to ms) compared with practical gate operation times (from ps to ns).

5. Charge qubits in semiconductors (e.g., electron position in double quantum wells) offer potential for extremely fast qubit (ps) operations.
1. Background impurity levels and disorder in semiconductor systems may lead to difficulties in device reproducibility. These issues are common with sub-100-nm devices in conventional microprocessors.

2. For spin qubits—single-spin readout is challenging.

3. For spin qubits in Si—the exchange interaction is predicted to oscillate as a function of donor separation, which may place stringent requirements on nanofabrication accuracy.

4. For charge qubits—decoherence likely to be dominated by voltage fluctuations on control gates and may be fast. Experiments on GaAs dots indicate dephasing times on the order of ns.

5. Most semiconductor-based schemes are based on linear qubit arrays. The extension to 2-D arrays will require via-gate techniques on the sub-100-nm scale, which is challenging.

6. A number of solid-state schemes are still at the conceptual phase. Detailed fabrication strategies still to be developed.