The proton radius puzzle

Proton

Hadron
Charge: +1
Composition: 2 up quarks, 1 down quark
Spin: 1/2
How to measure the (rms) radius of the proton

Method 1: Electron scattering

- Beam of electrons fired at hydrogen atoms
- Scattering pattern used to extract radius of the proton
G_E is the Sachs electric form factor of the proton and Q^2 is the negative of the square of the four-momentum transfer to the proton.

Proton radius determined from Q^2=0

Values of electric and magnetic form factors may be inaccurate due to extrapolation.
**Method 2: Hydrogen spectroscopy**

Hydrogen energy levels are quantized

Non-relativistic quantum mechanics:

- \( n \) is a principal quantum number
- \( L \) is orbital angular momentum quantum number, designated by letters
  - S: \( L=0 \)
  - P: \( L=1 \)
  - D: \( L=2 \)
**Lamb Shift**

Prediction of the relativistic (Dirac equation) theory for hydrogen energy levels:

This contradicts experimental observations which show a small difference between the energy levels of these states. Willis Lamb and Robert Retherford were the first to measure this shift in 1947 (1955 Nobel Prize).

**Quantum Electrodynamics (QED)**

In quantum electrodynamics, so-called "radiative corrections" to the Dirac theory are obtained by taking into account interactions of electrons with the quantized electromagnetic field.

Dominant QED contributions that cause the Lamb shift: **self-energy and vacuum polarization.**
Quantum Electrodynamics (QED)

Feynman diagrams for Lamb shift:
Element #1: Double line represents bound electrons
Element #2: Wavy line represents a virtual photon
Element #3: Double circle represents virtual electron-positron pairs.

Self-energy

Vacuum polarization

Finite radius of the proton → H energy level shifts
Measurement of transitions → measure nuclear size

1. Measure the transition energies between different levels
2. Calculate all corrections to these energies (need to calculate QED really well)
3. Extract the corrections to the energies due to a proton radius $\sim (Z\alpha) R_p^2 |\Psi(0)|^2$
4. Extract the rms radius
5. Repeat for many transitions and average

John Arrington, Argonne National Laboratory
Hydrogen spectroscopy (Lamb shift):

\[ L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz} \]

\[ E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3} \]

2 unknowns \(\Rightarrow\) 2 transitions

- Rydberg constant \(R_\infty\)
- Lamb shift \(L_{1S} \leftarrow r_p\)

Results for proton radius

- Electron scattering
- Slope of \(G_E\) at \(Q^2 = 0\)
- Hydrogen spectro.

Lamb shift (S-states)
Results of the measurements

Electron-proton scattering

Hydrogen spectroscopy

CODATA (2010)

Root-mean-square proton charge radius (femtometers)

Even better way to measure the proton radius: replace electron by a muon

\( \mu \) is 207 times heavier than \( e \)
Even better way to measure the proton radius

Probability for a lepton to be inside the proton $\propto$ to its mass cubed, $(207)^3 = 8\,869\,743$ enhancement for a muon!

Muonic hydrogen experiment
Paul Scherrer Institute (Switzerland)

Scientific American 310, 32 - 39 (2014)
Muonic hydrogen experiment

muonic hydrogen = $\mu^-p$  mass $m_\mu = 207m_e$

Bohr radius $\sim 1/m$

Finite size effects $\sim m^3 \approx 200^3 \approx 10^7$

Lamb shift in $\mu p$: $\Delta E(2P_{1/2} - 2S_{1/2}) = 206.0668(25) - 5.2275(10) \frac{r_p^2}{\hbar^2} [\text{meV}]$


Finite size contribution is 2% of the $\mu p$ Lamb shift

That's huge!

Laser spectroscopy to $10^{-5}$ gives $r_p$ to $10^{-3}$

$$\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4e^4}{3\hbar^2n^3} m_p^3 r_p^2 \delta_0$$

Muonic hydrogen experiment

"prompt" ($t \sim 0$)

$\mu^-$ stop in H$_2$ gas

$\Rightarrow \mu p^*$ atoms formed ($n \sim 1/4$)

99%: cascade to $\mu p(1s)$, emitting prompt $K_\alpha, K_\beta$ ...

1%: long-lived $\mu p(2s)$ atoms

lifetime $\tau_{2s} \approx 1 \mu s$ at 1 mbar H$_2$


"delayed" ($t \sim 1 \mu s$)

fire laser ($\lambda \approx 6 \mu m, \Delta E \approx 0.2 \text{ eV}$)

$\Rightarrow$ induce $\mu p(2s) \rightarrow \mu p(2p)$

$\Rightarrow$ observe delayed $K_{\alpha}$ x-rays

$\Rightarrow$ normalize delayed $K_{\alpha}$ x-rays prompt $K_{\alpha}$
Muonic hydrogen experiment

Timeline:
1997 Experiment proposed
1999 Experiment approved
2002 Assembly completed
2003 First “real” run
2006-2007 Major redesign, new runs
2009 Last chance to run

The proton accelerator at the Paul Scherrer Institute, which was used to create the muons used in this experiment.

Proton radius puzzle

Electron-proton scattering

\[
\{ \quad \text{Hydrogen spectroscopy} \quad \text{CODATA (2010)} \quad \text{Muonic hydrogen spectroscopy} \quad \}
\]

Root-mean-square proton charge radius (femtometers)
“This could be the discovery of the century. Depending, of course, on how far down it goes.”

Possible explanations

\[ \tilde{L}_{\mu p}^{\text{theo.}} (r_p^{\text{CODATA}}) - \tilde{L}_{\mu p}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases} \]

- \(\mu p\) theory wrong?
- \(\mu p\) experiment wrong?
- H theory wrong?
- H experiments wrong? \(\rightarrow R_\infty\) wrong?
- AND e-p scattering exp. wrong?
- Standard Model wrong?!?
The discrepancy is 4 linewidths!


Quantum Electrodynamics (QED)

Feynman diagrams for Lamb shift:

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Higher-order QED: examples

Some contributions to the $\mu P$ Lamb shift

Hydrogen Spectroscopy


New Physics?

2014 Constraints on muon-specific dark forces
2013 Advanced nucleon electromagnetic structure model and charge proton rms radius
2013 Precision Spectroscopy of Atomic Hydrogen
2013 Chiral perturbation theory of muonic hydrogen Lamb shift: polarizability contribution
2013 Effects of Noncommutativity on Light Hydrogen-Like Atoms and Proton Radius
2013 Studying the proton "radius" puzzle with μp elastic scattering
2013 Proton radius puzzle and quantum gravity at the Fermi scale
2013 Constraints to new physics models for the proton charge radius puzzle
from the decay K^+→μ^+ + u + e^- + e^-
2013 New measurements of the proton's size and structure using polarized photons
2013 Proton Radius Puzzle and Large Extra Dimensions
2013 Pure bound field corrections to the atomic energy levels and the proton size puzzle
2013 Proton polarizability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory
2013 Proton Radius, Darwin-Foldy Term and Radiative Corrections
2013 Can Large Extra Dimensions Solve the Proton Radius Puzzle?
2013 Nonidentical protons
2013 No radial excitations in low energy QCD. II. The shrinking radius of hadrons
2013 Proton polarizability contribution: Muonic hydrogen Lamb shift and elastic scattering
2012 Higher-order Corrections in the Proton Radius Extraction
2012 Why Three-Body Physics Does Not Solve the Proton-Radius Puzzle
2012 Proton polarizability contribution to the Lamb shift in muonic hydrogen at fourth order in chiral perturbation theory
New measurements of the 2S-4P transition frequency in H

New measurement of the 1S − 3S transition frequency of hydrogen: contribution to the proton charge radius puzzle, arXiv:1801.08816 (2018)
Conclusion

Proton radius puzzle persists.
New data needed!

Future measurements:

- Lamb shift in regular hydrogen
- Muonic helium
- More hydrogen measurements
- New low-$Q^2$ measurements on the proton
- Mainz low-$Q^2$ measurement on the deuteron
- MUSE (muon scattering experiment) @ PSI