Atomic Clocks and their Applications

Ingredients for a clock

1. Need a system with periodic behavior:
   it cycles occur at constant frequency

2. Count the cycles to produce time interval
3. Agree on the origin of time to generate a time scale

NOAA/Thomas G. Andrews

Ludlow et al., RMP 87, 637 (2015)
Ingredients for atomic clock

1. Atoms are all the same and will oscillate at exactly the same frequency (in the same environment): you now have a perfect oscillator!

2. Take a sample of atoms (or just one)

3. Build a device that produces oscillatory signal in resonance with atomic frequency

4. Count cycles of this signal

valentinagurarie.wordpress.com/tag/atom/  
Ludlow et al., RMP 87, 637 (2015)
**Current definition of a second**

1967: the second has been defined as the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

1997: the periods would be defined for a cesium atom at rest, and approaching the theoretical temperature of absolute zero (0 K).

**New world of ultracold**

1997 Nobel Prize Laser cooling and trapping

2001 Nobel Prize Bose-Einstein Condensation

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300K

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500nK
Making crystals from light: atoms in Optical Lattices

Spin state control

A gas of cesium atoms enters the clock's vacuum chamber. Six lasers slow the movement of the atoms and force them into a spherical cloud at the intersection of the laser beams.

The ball is tossed upward by two lasers through a cavity filled with microwaves. All of the lasers are then turned off.

Gravity pulls the ball of cesium atoms back through the microwave cavity. The microwaves partially alter the atomic states of the cesium atoms.

Cesium atoms that were altered in the microwave cavity emit light when hit with a laser beam. This fluorescence is measured by a detector (right). The entire process is repeated many times while the microwave energy in the cavity is tuned to different frequencies until the maximum fluorescence of the cesium atoms is determined. This point defines the natural resonance frequency of cesium, which is used to define the second.


Cesium atomic clock


NIST cesium atomic clock

ANIMATION: http://www.nist.gov/pml/div688/how-nist-f2-works.cfm
HOW GPS WORKS

GPS uses precise time signals transmitted by satellites to determine the location of a receiver on the surface of the Earth. The signals are transmitted from a constellation of satellites orbiting the Earth. The signals are received by a GPS receiver, which then calculates the location of the receiver by measuring the time it takes for the signals to travel from the satellites to the receiver.


https://www.youtube.com/watch?v=o4gYnbGXD6o

https://www.youtube.com/watch?v=QqLIIEW4ACw
Stability is a measure of the precision with which we can measure a quantity (think of how widely scattered a group of arrows at target might be), and is usually stated as a function of averaging time since for many noise processes the precision increases (i.e., the noise is reduced through averaging) with more measurements.

The stability is usually set by the combination of the inherent frequency purity of the physical system and the signal-to-noise ratio with which we can measure the system.

How good is a clock: uncertainty

In contrast, the (absolute) uncertainty for an atomic clock tells us how well we understand the physical processes that can shift the measured frequency from its unperturbed ("bare"), natural atomic frequency (think of how off-centre our group of arrows might be).

Small absolute uncertainty is clearly an essential part of a good primary frequency standard and requires extensive evaluation of all known physical shifts (usually called "systematic effects").


Clock instability

Let us first consider the formula for clock instability, $\sigma_y$, in the regime where it is limited by fundamental (as opposed to technical) noise sources, such as atomic statistics based on the number of atoms:

$$
\sigma_y(\tau) \approx \frac{\Delta \nu}{\nu_0} \sqrt{\frac{T_c}{\tau}} \sqrt{\frac{1}{N}}
$$

- $\Delta \nu$: spectroscopic linewidth of the clock system
- $\nu_0$: clock transition frequency
- $T_c$: the time required for a single measurement cycle
- $N$: the number of atoms or ions used in a single measurement
- $\tau$: the averaging period
How to build a better clock?

Cesium microwave atomic clock

9 $\times$ 10^9 periods per second

Strontium optical atomic clock

4.3 $\times$ 10^{14} periods per second

http://www.nist.gov/pml/div689/20140122_strontium.cfm
Problem: counting optical frequencies

Fastest electronic counters
\[ \Delta f \sim 10^{11} \text{ Hz} \]

Sr clock
4.3 \times 10^{14} \text{ periods per second}
\[ \Delta f \sim 10^{14} \text{ Hz} \]

Solution:
Femtosecond laser frequency comb

http://www.nist.gov/pml/div689/20140122_strontium.cfm

2005 Nobel Prize
Laser-based precision spectroscopy and the optical frequency comb technique

\[ f_n = n f_{\text{rep}} + f_0 \]

Theodor Hänsch
John Hall

www.laserfocusworld.com
What do we need to build a clock?

atomic oscillator

electronic signal → feedback control → electronic signal

ν
interrogation laser

E2

|2|
hν0

E1

|1|

atomic reference

counter

optical comb

Schematic view of an optical atomic clock: the local oscillator (laser) is resonant with the atomic transition. A correction signal is derived from atomic spectroscopy that is fed back to the laser. An optical frequency synthesizer (optical frequency comb) is used to divide the optical frequency down to countable microwave or radio frequency signals.

Requirements for atomic reference

(1) Metastable level
(2) Near optical transition

Strontium optical lattice neutral atom clock

Yb⁺ single trapped ion clock

Sr clock will lose 1 second in 15 billion years!

Nicholson et al., Nature Comm. 6, 6896 (2015) Sr: $2 \times 10^{-18}$
Optical vs. microwave clocks

Applications of Atomic Clocks

- Improved timekeeping and synchronization capabilities
- Design of absolute gravimeters and gravity gradiometers for geophysical monitoring and research and gravity aided navigation
- Search for variation of fundamental constants
- Search for topological dark matter
- Search for violation of local Lorenz invariance
- Exploration of strongly correlated quantum many-body systems
- Other …
Our understanding of the Universe and its fundamental physics laws is incomplete.

Precision atomic measurements: Do laws of physics hold within the experimental precision?

Precision atomic tests may discover new physics and will constrain new theories.

Being able to compare and reproduce experiments is at the foundation of the scientific approach, which makes sense only if the laws of nature do not depend on time and space.

The modern theories directed toward unifying gravitation with the three other fundamental interactions suggest variation of the fundamental constants in an expanding universe.

### TABLE I
An abbreviated list of the CODATA recommended values of the fundamental constants of physics and chemistry based on the 2014 adjustment.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Numerical value</th>
<th>Unit</th>
<th>Relative std. uncert. (u_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>speed of light in vacuum</td>
<td>(c)</td>
<td>299,792,458</td>
<td>(\text{m s}^{-1})</td>
<td>exact</td>
</tr>
<tr>
<td>magnetic constant</td>
<td>(\mu_0)</td>
<td>(4 \times 10^{-7})</td>
<td>(\text{N} \cdot \text{A}^{-2})</td>
<td>exact</td>
</tr>
<tr>
<td>electric constant (1/\mu_0 e^2)</td>
<td>(e_0)</td>
<td>(= \frac{12,566,370,614... \times 10^{-7}}{4 \pi} )</td>
<td>(\text{F m}^{-1})</td>
<td>exact</td>
</tr>
<tr>
<td>Newtonian constant of gravitation</td>
<td>(G)</td>
<td>(6.67408(31) \times 10^{-11})</td>
<td>(\text{m}^3 \text{kg}^{-1} \text{s}^{-2})</td>
<td>(4.7 \times 10^{-5})</td>
</tr>
<tr>
<td>Planck constant</td>
<td>(h)</td>
<td>(6.626069576(48) \times 10^{-34})</td>
<td>(\text{J} \cdot \text{s})</td>
<td>(1.2 \times 10^{-10})</td>
</tr>
<tr>
<td>(\hbar/2\pi)</td>
<td>(e)</td>
<td>(1.602176680(20) \times 10^{-19})</td>
<td>(\text{C})</td>
<td>(6.6 \times 10^{-9})</td>
</tr>
<tr>
<td>magnetic flux quantum (h/2\pi e)</td>
<td>(\Phi_0)</td>
<td>(2.0678338(13) \times 10^{-15})</td>
<td>(\text{Wb})</td>
<td>(6.1 \times 10^{-9})</td>
</tr>
<tr>
<td>conductance quantum (2e^2/h)</td>
<td>(G_0)</td>
<td>(7.74809130(18) \times 10^{-6})</td>
<td>(\text{S})</td>
<td>(2.3 \times 10^{-10})</td>
</tr>
<tr>
<td>electron mass</td>
<td>(m_e)</td>
<td>(9.10938356(11) \times 10^{-31})</td>
<td>(\text{kg})</td>
<td>(1.2 \times 10^{-8})</td>
</tr>
<tr>
<td>proton mass</td>
<td>(m_p)</td>
<td>(1.672621889(21) \times 10^{-27})</td>
<td>(\text{kg})</td>
<td>(1.2 \times 10^{-8})</td>
</tr>
<tr>
<td>proton-electron mass ratio</td>
<td>(m_p/m_e)</td>
<td>(4030.152783(37))</td>
<td>(\text{kg}^{-1})</td>
<td>(9.5 \times 10^{-11})</td>
</tr>
<tr>
<td>fine-structure constant (e^2/4\pi\varepsilon_0\hbar c)</td>
<td>(\alpha)</td>
<td>(7.292427(17) \times 10^{-3})</td>
<td>(\text{m}^{-1})</td>
<td>(2.3 \times 10^{-10})</td>
</tr>
<tr>
<td>inverse fine-structure constant (\alpha^{-1})</td>
<td>(\alpha^{-1})</td>
<td>(17526034)</td>
<td>(\text{m}^{-1})</td>
<td>(2.3 \times 10^{-10})</td>
</tr>
<tr>
<td>Rydberg constant (e^4\alpha^2/m_e c/2\hbar)</td>
<td>(R_n)</td>
<td>(10,973,731.568(65))</td>
<td>(\text{m}^{-1})</td>
<td>(5.9 \times 10^{-12})</td>
</tr>
<tr>
<td>Avogadro constant (N_A)</td>
<td>(N_A)</td>
<td>(6.022140)</td>
<td>(\text{mol}^{-1})</td>
<td>(1.2 \times 10^{-9})</td>
</tr>
<tr>
<td>Faraday constant (N_A e)</td>
<td>(F)</td>
<td>(96,485,332.89(59))</td>
<td>(\text{C} \cdot \text{mol}^{-1})</td>
<td>(6.2 \times 10^{-9})</td>
</tr>
<tr>
<td>molar gas constant</td>
<td>(R)</td>
<td>(8.314459(86))</td>
<td>(\text{J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1})</td>
<td>(5.7 \times 10^{-7})</td>
</tr>
<tr>
<td>Boltzmann constant (R/N_A)</td>
<td>(k)</td>
<td>(1.38064852(79) \times 10^{-3})</td>
<td>(\text{J} \cdot \text{K}^{-1})</td>
<td>(5.7 \times 10^{-7})</td>
</tr>
<tr>
<td>Stefan-Boltzmann constant ((\pi^2/60)k^4\hbar^2c^2)</td>
<td>(\sigma)</td>
<td>(5.670367(13) \times 10^{-8})</td>
<td>(\text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})</td>
<td>(2.3 \times 10^{-6})</td>
</tr>
</tbody>
</table>

Non-SI units accepted for use with the SI units:

- electron volt (eV) \(J\)
  - \(eV = 1.6021766208(19) \times 10^{-19} \text{J}\)
  - \(6.1 \times 10^{-9}\)

- unified atomic mass unit \(\text{u}\)
  - \(\text{u} = 1.660539044(80) \times 10^{-27} \text{kg}\)
  - \(1.2 \times 10^{-8}\)
The New International System of Units based on Fundamental Constants

\[ \alpha = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{\hbar c} \]
Search for the variation of the fine-structure constant $\alpha$

$$\alpha = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{\hbar c}$$

$\alpha \sim 1/137$
Life needs very specific fundamental constants!

If $\alpha$ is too big $\rightarrow$ small nuclei can not exist
Electric repulsion of the protons $>$ strong nuclear binding force

$\alpha \sim 1/137$ \hspace{1cm} $\alpha \sim 1/10$

$\alpha$ will blow carbon apart

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Life needs very specific fundamental constants!

Nuclear reaction in stars are particularly sensitive to $\alpha$.
If $\alpha$ were different by 4%: **no carbon produced by stars. No life.**
Life needs very specific fundamental constants!

No carbon produced by stars: No life in the Universe

\[ \alpha \sim \frac{1}{132} \]
How to test if $\alpha$ changed with time?

Atomic transition energies depend on $\alpha^2$

Simulated spectra show how changing $\alpha$ affects the absorption of near-ultraviolet light by various atomic species. The horizontal black lines represent absorbed wavelengths. Each type of atom or ion has a unique pattern of lines.
Quasar

**Quasar:** "quasi-stellar radio source":

Quasar: extremely bright source, luminosity can be 100 times greater than that of the Milky Way.

Compact region in the center of a massive galaxy surrounding a central supermassive (hundreds of thousands to billions of solar masses) black hole.


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**Looking for Changes in Quasar Light**

A distant gas cloud, known by a quasar, gives astronomers an opportunity to probe the process of light absorption—and therefore the value of the fine-structure constant—earlier in cosmic history.

1. Light from a quasar begins its journey to Earth billions of years ago, with a smooth spectrum.
2. On its way, the light passes through one or more gas clouds. The gas blocks specific wavelengths, creating a series of black lines in the spectrum. For studies of the fine-structure constant, astronomers focus on absorption by metals.
3. By the time the light arrives on Earth, the wavelengths of the lines have been shifted by cosmic expansion. The amount of shift indicates the distance of the cloud and, hence, its age.
4. The spacing of the spectral lines can be compared with values measured in the laboratory. A discrepancy suggests that the fine-structure constant used to have a different value.

Quasar spectrum, taken at the European Southern Observatory’s Very Large Telescope, shows absorption lines produced by gas clouds between the quasar (arrow point at right) and us. The position of the lines (arrow points at far right) indicates that the light passed through the clouds about 75 billion years ago.

Astrophysical searches for variation of fine-structure constant $\alpha$

Observed from quasar absorption spectra

Laboratory frequency

$$E_Z = E_0 + q \left( \frac{\alpha_Z}{\alpha_0} \right)^2 - 1$$

Conflicting results

$\Delta \alpha / \alpha = -0.64(36) \times 10^{-5}$
Murphy et al., 2007
Keck telescope, 143 systems, 23 lines, 0.2<z<4.2

$\Delta \alpha / \alpha = -0.06(0.06) \times 10^{-5}$
Srianand et al, 2004: VL telescope, 23 systems, 12 lines, Fe II, Mg I, Si II, Al II, 0.4<z<2.3

Molaro et al., 2007
$\Delta \alpha / \alpha = -0.12(1.8) \times 10^{-6}$
$Z=1.84$
$\Delta \alpha / \alpha = 5.7(2.7) \times 10^{-6}$
Indications of a Spatial Variation of the Fine Structure Constant

J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge

A Special Axis

Alpha changes the most along an apparently “special” axis through the universe—increasing at greater distances from Earth and decreasing in the opposite direction (shown as negative distances). Remote regions of our universe may have quite different values.

Sometimes It Changes, Sometimes Not

According to the authors’ theory, the fine-structure constant should have stayed constant during certain periods of cosmic history and increased during others. Future data may reveal that effect, but so far only a variation with location in the universe has tentatively been detected.
Can we look for variation of fundamental constants in a lab?

YES! YES! YES! YES!

NEED ULTRA-PRECISE ATOMIC CLOCKS

Laboratory searches for variation of fundamental constants

\[ \alpha = \frac{1}{4\pi e_0} \frac{e^2}{\hbar c} \]

Measure the ratio \( R \) of two clock frequencies

\[ R = \frac{\omega_1}{\omega_2} = A \times \left[ \alpha \right]^{\kappa_\alpha} \times \left[ \frac{m_e}{m_p} \right]^{\kappa_e} \times \left[ \frac{m_q}{\Lambda_{QCD}} \right]^{\kappa_q} \]

Ratio of mass of electron to mass of the proton

Ratio of mass of quark to quantum chromodynamics scale
Laboratory searches for variation of fundamental constants

\[ \alpha = \frac{1}{4\pi e_0^2 \hbar c} \]

Measure the ratio \( R \) of two clock frequencies

\[ R = \frac{\omega_1}{\omega_2} = A \times \left( \alpha \right)^{K_e} \times \left( \frac{m_e}{m_p} \right)^{K_{ep}} \times \left( \frac{m_q}{\Lambda_{QCD}} \right)^{K_{q}} \]

Measure the ratio \( R \) of two optical clock frequencies: sensitive only to \( \alpha \)-variation

\[ \omega = \omega_0 + q \left( \frac{\alpha^2}{\alpha_0^2} - 1 \right) \]

Calculate with good precision

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**Al+/Hg+ Comparison**

Frequency-comb locked to Hg+ measure beat with Al+

from Jim Bergquist’ talk

Science 319, 1808 (2008)
Constraints on temporal variations of $\alpha$ and $\mu$ from comparisons of atomic transition frequencies. Phys. Rev. Lett. 113, 210802 (2014)

**Th$^{3+}$ nuclear clock**

**Th nuclear clock:**
Nuclear isomer transition in 229 Thorium has been suggested as an etalon transition in a new type of optical frequency standard.
Other applications of atomic clocks

Hunting for topological dark matter with atomic clocks

A. Derevianko¹ and M. Pospelov²³

Instead of the usual assumption of a mostly homogenous distribution, dark matter might be clumped to form point-like monopoles, one-dimensional strings or two-dimensional sheets that are called domain walls.

Such topological dark matter may have formed when the early Universe cooled down after the Big Bang, similar to the domains formed in a ferromagnet below its Curie temperature.

If we assume that the size of the defects is comparable to the size of the Earth or smaller, and if they occur frequently enough so that the Earth will pass through one of them we can detect this with atomic clocks.
Topological dark matter may be detected by measuring changes in the synchronicity of a global network of atomic clocks, such as the Global Positioning System, as the Earth passes through the domain wall.
Atomic clocks for design of absolute gravimeters and gravity gradiometers for geophysical monitoring and research

Figure 1 | Concept of a dark-matter search using atomic clocks. By monitoring time discrepancies between two spatially separated clocks one could search for the passage of topological defects, such as the domain wall pictured here.

Clock can measure difference in height (gravitational redshift)

As predicted by relativity and the equivalence principle, if a gravitational potential difference exists between a source (one clock) and an observer (another clock, otherwise identical), the two clocks run at different rates.

On the surface of the Earth a clock that is higher by $\Delta h$ than another clock runs faster by

$$\frac{\delta f}{f_0} = \frac{g \Delta h}{c^2}$$

where $g$ is the local acceleration of gravity.

$\Delta h = 10$ cm, $\frac{\delta f}{f_0} \approx 10^{-17}$


Two time dilation effects:

1. Time passes faster at higher elevations—a curious aspect of Einstein's theories of relativity that previously has been measured by comparing clocks on the Earth's surface and a high-flying rocket.

Physicists at the National Institute of Standards and Technology (NIST) have measured this effect at a more down-to-earth scale of 33 centimeters, or about 1 foot. Current clocks can detect this effect to a few cm.

2. Time passes more slowly when you move faster—clocks can test it for speeds at 10m/s (36 km/h).

Cryogenic optical-lattice clocks will enable mapping Earth’s gravity via general relativity

Monitoring volcanoes with ground-based atomic clocks