Physics 313: Laboratory 7 - Fresnel Diffraction

Introduction: The wave nature of light has been clearly established. Given this fact, one must treat the propagation of light as a wave. When determining how waves interact, the principle of superposition indicates that waves must be added. These mathematical summations can be complex, but following a solid physical understanding of the problem, solutions are possible. The basic physics of diffraction is simple, whether using two slits or an arbitrary object. The underlying principle is the same: **ADD ALL THE WAVES TOGETHER.** For a few problems this can be handled analytically, e.g. problems with two narrow slits far from the screen are easy. Since the world is composed of more complicated problems, better approaches have to be formulated. One approach is to divide the aperture into Fresnel zones. Another approach is numerical calculations using high speed computers where summations can be used to solve complicated integrals with any desired precision. After finishing this lab you should:

- Understand how electromagnetic radiation (microwaves, light) diffracts
- Understand how Fresnel zone analysis and/or mathematical treatment can determine the physical result of diffraction
- Appreciate the power of computer calculations by theoretically solving a diffraction problem and testing the results

Method: Design and construct a Fresnel zone plate for use in the microwave wavelength. Observe near field focusing from the zone plate and measure its focal length. Measure the light diffraction pattern from a single slit and from a straight edge in the Fresnel, or near field, limit. For extra credit: write a computer program to determine the diffraction pattern from these two cases, and compare the two results. The following equipment is required to collect the data:

<table>
<thead>
<tr>
<th>optical table</th>
<th>HeNe laser</th>
<th>CCD camera</th>
<th>ruler</th>
</tr>
</thead>
<tbody>
<tr>
<td>lenses</td>
<td>razor blade on mount</td>
<td>single slit on mount</td>
<td>hand-held microscope</td>
</tr>
<tr>
<td>computer</td>
<td>Maple, mathematica, etc.</td>
<td>foil, paper, spray glue</td>
<td>microwave source, detector</td>
</tr>
</tbody>
</table>
Experiment 1: Microwave Fresnel Zone Plate

Each Fresnel zone is on average \( \sqrt{2} \) further from observation point P than the preceding zone. If we have plane waves reaching the aperture, the Fresnel zone radii are then defined by \( R^2 + r_0^2 = (r_0 + N\lambda/2)^2 \Rightarrow R = \sqrt{N r_0 \lambda} \). Figure 1 shows the geometry. Determine the first 10 radii if your observation point is 5 cm from the plate and the wavelength is that of microwaves of 10 GHz frequency.

At the observation point, the odd zones’ contribution will be 180 degrees out of phase with the even zone’s contribution. Counter-intuitively, you can increase the irradiance by blocking part of the path between the source and observation point.

Construct a zone plate for microwaves using foil on a cardboard backing. You can bond paper and foil using spray adhesive – lightly coat both sides, wait 30 seconds, then press together. Make a radius template using a CAD (computer assisted design) program, print it out and attach it. Use an X-Acto knife to remove the odd Fresnel zones.

Set up your microwave source away from reflecting metal objects. Leave the horn on and put the detector far away to approximate plane wave at the near aperture. Remove the detector’s horn to reduce its directionality. Adjust things so that you can barely detect microwaves, and place your zone plate 5 cm away. Do you observe a focusing of the microwave power at the predicted spot? In principle, a zone plate will have additional focal lengths closer in. Do you see these? Why or why not?
Experiment 2: Light Fresnel Diffraction Experimental Study

Set up an arrangement in the lab to observe light Fresnel diffraction from a slit. Use lenses to expand a 632 nm laser beam and form a line focus source. Cylindrical lenses work well for this purpose. Arrange a slit 10 cm after the line source. Use your CCD camera to record the illumination pattern, i.e. intensity, 100 cm from the slit (110 cm from the line source). Identify the locations of the minimum so that you can compare these to a calculation of the setup. Measure the slit width with a handheld microscope. Next, record the illumination pattern from an edge using a razor blade to provide the edge. An applet at www.geocities.com/piratord/Difra/difra.html calculates Fresnel diffraction patterns for slits and circular apertures (Figure 7). Are your observed patterns similar?

Application Problem 3: The Information Age

Record keeping and inventory is rapidly changing. Grocery stores and general merchandise stores can now keep detailed track of what product is selling at which price and from what supplier. Barcode scanners are a big part of this increased information push. The desired for more information has never been greater to increase efficiency and reduce overhead and storage costs. For example, grocery stores need to know not only the vendor and product but also the date of manufacture to make sure they are moving the correct stock and prevent spoilage. Shipping companies need to put the contents, weight, sender, send date, recipient, shipping cost, insurance, due date, etc.
Barcode scanners vary in complexity but a large number use a laser beam that reflects off a rotating mirror to scan across the product. A photosensitive detector measures the reflected light. When a valid sequence of data is measured (010010001) as the laser sweeps across the container, then the computer registers the product. Usually the information on the barcode is very simple, a vendor and its product number plus a termination sequence to indicate a valid data sequence measurement. A new UPC barcode is being proposed and as part of the development, you need to determine the maximum data density for a standard one-dimensional bar code. The barcode must be read within a distance of 10 cm to 50 cm from the code. What is the minimum spacing and width of the bars that can be used?

Figure 5: Sample barcode.
Experiment 4: Modeling Fresnel Diffraction on the computer

You will receive extra credit for doing this section. The setup of the analysis can be done in groups, but the computer implementation must be done independently and included with the report for full credit. Please document your program so it can be easily understood.

Use the computer to model the diffraction of light for the data you just took. Begin by working on the ‘simple’ problem of light incident upon an infinitely narrow slit. Then, move to modeling the diffraction from a slit with a finite width. Finally, modify the program to determine diffraction from an edge.

For the edge diffraction problem, place a line source, 632 nm, 10 cm behind a slit. Place a viewing screen 100 cm from the edge, 110 cm from the line source. Determine the illumination pattern, i.e. intensity, on the screen from 1 cm below the edge to 3 cm above the edge. Define 0 cm as the plane including the point source and the end of the blocking edge. See Fig. 4 for an example of the setup.

What is the physics you need to know to solve the problem? After the few simple physics concepts comes the application to this problem. What is the wave source here? How can the light at the slit be represented? How can the light at the screen be determined? Break the problem down into its physical steps as the wave diffraction occurs.
After your application of the physics to the general problem comes the detailed analytical application part of the problem solving. You must now find a mathematical representation of the original wave, the wave at the slit, and the wave at the screen in a coordinate system that will allow you to solve the problem easily. Do NOT try to solve the whole problem on the computer right away. Computers are only good for a few things – figuring out complex problems is not one of them and that is why you are taking this course.

First clearly solve as much of the problem analytically and simplify the problem. Then once you are sure you know what you want the computer to do, program it for a simple version of the task. For example, try computing the intensity for a few points on the screen to see if the calculation makes sense to you before calculating one hundred points. Also, do not get too worried about the user interface. KEEP THE PROGRAM SIMPLE! For example, if you can calculate $k$, do it rather than have the program do it hundreds of times.

Do not forget to add the obliquity factor to the elements $dE$. Note that to evaluate the intensity for one point on the screen, a full summation must be done of all the elements in the original source. Evaluate the summation for all $dE$ where the amplitude significantly contributes to the intensity at the point on the screen. At some point, $dE$ will be small enough that it need not be considered and you need not include it in the summation. Since your source is a line, you only need one spatial degree of freedom in your program.
In the end you need the intensity as a function of position rather than the electric field as a function of position and time. How can you convert your numerically summed electric field into an intensity? Again, it is best to do this on paper rather than complicating things by including the computer. Some complex math you may find helpful in your derivation is shown in Table 1.

In your code, confirm the accuracy in your answer by increasing the precision of your calculation, e.g., by decreasing the “step-size” in your integral, until the answer does not change by more than a few percent. This procedure confirms that your calculation has converged, which means that the numerical approximations are not limiting the physical answer. Plot the results normalized to an intensity that would be at the screen without the blocking edge. Determine the results to an accuracy of 10% in the intensity.

\[ <(A+iB)^* \exp(i\omega t) \exp(i\theta) > = \frac{A^2 + B^2}{2} \]

since \[ <A(t)> = \int A(t) \, dt / \int dt \]
and
\[ \int (A+iB)^* \exp(i\omega t) \, dt = \]
\[ = \int (\text{Real} \, |z| \exp(i\theta) \exp(i\omega t)) \, dt \]
use \[ |z| = \sqrt{A^2 + B^2} \] and \[ \theta = \text{arctan}(B/A) \]
\[ = \int (\text{Real} \, |z| \exp(i\omega t + i\theta)) \, dt \]
\[ = |z|^2 \int (\cos(\omega t + \theta))^2 \, dt \]
use \[ \tau = t + \theta/\omega \] and \[ d\tau = dt \]
\[ = |z|^2 \int (\cos(\omega \tau))^2 \, d\tau \]
therefore:
\[ <(A+iB)^* \exp(i\omega t) \exp(i\theta) > = |z|^2 \int (\cos(\omega \tau))^2 \, d\tau / \int d\tau \]
\[ = \frac{|z|^2}{2} \]
\[ = \frac{A^2 + B^2}{2} \]

Table 1: Derivation showing how to determine the time averaged real part of a complex function.

Figure 7: Fresnel diffraction applet available at www.geocities.com/piratord/Difra/difra.html