A RIGID BODY IN STATIC EQUILIBRIUM

TORQUE DEFINED

\[ \tau = r \times F \] (See your textbook.)

STATIC EQUILIBRIUM

Many structures in our everyday lives such as ladders, hanging signs, seesaws, buildings, bridges, etc. are rigid frameworks which have a variety of forces acting on them. The fact that these extended bodies remain stationary, both translational and rotational, tells us something about the total of the forces acting. The first condition of Newton's First Law, \( \Sigma F = 0 \), informs us that if an object is in translational equilibrium, the sum of the forces acting on it is zero.

There is a similar role for rotation, called the second condition of Newton's Law in rotational form. It says, first, if an object is in rotational equilibrium — that is, it is not changing its angular velocity — then it must be true that the sum of the torques acting on the object is zero.

Each of the two laws stated is a vector equation and therefore is true for each direction in space chosen. Since we live in three dimensions, the equations therefore have three independent components for which the laws are true.

\[
\begin{align*}
\Sigma F_x &= 0 \\
\Sigma F_y &= 0 \\
\Sigma F_z &= 0 \\
\Sigma r_x &= 0 \\
\Sigma r_y &= 0 \\
\Sigma r_z &= 0
\end{align*}
\]

(1)

In our actual experiment, however, our bar is being pulled in only two independent directions, \( x \) and \( y \), and not in the third direction, \( z \), perpendicular to the length of the bar itself. So that bar in the figure below rotates only in the x-y plane, i.e., in the z direction. Hence, in our experiment, the above 6 component equations reduce to three: a single torque equation and two \((x \text{ and } y)\) components of the forces.

PROCEDURE

In this lab, the rigid body is a long metal bar. It has four attached hooks and a protractor attached to the bar at each end so that the angle can be read. If the bar is suspended from two spring scales, three forces are acting on the bar when it hangs at rest. (The third force is that of gravity acting down on its center of mass.) A string attached to the fourth hook and pulled over a pulley by known weights can even contribute a fourth force acting on the metal bar.

**Part 1.** Suspend the bar using the two spring scales. You may neglect the extra, fourth hook on the underside, if you wish. Before you suspend the bar, make sure that your two spring scales read zero when there is no force acting on them. When you have a nice equilibrium result (see sample sketches), take all the pertinent data: the magnitude of the forces acting on each
hook, the angle each of those forces makes with the bar, and the distance from the pivot point to each of the forces acting. The pivot point you choose is arbitrary. Your instructor might suggest one. If you are allowed to choose your own, a smart choice can eliminate one of the forces. For the force pointing down at the middle of the bar, you will need to find the weight of the bar. Either measure it on a balance, or use one of your spring balances to hang from by itself. If you are allowed to choose your own, a smart choice can eliminate one of the forces. Do the calculation in the next section, if you have time, before moving on to Part 2. If the calculation does not work, you might play with the suspended bar to see if you can discover a problem or see how much error is possible in varying things.

Part 2. Suspend the bar again in a new fashion. This time be sure to use all four forces. You might even hang weights on to the middle hook so that it has more than just the bar's weight acting there (but it is not required unless your instructor chooses). Again record all the data.

**CALCULATIONS**

In each part, we want to verify that the sum of the torques truly is zero according to all the numbers you have recorded. You can insert all the numbers into \( r_1 F_1 \sin \theta_1 \) + \( r_2 F_2 \sin \theta_2 \) + ... and see if both sides come to equal the same value. Or you can pick on of the force magnitudes or directions as an unknown and solve for it, to see if the calculation yields the result that you recorded for that value experimentally.

Further calculations should also work out: the sum of all the \( x \) components of the forces acting should be zero. The sum of all the \( y \) components of the forces should be zero. The choice of \( x \) and \( y \) directions are essentially arbitrary in that one might conveniently choose the length of the bar as the \( x \) direction and so the \( y \) direction is perpendicular to it. Notice that if the bar is not suspended perfectly horizontally, then these \( x \) and \( y \) directions are tilted from the horizontal to the earth's surface and vertical to the earth's surface that are often chosen in physics problems. It does not matter, does it?
QUESTIONS

Answer the above question. Did your torque equation numbers work out as expected? Did each of the force component equations work out? When the bar is stationary, does it matter what pivot point is chosen? If the total torque on an object is zero, does that necessarily mean that the object is stationary?